Engineering Statistics

STATISTICAL FORMULAE

1. For X having the Binomial distribution, $B(n, p)$: $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$, for $k = 0, 1, 2, \ldots, n$ and X has mean np and variance $np(1-p)$.

2. For Y having the Poisson distribution, parameter λ: $P(Y = k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$, for $k = 0, 1, 2, \ldots$ and Y has mean λ and variance λ.

3. For the sample $x_1, x_2, \ldots, x_n$, the sample mean is $\bar{x} = \sum_{i=1}^{n} x_i / n$ and the sample variance is $s^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2 / (n-1) = \left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right) / (n-1)$.

4. Observations $x_1, x_2, \ldots, x_m$ occur with frequencies $f_1, f_2, \ldots, f_m$, the sample mean is $\bar{x} = \dfrac{\sum_{i=1}^{m} f_i x_i}{n}$ and the sample variance is $s^2 = \dfrac{\sum_{i=1}^{m} f_i (x_i - \bar{x})^2}{n-1} = \dfrac{\sum_{i=1}^{m} f_i x_i^2 - n\bar{x}^2}{n-1}$.

5. Random sample $X_1, X_2, \ldots, X_n$ from a $N(\mu, \sigma^2)$ distribution, then $\dfrac{\bar{X} - \mu}{\sqrt{\sigma^2 / n}}$ has a $N(0, 1)$ distribution and $\dfrac{\bar{X} - \mu}{\sqrt{s^2 / n}}$ has a t_{n-1} distribution.

6. $B(n, p)$ is approximated by $N(np, np(1-p))$, when n is large and np is not too close to 0 or n. Poisson, λ, is approximated by $N(\lambda, \lambda)$, when λ is large.

7. The linear regression line is estimated by $y = \hat{a} + \hat{b}x$ where $\hat{b} = S_{xy} / S_{xx}$,

$$\hat{a} = \bar{y} - \hat{b}\bar{x}, \quad \hat{\sigma}^2 = \frac{S_{yy} - \hat{b}S_{xy}}{n-2}, \quad S_{xx} = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}, \quad S_{yy} = \sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}$$ and

$$S_{xy} = \sum x_i y_i - \frac{\sum x_i \sum y_i}{n}.$$

$\dfrac{\hat{b} - b}{\sqrt{\hat{\sigma}^2 / S_{xx}}}$ has a t_{n-2} distribution. The mean value of y at x_0, $a + bx_0$, has confidence interval $\hat{a} + \hat{b}x_0 \pm t_{n-2}\sqrt{\hat{\sigma}^2\left(\dfrac{1}{n} + \dfrac{(x_0 - \bar{x})^2}{S_{xx}}\right)}$.

A confidence interval for a single response at x_0 is

$$\hat{a} + \hat{b}x_0 \pm t_{n-2}\sqrt{\hat{\sigma}^2\left(1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right)}.$$

The (Pearson product-moment) sample correlation is $r = \dfrac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$.