Statistics for Engineers

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http://cosmologist.info/teaching/STAT/

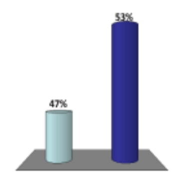
Student response systems

- Not assessed
- Not individually monitored
- Not identified

Sample question

Have you previously done any statistics?

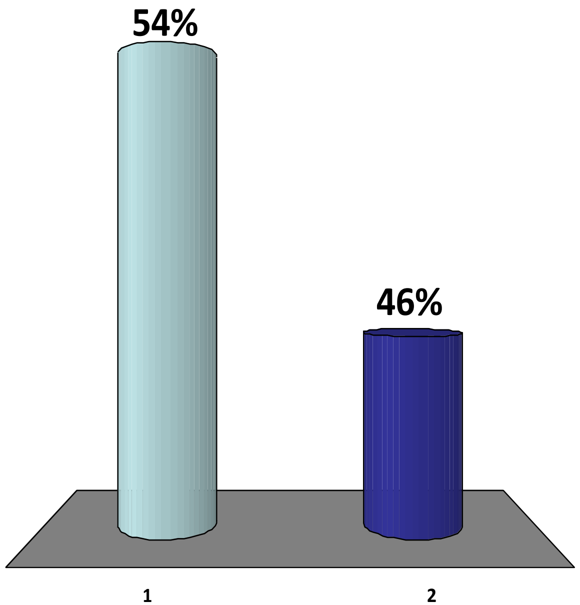
1. Yes
2. No



Starter question

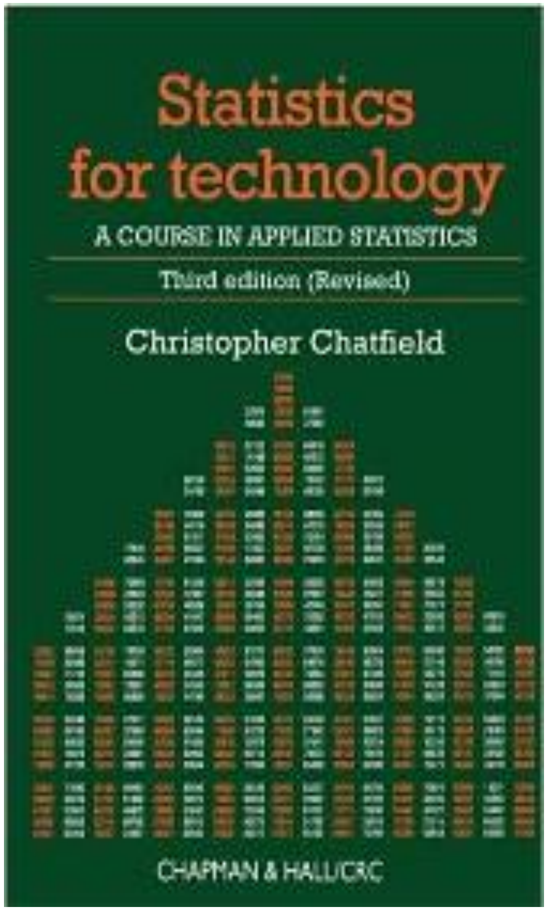
Have you previously done any statistics?

1. Yes
2. No

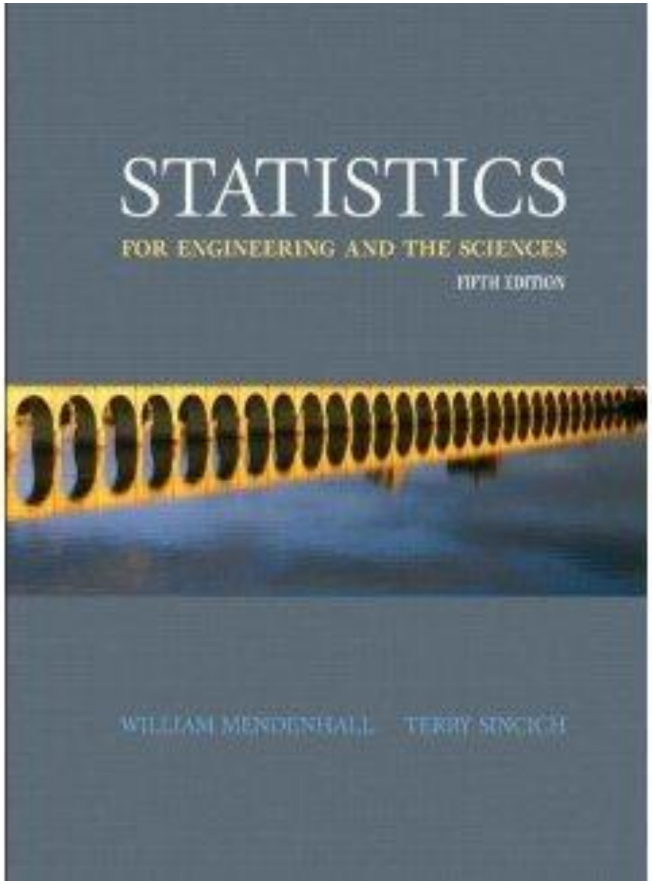


BOOKS

Chatfield C, 1989. *Statistics for Technology*, Chapman & Hall, 3rd ed.

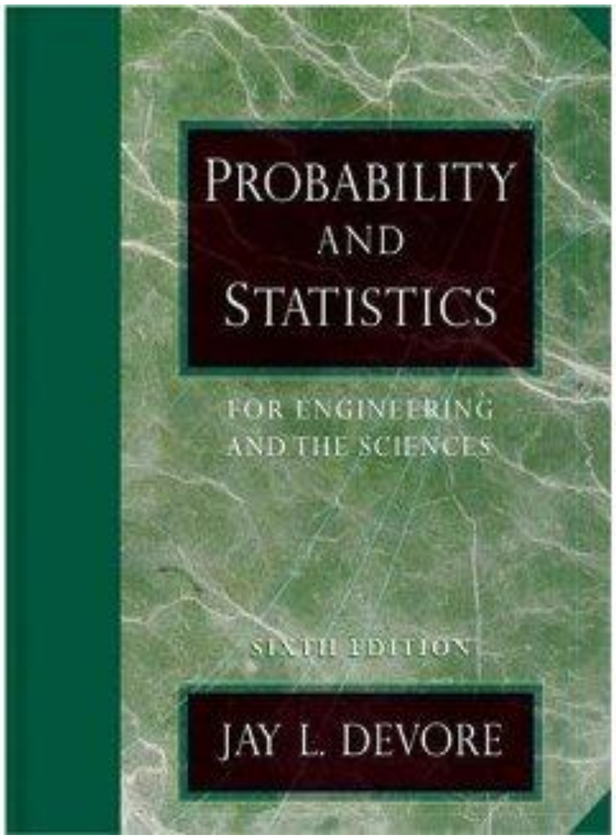


Mendenhall W and Sincich T, 1995. *Statistics for Engineering and the Sciences*



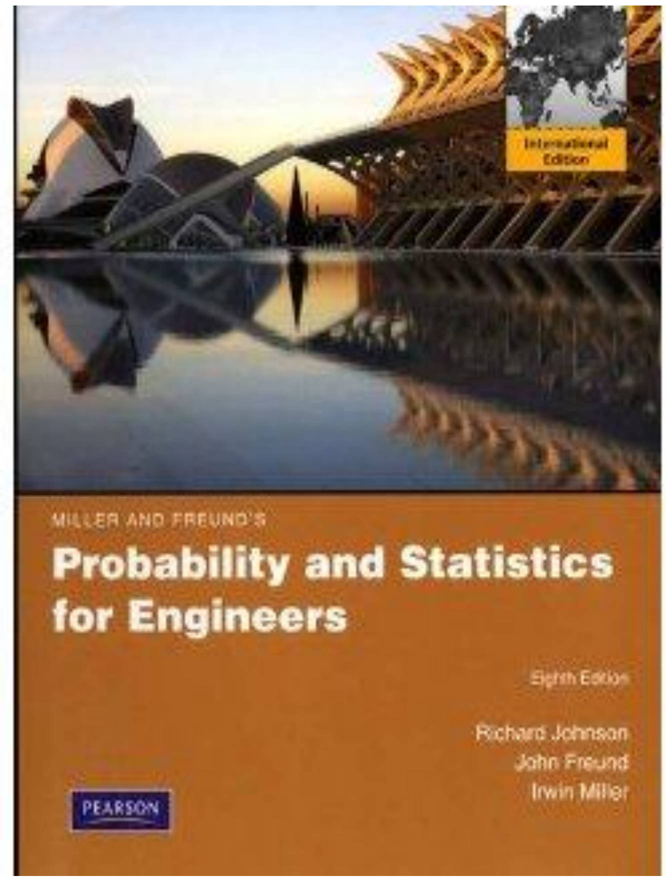
Books

Devore J L, 2004.
Probability and Statistics for Engineering and the Sciences,
Thomson, 6th ed.



Richard A. Johnson

Miller and Freund's Probability and Statistics for Engineers



Wikipedia also has good articles on many topics covered in the course.

Workshops

- Doing questions for yourself is very important to learn the material

- Hand in questions at the workshop, or ask your tutor when they want it for next week (hand in at the maths school office in Pevensey II).

- Marks do not count, but good way to get feedback

Probability

Event: a possible outcome or set of possible outcomes of an experiment or observation. Typically denoted by a capital letter: *A*, *B* etc.

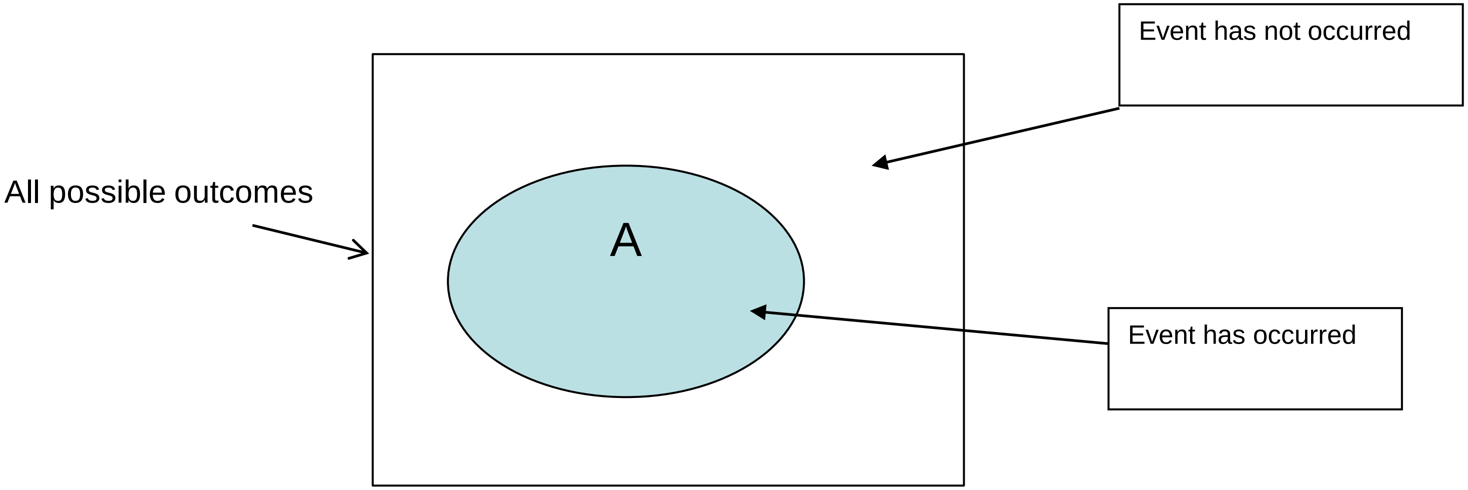
E.g. The result of a coin toss

Probability of an event *A*: denoted by P(*A*).

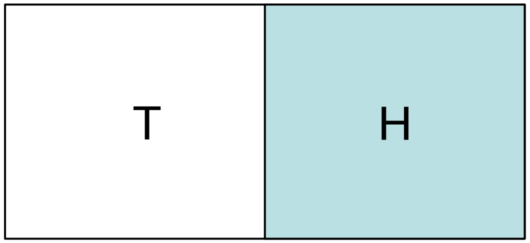
Measured on a scale between 0 and 1 inclusive. If A is impossible P(*A*) = 0, if A is certain then P(A)=1.

E.g. P(result of a coin toss is heads)

If there a fixed number of equally likely outcomes $P(A)$ is the fraction of the outcomes that are in *A*.



E.g. for a coin toss there are two possible outcomes, Heads or Tails



P(*result of a coin toss is heads*) = 1/2.

Intuitive idea: P(*A*) is the typical fraction of times *A* would occur if an experiment were repeated very many times.

Probability of a statement S:
P(S) denotes degree of belief that S is true.

E.g. P(tomorrow it will rain).

Conditional probability: P(A|B) means the probability of A given that B has happened or is true.

e.g. P(result of coin toss is heads | the coin is fair) =1/2

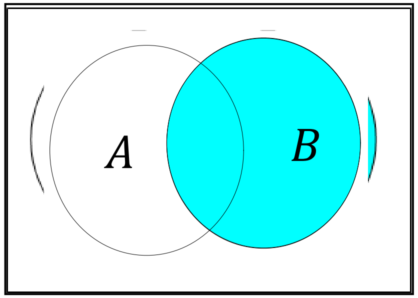
P(Tomorrow is Tuesday | it is Monday) = 1

P(card is a heart | it is a red suit) = 1/2

Conditional Probability

In terms of P(B) and P(A and B) we have

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



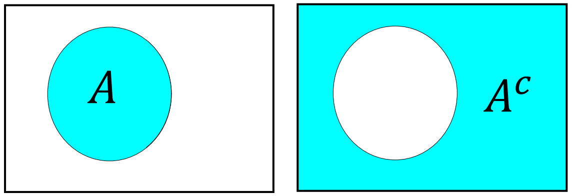
$P(B)$ gives the probability of an event in the B set. Given that the event is in B, $P(A|B)$ is the probability of also being in A. It is the fraction of the B outcomes that are also in A

Probabilities are always conditional on something, for example prior knowledge, but often this is left implicit when it is irrelevant or assumed to be obvious from the context.

Rules of probability

1. Complement Rule

Denote "all events that are not A" as A^c.



Since either A or not A must happen, $P(A) + P(A^c) = 1$.

Hence

P(Event happens) = 1 - P(Event doesn't happen)

so

$$P(A) = 1 - P(A^c)$$
$$P(A^c) = 1 - P(A)$$

E.g. when throwing a fair dice, P(not 6) = 1-P(6) = 1 – 1/6 = 5/6.

2. Multiplication Rule

We can re-arrange the definition of the conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A|B)P(B)$$

or

$$P(A \cap B) = P(B|A)P(A)$$

You can often think of $P(A \text{ and } B)$ as being the probability of first getting A with probability $P(A)$, and then getting B with probability $P(B|A)$.

This is the same as first getting B with probability $P(B)$ and then getting A with probability $P(A|B)$.

Example:

A batch of 5 computers has 2 faulty computers. If the computers are chosen at random (without replacement), what is the probability that the first two inspected are both faulty?

Answer:

P(first computer faulty AND second computer faulty)

Use $P(A \cap B) = P(A)P(B|A)$

= P(first computer faulty) × P(second computer faulty | first computer faulty)

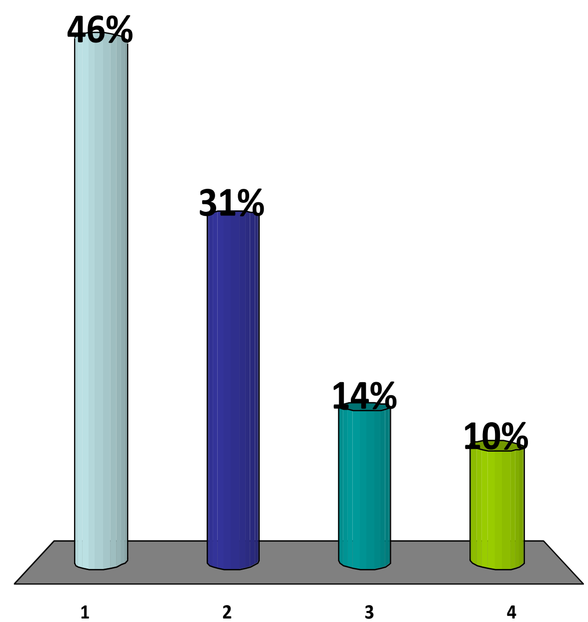
$$\underbrace{\text{P(first computer faulty)}}_{\frac{2}{5}} \times \underbrace{\text{P(second computer faulty | first computer faulty)}}_{\frac{1}{4}}$$

$$= \frac{2}{5} \times \frac{1}{4} = \frac{2}{20} = \frac{1}{10}$$

Drawing cards

Drawing two random cards from a pack without replacement, what is the probability of getting two hearts?
[13 of the 52 cards in a pack are hearts]

1. 1/16
2. 3/51
3. 3/52
4. 1/4



Drawing cards

Drawing two random cards from a pack without replacement, what is the probability of getting two hearts?

To start with 13/52 of the cards are hearts.

After one is drawn, only 12/51 of the remaining cards are hearts.

So the probability of two hearts is

$$P(\text{first is a heart AND second is a heart}) = P(\text{first is a heart}) \times P(\text{second is a heart} \mid \text{first is a heart})$$

$$= \frac{13}{52} \times \frac{12}{51} = \frac{1}{4} \times \frac{12}{51} = \frac{3}{51}$$

Special Multiplication Rule

If two events *A* and *B* are *independent* then P(A| B) = P(A) and P(B| A) = P(B): knowing that *A* has occurred does not affect the probability that *B* has occurred and vice versa.

In that case

$$\text{P}(A \text{ and } B) = P(A \cap B) = P(A)P(B|A) = P(A)P(B)$$

Probabilities for any number of independent events can be multiplied to get the joint probability.

E.g. *A fair coin is tossed twice, what is the chance of getting a head and then a tail?*

$$\text{P(H1 and T2)} = \text{P(H1)P(T2)} = \tfrac{1}{2} \times \tfrac{1}{2} = \tfrac{1}{4}.$$

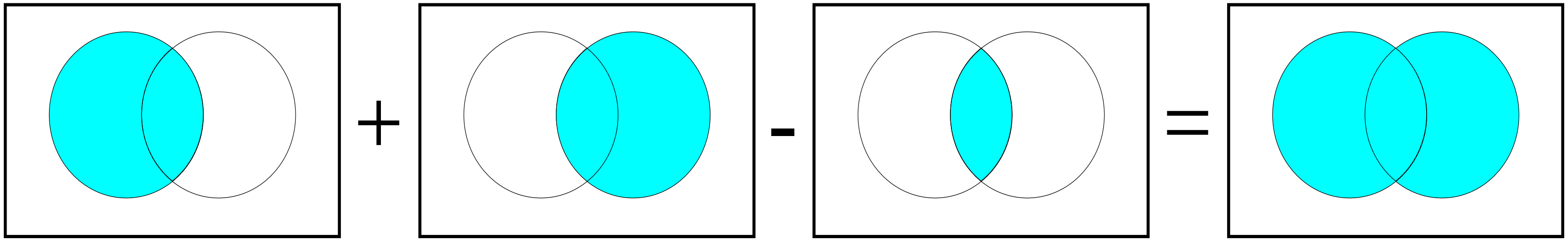
E.g. *Items on a production line have 1/6 probability of being faulty. If you select three items one after another, what is the probability you have to pick three items to find the first faulty one?*

$$P(\text{1st OK})P(\text{2nd OK})P(\text{3rd faulty}) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216} = 0.116..$$

3. Addition Rule

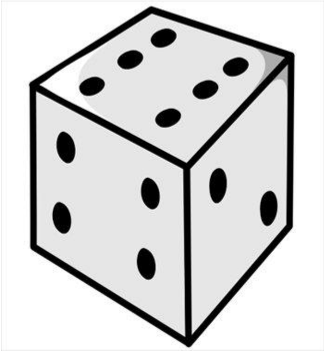
For any two events A and B,

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



Note: "*A* or *B*" = $A \cup B$ includes the possibility that both *A* and *B* occur.

Throw of a die



Throwing a fair dice, let events be

A = get an odd number
B = get a 5 or 6

What is *P(A* or *B)?*

1. 1/6
2. 1/3
3. 1/2
4. 2/3
5. 5/6

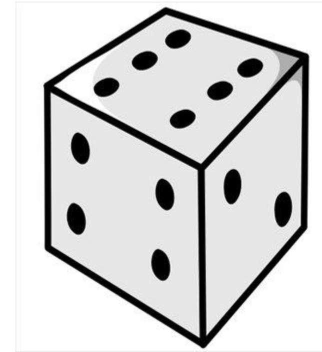
Q

Throw of a die

Throwing a fair dice, let events be

A = get an odd number
B = get a 5 or 6

What is *P(A* or *B)?*



A

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

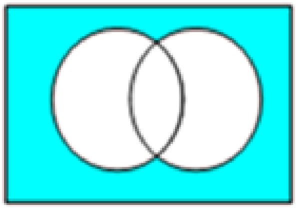
$$= P(\text{odd}) + P(5 \text{ or } 6) - P(5)$$

$$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

This is consistent since $P(A \cup B) = P(\{1,3,5,6\}) = \frac{4}{6} = \frac{2}{3}$

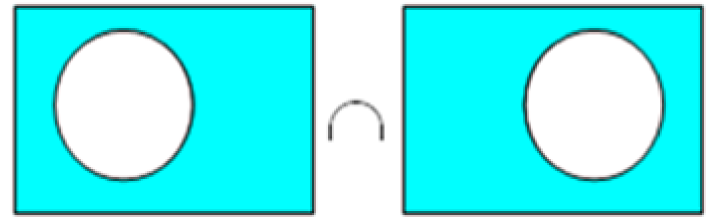
Alternative

"Probability of not getting either A or B = probability of not getting A and not getting B"



$(A \cup B)^c$

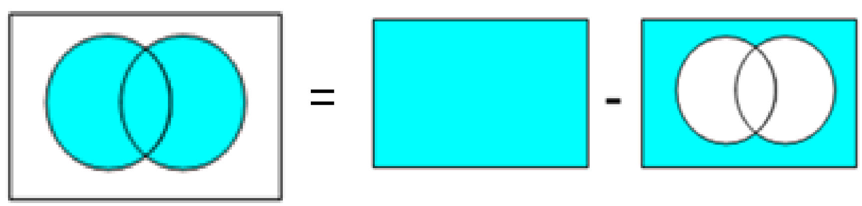
=



A^c B^c

$$(A \cup B)^c = \mathrm{A}^c \cap B^c$$

Complements Rule $\Rightarrow P(A \cup B) = 1 - P(A^c \cap B^c)$



i.e. P(A or B) = 1 – P("not A" and "not B")

Throw of a dice

Throwing a fair dice, let events be

A = get an odd number
B = get a 5 or 6

What is *P(A* or *B)?*

Alternative answer

A^c={2,4,6}, B^c = {1,2,3,4} so $A^c \cap B^c$ ={2,4}.

Hence

$$P(A \text{ or } B) = 1 - P(A^c \cap B^c)$$

$$= 1 - P(\{2,4\})$$

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

Lots of possibilities

This alternative form has the advantage of generalizing easily to lots of possible events:

$$P(A_1 \text{ or } A_2 \text{ or } \dots \text{ or } A_k) = 1 - P(A_1^c \cap A_2^c \cap \cdots \cap A_k^c)$$

Remember: for independent events, $\mathrm{P}(A \cap B \cap C \dots) = P(A) \times P(B) \times P(C)$.

Example: There are three alternative routes A, B, or C to work, each with some probability of being blocked. What is the probability I can get to work?

The probability of me not being able to get to work is the probability of all three being blocked. So the probability of me being able to get to work is

P(A clear or B clear or *C clear) = 1 – P(A blocked* and *B blocked* and *C blocked).*

e.g. if $P(A\ blocked) = \frac{1}{10}, P(B\ blocked) = \frac{3}{5}, P(C\ blocked) = \frac{5}{9}$

then
P(can get to work) = P(A clear or B clear or C clear)

= *1 – P(A blocked* and *B blocked* and *C blocked*

$$= 1 - \frac{1}{10} \times \frac{3}{5} \times \frac{5}{9} \quad = 1 - \frac{1}{30} = \frac{29}{30}$$

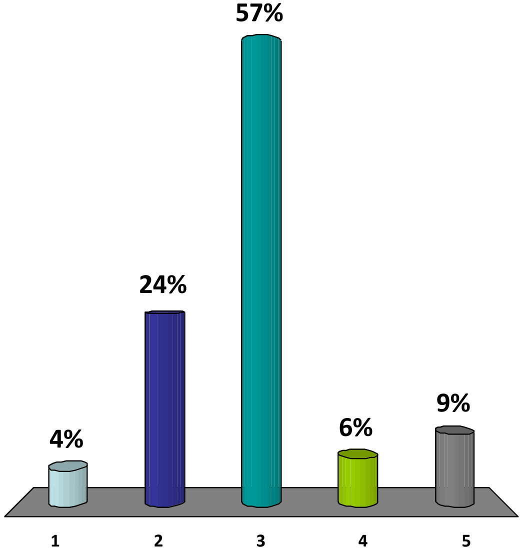
Problems with a device

There are three common ways for a system to experience problems, with independent probabilities over a year

A = overheats, P(A)=1/3
B = subcomponent malfunctions, P(B) = 1/3
C = damaged by operator, P(C) = 1/10

What is the probability that the system has one or more of these problems during the year?

1. 1/3
2. 2/5
3. 3/5
4. 3/4
5. 5/6



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A = overheats, P(A)=1/3
B = subcomponent malfunctions, P(B) = 1/3
C = damaged by operator, P(C) = 1/10

What is the probability that the system has one or more of these problems during the year?

$$P(\text{has a problem}) = P(A \cup B \cup C) = 1 - P(A^c \cap B^c \cap C^c)$$

$$= 1 - \frac{2}{3} \times \frac{2}{3} \times \frac{9}{10}$$

$$= 1 - \frac{4}{10} = \frac{3}{5}$$

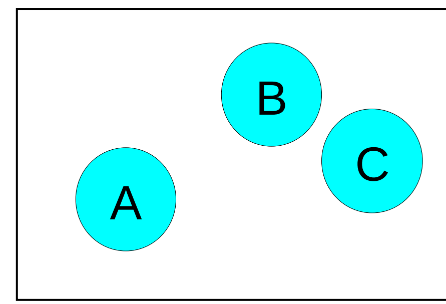
Special Addition Rule

If $P(A \cap B) = 0$, the events are *mutually exclusive*, so

$$P(A \text{ or } B) = P(A \cap B) = P(A) + P(B)$$

In general if several events $A_1, A_2, \dots, A_k$, are mutually exclusive (i.e. at most one of them can happen in a single experiment) then

$$P(A_1 \text{ or } A_2 \text{ or } \dots \text{ or } A_k) = P(A_1 \cup A_2 \cup \cdots \cup A_k)$$
$$= P(A_1) + P(A_2) + \cdots + P(A_k) = \sum_k P(A_k)$$



E.g. Throwing a fair dice,

P(getting 4,5 or 6) = P(4)+P(5)+P(6) = 1/6+1/6+1/6=1/2

Rules of probability recap

- Complements Rule: $P(A^c) = 1 - P(A)$

  Q. *What is the probability that a random card is not the ace of spades?*
  A. *1-P(ace of spades) = 1-1/52 = 51/52*

- Multiplication Rule:
  $P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$

  Q *What is the probability that two cards taken (without replacement) are both Aces?*
  A $P(\text{first ace})P(\text{second ace}|\text{first ace}) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$

- Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

  Q *What is the probability of a random card being a diamond or an ace?*
  A $P(\text{diamond}) + P(\text{ace}) - P(\text{diamond and ace}) = \frac{1}{4} + \frac{1}{13} - \frac{1}{52} = \frac{4}{13}$