

Early Universe : Unassessed Problem Sheet 3

1 Gravitational waves [18]

Consider harmonic expansion of the gravitational wave distribution with H_{\pm} of either helicity.

- (i) Starting from the field equation in the notes, show that $u \equiv aH_{\pm}$, satisfies the equation

$$u'' + \left(k^2 - \frac{a''}{a} \right) u = 0.$$

- (ii) Show that for $k \gg \mathcal{H}$ (specifically $a''/a \ll k^2$) that the amplitude of the sub-horizon tensor perturbations decays as $1/a$.
- (iii) Find the solution for H_{\pm} in radiation domination [now for any k/\mathcal{H}], with $H_{\pm} = 1$ at $\eta = 0$ (i.e. the transfer function).
- (iv) Assume the spectrum of primordial gravitational waves is scale-invariant, so at $\eta = 0$ we have $\mathcal{P}_T(k, 0) = A_T$.
- (a) Find the power spectrum at matter-radiation equality (in terms of the conformal time η_{eq}), approximating the universe as radiation dominated.
- (b) Sketch it as a function of $\ln k$, labelling any relevant scales.