Early Universe : Self-Assessed Warmup Problem

1 Age of the universe and time of recombination [15]

Consider a flat FRW model containing pressureless matter and a positive cosmological constant, with $\Omega_m + \Omega_\Lambda = 1$, $\Omega_\Lambda > 0$. Neglect any effects from the radiation energy density and pressure until part 5. Recall that if $a = 1$ today a solution for the scale factor is given by

$$a(t) = \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3} \left(\sinh\left[\frac{3}{2}\sqrt{\Omega_\Lambda} H_0 t\right]\right)^{2/3}.$$

1. Give a general result for the age of the universe today. What is the age of the universe (in billions of years) for $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$, $H_0 = 72\mathrm{km\,s^{-1}\,Mpc^{-1}}$? [2]

2. Given that the temperature of the CMB today is 2.725K, what was the scale factor when the photon temperature was 3000K at recombination? How old was the universe at recombination using the same parameters as Part 1? Given that we have neglected radiation, should this be an overestimate or underestimate? [3]

3. Show that $a(t) \approx Ct^{2/3}$ for $t \ll (\sqrt{\Omega_\Lambda} H_0)^{-1}$ and find the constant C. [2]

4. Using this approximation, what is the maximum comoving distance that radiation could have travelled from the big bang till recombination? Use the same parameters as Part 1 and give the result in Megaparsecs. [3]

5. At high redshifts ($z \gg 1$) the cosmological constant density should be negligible. Now accounting for radiation (but neglecting changes in g_* for $T > 1\mathrm{MeV}$) show that for $z \gg 1$

$$H_0 t = \frac{2}{3\Omega_m^2}\left(\sqrt{\Omega_m a + \Omega_r}(\Omega_m a - 2\Omega_r) + 2\Omega_r^{3/2}\right)$$

   is a solution to the Friedmann equation. Hence calculate a refined estimate for the age and conformal time at recombination using the same parameters as before. [5]