THE UNIVERSITY OF SUSSEX

MSc PHYSICS AND ASTRONOMY EXAMINATION MAY 2013

COSMOLOGY

Credit will given for the best **TWO** *answers only.*
Total time allowed: **ONE and a HALF** *hours.*

Each question carries 20 marks. The approximate allocation of marks is shown in brackets by the questions.

A list of **physical constants** *is provided.*

Turn over/

1. Consider that instead of a cosmological constant, dark energy might be due to an exotic fluid with an equation of state $P_{de} = -(5/6)\rho_{de}$ that does not interact with anything else (except via gravity). If a homogenous spatially-flat universe with scale factor a contains only this form of dark energy and pressureless matter with density ρ_m:

   (i) Use the energy conservation equation for an uninteracting fluid with density ρ and pressure P

   $$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0$$

   (in natural units, where an overdot denotes a time derivative) to show that the energy density of the dark energy fluid is given by

   $$\rho_{de}(a) = \frac{\rho_{de,0}}{\sqrt{a}}$$

   where $\rho_{de,0}$ is the energy density today (take $a_0 = 1$ today). [4]

   (ii) If $\Omega_{de,0} = 0.7$, what was ρ_{de}/ρ_m at redshift 3? [2]

   (iii) Give an equation for the Hubble parameter as a function of scale factor in terms of the Hubble parameter today H_0, and the ratios of the energy densities to the critical density today, $\Omega_{m,0}$ and $\Omega_{de,0}$. [1]

   (iv) Explain briefly what is meant by *luminosity distance* $d_L(z)$ and why it is useful to relate observations to theoretical models [3]

   (v) For a flat universe

   $$d_L(z) = (1+z)\chi(z)$$

   and $\chi(z)$ is the comoving radial distance travelled by light from a source at redshift z.

   A supernova of known intrinsic luminosity (and no peculiar velocity) is observed at redshift 2. If $\Omega_{de,0} = 3/4$ and H_0 is also measured accurately, what is the expected ratio of the observed flux to the flux that would be observed in a universe where the dark energy is a cosmological constant ($P_{de}/\rho_{de} = -1$)? You might find the following integrals useful:

   $$\int_{1/3}^{1} \frac{dx}{\sqrt{3x^{7/2} + x}} \approx 0.6068 \qquad \int_{1/3}^{1} \frac{dx}{\sqrt{3x^4 + x}} \approx 0.6330$$

   [8]

   (vi) Sketch $d_L(z)$ in the two possible cosmologies (one with $P_{de} = -(5/6)\rho_{de}$, one with $P_{de} = -\rho_{de}$, assuming $\Omega_{de,0}$ and H_0 are the same in both cases) [2]

2. (i) In the absence of Λ, the Friedmann equation the Friedmann equation is given by

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$$

where ρ is the density, H is the Hubble parameter and K is a constant. State what is meant by the *critical density* ρ_c, and use it to define the density parameter Ω. If the Hubble parameter today is $H_0 = 100h\text{km}s^{-1}\text{Mpc}^{-1}$ find the value of the critical density today (in Kilograms per cubic metre). [4]

(ii) Describe the particles that dominate the energy density of the early universe for temperature T is the range $100\text{MeV} \gg T \gg 0.5\text{MeV}$. Which of the particles are fermions, and how are the energy densities of the fermionic particles related? [3]

(iii) For most of the evolution of the universe the temperature of the photons scales with scale factor a as $T_\gamma \propto 1/a$. What happens around $T \sim 0.5\text{MeV}$ to change this? Why does the neutrino temperature behave differently? [3]

(iv) The effective degrees of freedom in entropy for particles in equilibrium at the same temperature is

$$g_{*S} = \sum_{i,\text{bosons}} g_i + \frac{7}{8} \sum_{i,\text{fermions}} g_i,$$

where g_i are the degeneracy factors. The entropy density is then given in terms of the photon temperature T_γ by $s = \frac{2\pi^2}{45} g_{*S} T_\gamma^3$.

Using conservation of entropy show that at $T \ll 0.5\text{MeV}$ the photon and neutrino temperatures are related by $T_\gamma = \left(\frac{11}{4}\right)^{1/3} T_\nu$. [5]

(v) The CMB temperature today is $T_{\gamma,0} \approx 2.726K$. Using the Stefan-Boltzmann relation for the photon energy density $\rho_\gamma = a_R T_\gamma^4$, and the effective degrees of freedom for the total radiation density $g_* = 3.36$, find $\Omega_{R,0}$, the total (photon + neutrino) radiation density parameter today. [3]

(vi) If $\Omega_m = 0.3$ and the Hubble parameter today is given by $h = 0.7$, find z_{eq}, the redshift at which the densities in matter and radiation were equal. [2]

[Throughout the question you can take the following if needed:
$a_R = 7.5657 \times 10^{-16} Jm^{-3}K^{-4}$, $1\text{Mpc} = 3.086 \times 10^{22} m$, $c = 3 \times 10^8 ms^{-1}$,
$G = 6.67 \times 10^{-11} m^3 kg^{-1} s^{-1}$, $m_e \approx 0.51\text{MeV}$].

3. (i) Briefly review the thermal condition and scattering processes that prevailed, and the changes that occurred in the Universe around the epoch of photon decoupling. [5]

(ii) Compare the formation of the cosmic microwave background with the formation of the light elements in terms of the following; the timescale after the Big Bang, the ionisation state of the universe, and the type of material dominating the energy density of the universe. [3]

For parts *iii* and *iv*, assume the universe has a flat geometry, zero cosmological constant, and is always dominated by non-relativistic matter, *i.e.* with scale factor given by $a(t) = \left(\frac{t}{t_0}\right)^{2/3}$.

(iii) If $H_0 = 67\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, show that the corresponding age of the universe is $t_0 \simeq 10^{10}$ years. Calculate the comoving distance that light could have travelled in the time between the Big Bang and the present day (express your answer in Mpc).
[1 year = 3.156×10^7 s, 1pc = 3.086×10^{16}m, c = 3.076×10^{-7} Mpc yr^{-1}] [4]

(iv) Write down an expression relating redshift and scale factor. If decoupling occurred at $z = 1000$, demonstrate that the maximum comoving distance that light could have travelled in the time between the Big Bang and the epoch of decoupling is approximately 292 Mpc. What angle would a line of this radius at recombination subtend when observed today? (You can assume the current distance to the surface of last scattering is given by your answer to part *iii*). [5]

(v) With reference to your answer to part *iv*, comment on how an early period of rapid expansion might solve the so-called Horizon problem. [3]

End of Paper