Cosmology: Problem Sheet 4

Deadline: 11 Dec (Week 13), Wednesday 12:00 (School Office hand in)

I. IN YOUR OWN WORDS [28]

A local radio or TV show has contacted you to ask you some questions about our universe. You should be specific, *concise* and clear in answering the questions, and try to explain things using as little scientific jargon as possible. Each part should be answered in less than a page, much less should usually be enough. Here are the questions:

1. Is our Universe infinite?
2. How do we know the universe is expanding?
3. What is inflation and do we need it?
4. What will happen in the future?

II. HORIZONS AND THE HORIZON PROBLEM [30]

Consider a photon (test particle) moving through a flat universe filled with a single fluid whose properties are described by an arbitrary constant equation of state, i.e. $w = P/\rho = \text{const}$. Assume $w \geq -1$, but carefully consider the distinct cases $w = -1$, $w < -1/3$, $w > -1/3$ separately where necessary. Give your results in terms of w and t_0 or H_0 where the answer is well defined.

1. If $a(t_0) = 1$ find $a(t)$ in this universe [5]
2. Find the comoving distance travelled by a photon between the big-bang (at $t = 0$) and a fixed time t_0. [5]
3. Find the maximum comoving distance that the photon can ever reach if it is emitted at t_0. [6]
4. Comment on the results of (2) and (3) in relation to the problems with the hot big bang model. How does the equation of state of the dark energy effect what we would be able to see in the distant future? [5]
5. Show that $P < -\rho/3$ implies that $\ddot{a} > 0$ and that

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0.$$

   Interpret this result in terms of the horizon problem. [9]

III. INFLATION [21]

Assume for simplicity that a period of inflation happened in the early universe with a nearly-constant Hubble parameter H (so $w = P/\rho \approx -1$), ending by sudden reheating to a hot big bang at temperature T_r with initial expansion rate $H_r = H$.

1. If $H = 10^{10}$GeV, roughly how many e-foldings of inflation are required for perturbations of comoving size 10Gpc to have been in causal contact at some point in the past? If inflation lasted just long enough to solve the horizon problem, how long was the period of inflation in seconds? [9]

2. The CMB anisotropies probe perturbations from very large scales ($\lambda \sim$ 10Gpc) down to about $\lambda \sim$ 10Mpc (distances are comoving). Assuming the perturbation properties are determined by the time during inflation when they left the horizon ($\lambda \sim (aH)^{-1}$), what range of e-foldings before the end of inflation are the CMB observations probing? [6]

3. In a previous question sheet we found that μ-distortions can develop in the CMB frequency spectrum by deposition of energy. If this energy is coming from damping of very small scale perturbations, down to size $\lambda \sim$ 1kpc, does observation of the μ-distortion signal potentially allow you to probe earlier or later in inflation, and by roughly how many e-foldings? [6]

IV. LEARNING FROM OBSERVATIONS [21]

In this question your answers should be a bit more technical than in the first question, and can take the form of brief notes. You should write your answers to be understood by a fellow MSc/MPhys student who is not taking the cosmology course.

1. How do supernovae observations give evidence for dark energy?

2. Explain what ‘acoustic oscillations’ are, and why the CMB power spectrum has ‘acoustic peaks’. What determines the observed angular scale of the first peak?

3. Discuss how one other observable (other than supernovae) could also help constrain dark energy.