Cosmology: Problem Sheet 3

Deadline: 27 Nov (Week 11), Wednesday 12:00
(School Office hand in; 24hr max lateness penalty; feedback the following Monday/Tuesday)

I. FREEZE OUT OF MUONS [20]

Muons μ^- are essentially identical to electrons, except that they are heavier ($m_\mu = 106MeV$); other than that, they also have the same charge and spin as the electron, and there is an antimuon μ^+ analogous to the positron.

1. What is the value of the effective g_* for muons when they are relativistic? [4]
2. When the temperature T is a little above 106MeV, what particles besides the muons are contained in the thermal radiation that fills the universe? What is the total effective g_*? [6]
3. As T falls below 106MeV, the muons disappear from the thermal equilibrium radiation. At these temperatures all of the other particles in the black-body radiation are interacting fast enough to maintain equilibrium, so the heat given off from the muons is shared among all the other particles. Letting a denote the FRW scale factor, by what factor does the quantity aT increase when the muons disappear? [In case you worry about it, ignore pions and QCD] [10]

II. NEUTRINO BACKGROUND [20]

1. Given the CMB temperature today is $2.726K$, what is the number density of neutrinos today? [7]
2. If two neutrinos are massless, but one has a mass of $0.05eV$ estimate Ω_ν, the contribution of neutrinos to the critical density today today for $h = 0.7$. [assume the massive neutrino is non-relativistic today, and neglect the energy density of the massless neutrinos] [6]
3. If instead there are three massive neutrinos with total mass $\sum m_\nu$ show that

$$\Omega_\nu h^2 = \frac{\sum m_\nu}{A}$$

and find the constant A (in eV mass units). [7]

III. THOMSON SCATTERING AND THE SAHA EQUATION [30]

The mean free path (typical distance between interactions) of photons through a plasma with electron number density n_e is given by $d \sim 1/(n_e \sigma_T)$ where σ_T is the Thomson scattering cross-section.

1. Estimate the photon mean free path when the scale factor was one millionth of its current value (the number density of electrons today is $\sim 0.2m^{-3}$). [6]
2. Calculate the typical time between photon interactions when $a = 10^{-6}$. How does it compare with the value for the Hubble time at this epoch? What is the significance of this comparison? [8]
3. In thermal equilibrium, at temperatures less than m_i, the number density is given by

$$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_i - m_i}{T}\right),$$

where $i = e, p, H$ (electrons, positrons and Hydrogen), m_i is the mass of species i and μ_i is the chemical potential of i. In chemical equilibrium, due to the process $p + e \to H + \gamma$, the chemical potentials satisfy: $\mu_p + \mu_e = \mu_H$. The binding energy for Hydrogen is given by $m_H = m_p + m_e - B$, and it is about $13.6eV$. [8]

   (a) Using this information (and noting the charge neutrality of the Universe), show that n_H can be expressed in terms of n_p and n_e as follows:

$$n_H = \frac{g_H}{g_p g_e} n_p n_e \left(\frac{m_e T}{2\pi}\right)^{-3/2} \exp\left(\frac{B}{T}\right)$$

   [Note that in the pre-exponential factor we have assumed $m_p \sim m_H$]

(b) The fractional ionization is defined as $X_e \equiv \frac{n_p}{n_B} = \frac{n_p}{n_p+n_H}$. Defining the baryon-to-photon ratio as $\eta \equiv \frac{n_B}{n_\gamma}$, obtain the Saha equation:

$$\frac{1-X_e}{X_e^2} = \frac{4\sqrt{2}\zeta(3)}{\sqrt{\pi}}\eta\left(\frac{T}{m_e}\right)^{3/2}\exp\left(\frac{B}{T}\right)$$

[8]

IV. NEUTRON-PROTON RATIO AND BBN [30]

1. When the temperature of the early universe was 5×10^{10}K, what was the ratio of neutrons to protons? You may assume thermal equilibrium, and that the mass difference is given by 1.293 MeV. [6]

2. The ratio of neutrons-to-protons freezes out of equilibrium at $T \sim 0.7$MeV. Calculate the neutron-to-proton rate at this temperature. [3]

3. However, we know that nucleosynthesis does not start until $T \sim 75$keV. Due to the neutron decay, the ratio of neutron-to-proton has kept decreasing. Estimate the neutron-to-proton ratio (the neutron life-time is 889 seconds) at $T \sim 75$keV. [7]

4. What would the neutron-to-proton ratio be at $T \sim 75$keV, if they were still in equilibrium? [2]

5. The calculations of big bang nucleosynthesis depend on a number of measured parameters. Below you are asked to qualitatively describe the effects of changing some of these parameters. Explain your answers.

   (a) Suppose an extra neutrino species is added to the calculation. Would the predicted helium abundance go up or down?

   (b) Suppose the weak interactions were stronger than they actually are, so that the thermal equilibrium distribution between neutrons and protons were maintained until $T \sim 0.25$ MeV. Would the predicted helium abundance be larger or smaller than in the standard model?

   (c) Suppose the proton-neutron mass difference were larger than the actual value of 1.29 MeV. Would the predicted helium abundance be larger or smaller than in the standard calculation? [12]