Cosmology: Problem Sheet 1

Approximate marks in [] Deadline: 16th Oct (Week 5), Wednesday 12:00 (School Office)

I. ANGULAR DIAMETER DISTANCE [20]

Consider a population of spherical galaxies of a constant fixed physical diameter D in a flat matter-dominated universe ($\Lambda = P = K = 0$, $\Omega_m = 1$).

1. Write down the relationship between the observed angular size θ of a galaxy, and the radial comoving distance χ and redshift z of the galaxy. By integrating to find χ show that

$$\theta = A\frac{(1+z)^{3/2}}{[\sqrt{1+z}-1]},$$

   where A should be given in terms of D and the Hubble parameter today H_0. [2,10]

2. Show that the observed angular size of a galaxy at first decreases with its distance, but then becomes larger again. What is the redshift at which a galaxy looks smallest? [8]

II. EVOLUTION OF A CLOSED MATTER-DOMINATED UNIVERSE [25]

1. Show that the first Friedmann equation for a pressureless matter-dominated universe takes the form

$$\dot{a}^2 + K = \frac{8\pi G\rho_m^0}{3a} \tag{1}$$

   where ρ_m^0 is the matter density today. [8]

2. Show that in a closed universe ($K > 0$), a solution to this equation is given by the following parametric solution

$$t = \alpha(\theta - \sin\theta) \tag{2}$$

$$a = \sqrt{K}\alpha(1-\cos\theta) \tag{3}$$

   in terms of the *development angle* θ, where[1] [10]

$$\alpha = \frac{1}{2}\frac{1}{H_0}\frac{\Omega_m}{(\Omega_m - 1)^{3/2}} \tag{4}$$

3. What is the maximum value of a? At what time will the evolution reach that value of a? What will happen after that? [4]

4. Find the Hubble parameter as a function of θ. What is the value of H at the point where a is maximum? [3]

III. CONTENT AND EXPANSION [30]

- Observational constraints on the baryon density are often expressed in terms of $\Omega_b h^2$. Why do you think this is better measured than Ω_b? [think about the relationship to the baryon density ρ_b]. [4]
- Assume that we know H_0 today and that the universe is spatially flat: would a larger cosmological constant make the inferred age of the universe shorter or longer? [6]

[1] Recall that without qualification Ω_m means the value of $\Omega_m(t)$ today $\Omega_m \equiv \Omega_{m,0} \equiv \Omega_m(t_0)$.

- Assume that a galaxy has a spherically symmetric distribution of dark matter which gravitationally dominates the baryons (so you can neglect the gravitational force of the galactic disk). Find the density profile $\rho(r)$ that would give a flat rotation curve. Would a cosmological constant with $\Omega_\Lambda = 0.7$ significantly change this? [8]
- If a flat universe contains only cosmological constant, give $a(t)$ in terms of H_0 taking $a = 1$ today. (a) Show that light received from the past could have come from an arbitrarily long comoving distance away. (b) If we emit some radiation today, what is the maximum comoving distance that it could ever travel in the future? (c) What is the recession velocity of a galaxy a distance $10H_0^{-1}$ away, and is this compatible with relativity? [12]

IV. DARK ENERGY AND LUMINOSITY DISTANCE [25]

It has been suggested that instead of a cosmological constant the dark energy might be due to an exotic fluid with an equation of state $P_{de} = -(2/3)\rho_{de}$ that does not interact with anything else (except via gravity). If a homogenous spatially-flat universe with scale factor a contains only this form of dark energy and pressureless matter with density ρ_m:

1. Write down the energy conservation equation for the dark energy fluid, and show that its energy density is proportional to $1/a$. If $\Omega_{de,0} = 0.7$, what was ρ_{de}/ρ_m at redshift 1? [8,4]
2. Give an equation for the Hubble parameter as a function of scale factor in terms of the Hubble parameter today H_0, and the ratios of the energy densities to the critical density today, $\Omega_{m,0}$ and $\Omega_{de,0}$. [3]
3. A supernova of known intrinsic luminosity (and no peculiar velocity) is observed at redshift 1. If $\Omega_{de,0} = 3/4$ and H_0 is also measured accurately, what is the expected ratio of the observed flux to the flux that would be observed in a universe where the dark energy is a cosmological constant ($p_{de}/\rho_{de} = -1$)? [10]

Results that you may find useful

$$\int_{1/2}^{1} \frac{dx}{\sqrt{3x^3 + x}} \approx 0.3652 \qquad \int_{1/2}^{1} \frac{dx}{\sqrt{3x^4 + x}} \approx 0.3979$$