Detecting CMB tensor modes

AML

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Abstract

Can we detect tensor modes with cosmic-variance limited CMB temperature and E polarization observations?

1 Introduction

The tensor temperature power spectrum is roughly flat up to l=40 or so (except for l=2, which can be significantly higher), and then falls off rapidly as shown in Fig. 1. In the same region the scalar power spectrum is also approximately flat, so the effect of some tensor modes is just to shift the amplitudes of the low l plateaux relative to those of the peaks. At the moment the situation is hopeless as we have no results covering large and small scales, however this will change with MAP and Planck. (Current results include a large uncertainty in the absolute amplitudes of the anisotropies).

Future results will be essentially cosmic-variance limited on these large scales, so we can neglect the noise. The effect of partial sky coverage is to reduce the effective number of modes by the fraction of sky observed - one can only hope for about 70% of the modes due to galactic contamination. This gives about $N = 0.7 \sum_{l=2}^{38} 2l + 1 \approx 1000$ independent modes.

The chi-squared statistic is given by $X = \sum_{lm} |a_{lm}|^2/C_{l,0}$ where $C_{l,0}$ is the power spectrum in the null-hypothesis that there are no tensors, and is a good statistic with the above assumptions. If the true power spectrum is $(1+r)C_{l,0}$ the distribution of X is $P_r(X) = \chi^2(X/(1+r))/(1+r)$, so $\langle X \rangle = (1+r)N$. In the null-hypothesis the variance is $2N$, so for the expected value to be more than one sigma away from the mean in the null-hypothesis we need $r > \sqrt{2/N} \approx 0.05$, i.e. where one has about a 50% chance of getting a detection at one sigma or better. More detailed numerical results are shown in the figures assuming perfect knowledge of a particular concordance cosmological model.

The plots have fudged the partial sky effect by using $f_{\text{sky}}(2l+1)$ modes at each l, though it would be quite straightforward to do it properly using the cut-sky harmonics. So with only temperature observations we'd have to be lucky to detect anything below $r = 0.1$ using just the temperature (in broad agreement with Ref. [1]). Note that we have assumed Gaussianity. The E polarization power spectrum is rather more constraining - we can lower this to around $r = 0.025$ - though in this case the assumption of cosmic variance limiting and perfect knowledge are rather more optimistic (the spectrum is much more sensitive to reionization, and polarization measurements are much more difficult). Even so, with the most optimistic assumptions, one could only in principle detect $A_T \approx 10^{-10}$ with temperature and E measurements. (I don't think the cross tells you much, though might be worth thinking about more carefully). When constraining the tensor modes from B measurements one should obviously do a full likelihood analysis include the E and T information. Planck constraints on tensors will probably come about equally from E and B information, future experiments will rely almost entirely on the non-cosmic-variance limited B modes.

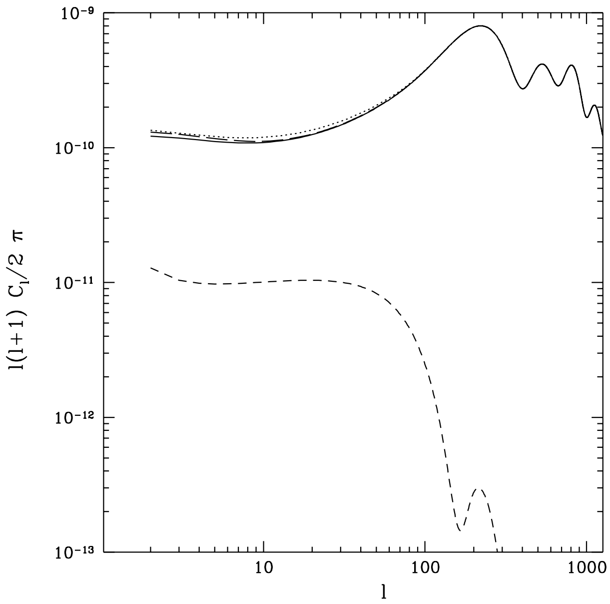

Figure 1: The scalar (red), tensor and total (blue) CMB power spectra for a typical model with $A_T = 4 \times 10^{-10}$. The black dashed line shows the equivalent scalar-only result if reionization is at $z = 10$ rather than $z = 6$. Spectra are normalized to be equal at high l.

There is the additional problem that there is a partial degeneracy with the reionization fraction. As shown in the figure this will give a similar increase[1] in the low l power. However for reionization at $z < 10$ this really is only on the largest scales - the argument above can be applied for modes in the range $10 < l < 40$, which is only a small loss of modes. The outlook is also very promising to constrain the reionization redshift[2] to good accuracy in the near future with other observation. Other ambiguities, such as different models of quintessence, also only generally effect the largest scales, which in any case have very large cosmic variance, leaving the same window in which to attempt to detect tensor modes over cosmic variance. However uncertainities and degeneracies with other cosmological parameters will undoubtedly reduce the limits that can be placed on the tensor amplitude significantly.

All the temperature spectrum will ever tell us is whether or not there is an excess of large scale power relative to what we expect with no-tensors. It is quite possible for the extra power to have other reasons. For example, as shown in Fig. 4, if one increases the power in the initial power spectrum on large scales one can construct a model which is essentially exactly degenerate with the tensor model.

There is therefore very good motivation for having analysed the real smoking gun - magnetic polarization[2]

[1] High l modes decrease by a factor very closely equal to $e^{-2\tau}$, but we normalize to get the same amplitude at high l. So the low l increase is both due to the change in normalization, and to some complicated scattering at reionization.

[2] For the CMB we mean the redshift at which the ionization fraction is one half. The power spectra are quite insensitive to how fast reionization occurs - though this is currently the main uncertainty in fixing the reionization fraction=half redshift from observations.

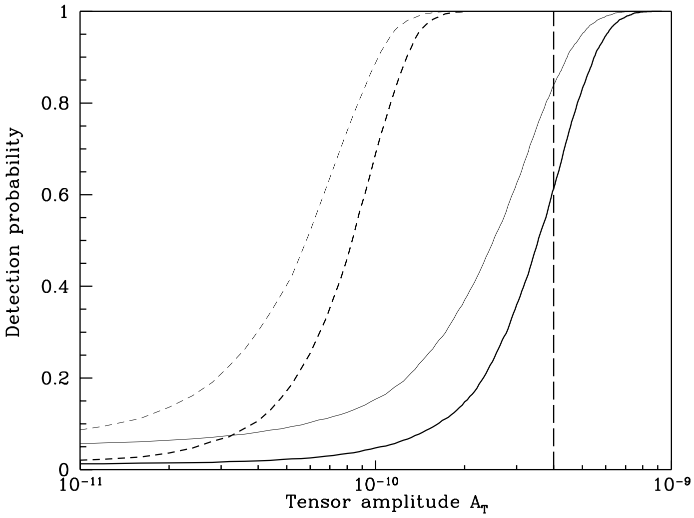

Figure 2: The probability of detecting tensor modes at 95% (thin lines) and 99% (thick lines) confidence as a function of the amplitude A_T assuming zero noise and full sky coverage, using temperature only (solid lines) and E polarization only (dashed lines).

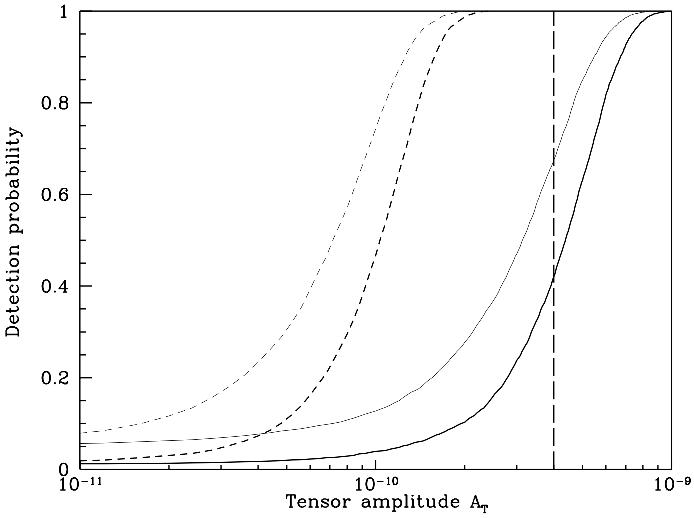

Figure 3: The probability of detecting tensor modes at 95% (thin lines) and 99% (thick lines) confidence as a function of the amplitude A_T assuming zero noise and 70% sky coverage, using temperature only (solid lines) and E polarization only (dashed lines).

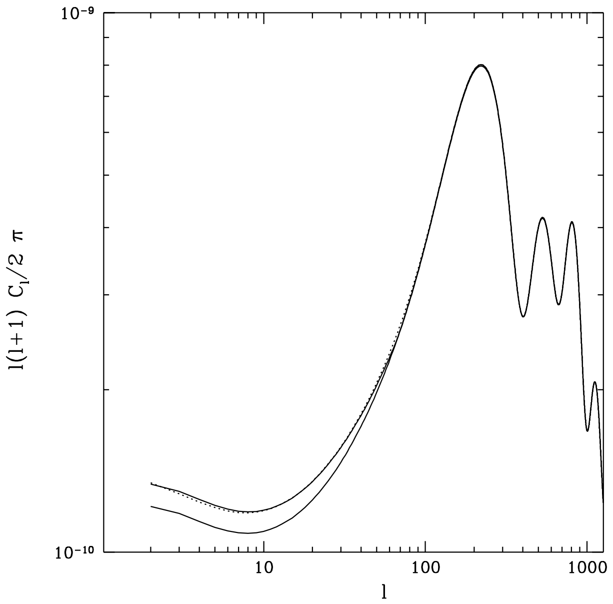

Figure 4: The $r = 0.1$ CMB power spectrum (blue dots) and a $r = 0$ spectrum with a non-flat initial power spectrum $P(k) = A_s(1 + \theta(0.005\text{Mpc}^{-1} - k)/10)$. ($\theta$ is the unit step function, black line), compared with the standard $r = 0$ model (red line).

(though it could also be generated by defects, other sources of vector modes, etc).

References

[1] W. H. Kinney. Constraining inflation with cosmic microwave background polarization. *Phys. Rev.*, D58:123506, 1998.

[2] A. Lewis, A. Challinor, and N. Turok. Analysis of CMB polarization on an incomplete sky. *Phys. Rev. D*, 65:023505, 2002.

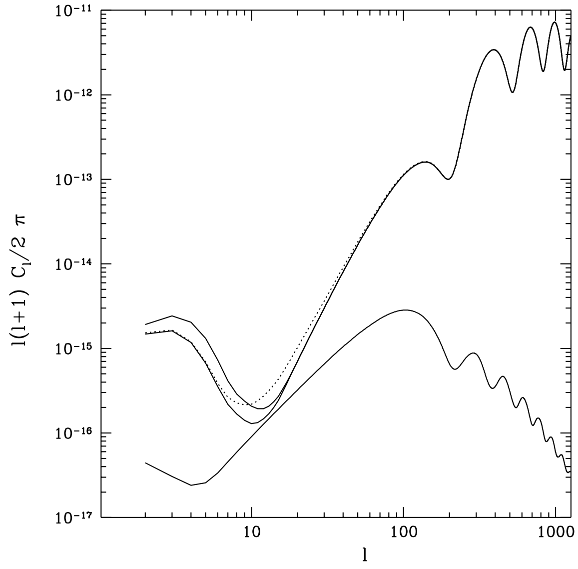

Figure 5: The $r = 0.1$ CMB E power spectrum (blue), the $r = 0$ spectrum (red) and total (blue dots), all with reionization at $z = 6$. The black curve shows an $r = 0$ spectrum with reionization at $z = 7$.

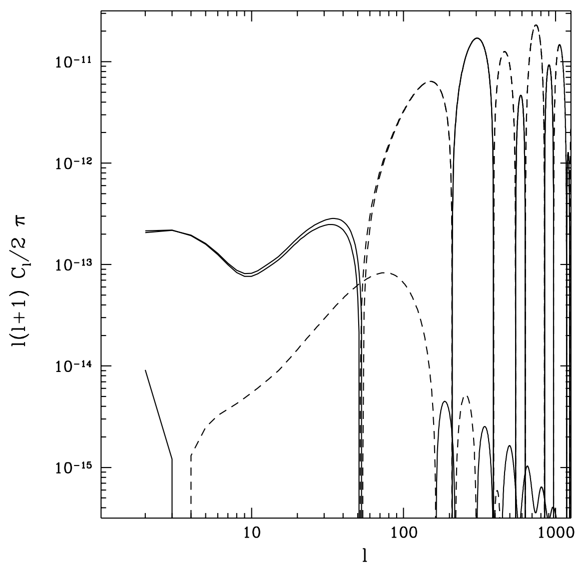

Figure 6: The $r = 0.1$ CMB cross-correlation power spectrum and $r = 0$ spectrum .