Geometric Algebra Identities

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Vectors

$$a\cdot(b\wedge c) = a\cdot bc - a\cdot cb$$
$$(a\wedge b)\cdot(c\wedge d) = c\cdot ba\cdot d - c\cdot ab\cdot d$$
$$a\cdot b = ia\wedge(ib)$$

Bivectors and vectors

$$a\cdot(b\wedge B) = a\cdot bB - b\wedge(a\cdot B)$$
$$(a\wedge b)\cdot B = a\cdot(b\cdot B) = (b\cdot B)\cdot a = b\cdot(B\cdot a)$$
$$(B\cdot\partial_a)\wedge a = 2B$$
$$\partial_a\wedge\partial_b(b\wedge a)\cdot B = 2B$$

Vector and grade r,s multivectors

$$a\cdot(A_r\wedge B_s) = (a\cdot A_r)\wedge B_s + (-1)^r A_r\wedge(a\cdot B_s)$$
$$a\wedge(A_r\cdot B_s) = (a\cdot A_r)\cdot B_s + (-1)^r A_r\cdot(a\wedge B_s)$$
$$\partial_a A_r a = (-1)^r(n-2r)A_r$$
$$\partial_a a\cdot A_r = rA_r$$

Multivector relations

$$A\times(B\times C) + B\times(C\times A) + C\times(A\times B) = 0$$
$$B\times(A\cdot C) = (B\times A)\cdot C + A\cdot(B\times C)$$
$$A_r\cdot(B_s\cdot C_t) = (A_r\wedge B_s)\cdot C_t \qquad r+s\leq t,\quad r,s>0$$

Linear functions

$$\underline{f}^{-1}(A) = \det\underline{f}^{-1}\bar{f}(AI)I^{-1}$$
$$\bar{f}^{-1}(A) = \det\underline{f}^{-1}I^{-1}\underline{f}(IA)$$
$$\partial_{\bar{h}(a)}\det\underline{h}^{-1} = -\det\underline{h}^{-1}\underline{h}^{-1}(a)$$
$$\partial_{\bar{h}(a)}\underline{h}^{-1}(b)\cdot v = -\underline{h}^{-1}(a)\cdot v\,\underline{h}^{-1}(b)$$