Polarization from Rayleigh scattering
Blue sky thinking for future CMB observations

Previous work: Takahara et al. 91, Yu, et al. astro-ph/0103149


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Classical dipole scattering

Oscillating dipole $p = p_0 \sin(\omega t) \hat{z} \Rightarrow$ radiated power $\propto \omega^4 p_0^2 \sin^2 \theta \, d\Omega$

**Thomson Scattering**

\[
m_e \ddot{z} = -eE_z \sin \omega t
\]

\[
\Rightarrow p = \frac{-e^2 E_z}{m_e \omega^2} \sin \omega t \, \hat{z}
\]

**Rayleigh Scattering**

\[
m_e \ddot{z} = -eE_z \sin \omega t - m_e \omega_0 z
\]

\[
\Rightarrow p = \frac{-e^2 E_z}{m_e (\omega^2 - \omega_0^2)} \sin \omega t \, \hat{z}
\]

Frequency independent

Given by fundamental constants:

\[
\sigma_T = \frac{8\pi}{3} \left( \frac{e^2}{4\pi \epsilon_0 m_e c^2} \right)^2
\]

Frequency dependent

Depends on natural frequency $\omega_0$ of target

\[
\sigma_R \approx \frac{\omega^4}{\omega_0^4} \sigma_T \quad (\omega \ll \omega_0)
\]

Both dipole scattering: same angular and polarization dependence (Boltzmann equations nearly the same)
Rayleigh scattering details

Early universe ⇒ Neutral hydrogen (+small amount of Helium)

After recombination, mainly neutral hydrogen in ground state:

\[ \sigma_R(\nu) = \left[ \left( \frac{\nu}{\nu_{\text{eff}}} \right)^4 + \frac{638}{243} \left( \frac{\nu}{\nu_{\text{eff}}} \right)^6 + \frac{1626820991}{136048896} \left( \frac{\nu}{\nu_{\text{eff}}} \right)^8 + \ldots \right] \sigma_T \]

(Lee 2005: Non-relativistic quantum calculation, for energies well below Lyman-alpha)

\[ \nu_{\text{eff}} \equiv \sqrt{\frac{8}{9}} c R_A \approx 3.1 \times 10^6 \text{GHz} \]

Only negligible compared to Thomson for \( n_H \left( \frac{(1+z)\nu_{\text{obs}}}{3 \times 10^6 \text{GHz}} \right)^4 \ll n_e \)
Scattering rate

Total cross section \( \approx \Gamma(\nu) = n_e \sigma_T + \sigma_R(\nu) [n_H + R_{He} n_{He}] \)

\( R_{He} \approx 0.1 \)

\[ \dot{\tau} = \Gamma/(1 + z) \]
In principle Rayleigh scattering could do tomography of last scattering and beyond
Expected signal as function of frequency

Zero order: uniform blackbody not affected by Rayleigh scattering (elastic scattering, photons conserved)

1st order: anisotropies modified, no longer frequency independent

May be detectable signal at $200\text{GHz} \leq \nu \leq 800\text{GHz}$
Effects of Rayleigh scattering

• Moves total visibility to lower redshift: larger sound horizon, so shift in peak scales

• Total visibility function broader: additional scattering to lower redshifts $\Rightarrow$ more Silk damping

• More total baryon-photon coupling: frequency independent longer tight coupling ($< 0.04\%$ - neglect)
Rayleigh signal

Total (frequency $i$) = primary blackbody + Rayleigh signal

$$X^i_{lm} \approx X_{lm} + \left(\frac{\nu_i}{\nu_0}\right)^4 \Delta X_{4,lm} + \left(\frac{\nu_i}{\nu_0}\right)^6 \Delta X_{6,lm}$$

$X, Y \in \{T, E, B\}$

Gaussian sky at multiple frequencies: sufficient statistics are

$$C^X_i Y^j_l = \langle X^i_{lm} Y^j_{lm} \rangle$$

$$C^X_i Y^j_l \approx C^X_i Y^j_l + \left(\frac{1}{\nu_0}\right)^4 \left[ \nu_j^4 C^X_i Y^j_l + \nu_i^4 C^\Delta X^i_4 Y + \nu_i^6 C^\Delta X^i_6 Y \right]$$

$$+ \left(\frac{1}{\nu_0}\right)^6 \left[ \nu_j^6 C^X_i Y^j_l + \nu_i^6 C^\Delta X^i_6 Y \right]$$

$$+ \left(\frac{\nu_i \nu_j}{\nu_0^2}\right)^4 C^\Delta X^i_4 \Delta Y^j_4 + \ldots$$
Large scales

Temperature

Rayleigh signal only generated by sub-horizon scattering
(no Rayleigh monopole background to distort by anisotropic photon redshifting)

\[
\frac{\Delta T_0}{T}(\hat{n}) = \frac{\Delta \gamma(\eta_*)}{4} + \Psi(\eta_*) - \Psi_0 + \hat{n} \cdot (\mathbf{v}_o - \mathbf{v}) + \int_{\eta_*}^{\eta_0} d\eta (\Psi' + \Phi')
\]

Temperature perturbation at recombination
(Newtonian Gauge)

Sachs-Wolfe  Doppler  ISW

Polarization

Generated by scattering of the blackbody quadrupole: similar to primary, but larger sound horizon

\[
E_l^\pm(\eta_0) = \frac{l(l-1)}{l+1} \int_{\eta_*}^{\eta_0} d\eta S_n e \sigma_T e^{-\tau} \left[ \frac{1}{\chi} \frac{d j_l(\chi)}{d \chi} + \frac{j_l(\chi)}{\chi^2} \right] \zeta^\pm
\]

\[
B_l^\pm(\eta_0) = -\frac{l(l-1)}{l+1} \int_{\eta_*}^{\eta_0} d\eta S_n e \sigma_T e^{-\tau} \frac{j_l(\chi)}{\chi} \zeta^\mp
\]
Small scales

Primary signal

Primary + Rayleigh signal

Rayleigh difference signal: photons scattered in to line of sight - scattered out

\[ \sim \tau_R \Delta T \]
Hot spots are red, cold spots are blue

Polarization is scattered and is red too
Rayleigh temperature power spectrum

\[(\text{Primary} + \text{Rayleigh})^2 = \text{Primary}^2 + 2 \text{Primary} \times \text{Rayleigh} + \text{Rayleigh}^2\]

**Solid:** Rayleigh × Primary

**Dot-dashed:** Rayleigh × Rayleigh

**Dots:** naïve Planck sensitivity to the cross per \(\Delta l/l = 10\) bin

Small-scale signal is highly correlated to primary

Can hope to isolate using Low frequency × High frequency

Note: not limited by cosmic variance of primary anisotropy
– multi-tracer probe of same underlying perturbation realization
Rayleigh polarization power spectra

Solid: primary  Dashed: primary + Rayleigh (857GHz)
Fractional differences at realistic frequencies

\[ \frac{\Delta C_l}{C_l} \quad \text{TT, EE, BB:} \]

\[ \frac{\Delta C_{l}^{TE}}{\sqrt{C_{l}^{EE}C_{l}^{TT}}} \quad \text{TE:} \]
Detectability

Blue sky thinking ≡ ignore foregrounds

\[ S/N \text{ per } l \text{ for primary } \times \text{Rayleigh cross-spectra (single channel)} \]

May be just detectable by Planck; strong signal in future CMB missions

e.g. PRISM: Rayleigh amplitude measured to 0.4%, EE detected at 20\(\sigma\) (several channels)
Measure new primordial modes?

In principle could double number of modes compared to T+E!

BUT: signal highly correlated to primary on small scales; need the uncorrelated part
Rayleigh-Primary correlation coefficient

Rayleigh only from sub-horizon (mainly Doppler)

Scattering from same quadrupole

Damping of the primary
Number of new modes with PRISM

Define

\[ n_l \equiv (2l + 1) f_{\text{sky}} \text{Tr} \left( ([C_l + N_l]^{-1} C_l)^2 \right) \]

New modes almost all in the \( l \leq 500 \) temperature signal: total \( \approx 10,000 \) extra modes

More horizon-scale information (disentangle Doppler and Sachs-Wolfe terms?)

Would need much higher sensitivity to get more modes from polarization/high \( l \).
Large scale $\frac{\Delta T}{T} + \Phi + \text{ISW}$

(anisotropic redshifting to constant temperature recombination surface)
Large scale $\hat{n} \cdot v_b$: Doppler

Doppler (Rayleigh, Primary)
Polarization
(Rayleigh, Primary)

Large scale quadrupole scattering

Three different perturbation modes being probed
Conclusions

- Significant Rayleigh signal at $\nu \geq 200$ GHz; several percent on T, E at $\nu \geq 500$GHz
- Strongly correlated to primary signal on small scales (mostly damping) – robust detection via cross-correlation?
- Multi-tracer probe of last-scattering
  - limited by noise/foregrounds, not cosmic variance
- Boosts large-scale polarization (except B modes from lensing)
- Must be modelled for consistency
- Powerful consistency check on recombination physics/expansion
- May be able to provide additional primordial information
  - mostly large-scale modes at recombination?

Question:

How well can foregrounds be removed/how large is uncorrelated foreground noise? (CIB, dust..)