Polarization from Rayleigh scattering

Blue sky thinking for future CMB observations

Previous work: Takahara et al. 91, Yu, et al. astro-ph/0103149

http://en.wikipedia.org/wiki/Rayleigh_scattering

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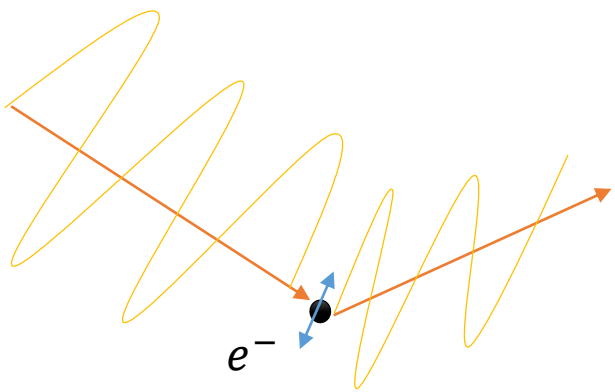
Classical dipole scattering

Oscillating dipole $\boldsymbol{p} = p_0 \sin(\omega t)\,\hat{\boldsymbol{z}} \Rightarrow$ radiated power $\propto \omega^4 p_0^2 \sin^2\theta \, \mathrm{d}\Omega$

Thomson Scattering

$$m_e \ddot{z} = -eE_z \sin \omega t$$

$$\Rightarrow \boldsymbol{p} = \frac{-e^2 E_z}{m_e \omega^2} \sin \omega t \;\; \hat{\boldsymbol{z}}$$



Frequency independent

Given by fundamental constants:

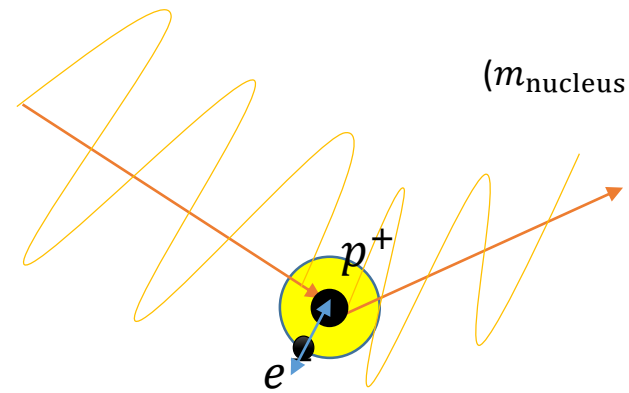
$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\epsilon_0 m_e c^2}\right)^2$$

Rayleigh Scattering

$$m_e \ddot{z} = -eE_z \sin \omega t - m_e \omega_0 z$$

$$\Rightarrow \boldsymbol{p} = \frac{-e^2 E_z}{m_e(\omega^2 - \omega_0^2)} \sin \omega t \;\; \hat{\boldsymbol{z}}$$

$(m_{\text{nucleus}} \gg m_e)$



Frequency dependent

Depends on natural frequency ω_0 of target

$$\sigma_R \approx \frac{\omega^4}{\omega_0^4}\sigma_T \qquad (\omega \ll \omega_0)$$

Both dipole scattering: same angular and polarization dependence (Boltzmann equations nearly the same)

http://farside.ph.utexas.edu/teaching/em/lectures/node95.html

Rayleigh scattering details

Early universe ⇒ Neutral hydrogen (+small amount of Helium)

After recombination, mainly neutral hydrogen in ground state:

$$\sigma_R(\nu) = \left[\left(\frac{\nu}{\nu_{\rm eff}}\right)^4 + \frac{638}{243}\left(\frac{\nu}{\nu_{\rm eff}}\right)^6 + \frac{1626820991}{136048896}\left(\frac{\nu}{\nu_{\rm eff}}\right)^8 + \ldots\right]\sigma_T$$

(Lee 2005: Non-relativistic quantum calculation, for energies well below Lyman-alpha)

$$\nu_{\rm eff} \equiv \sqrt{\frac{8}{9}}\, {\rm c\, R_A} \approx 3.1 \times 10^6 {\rm GHz}$$

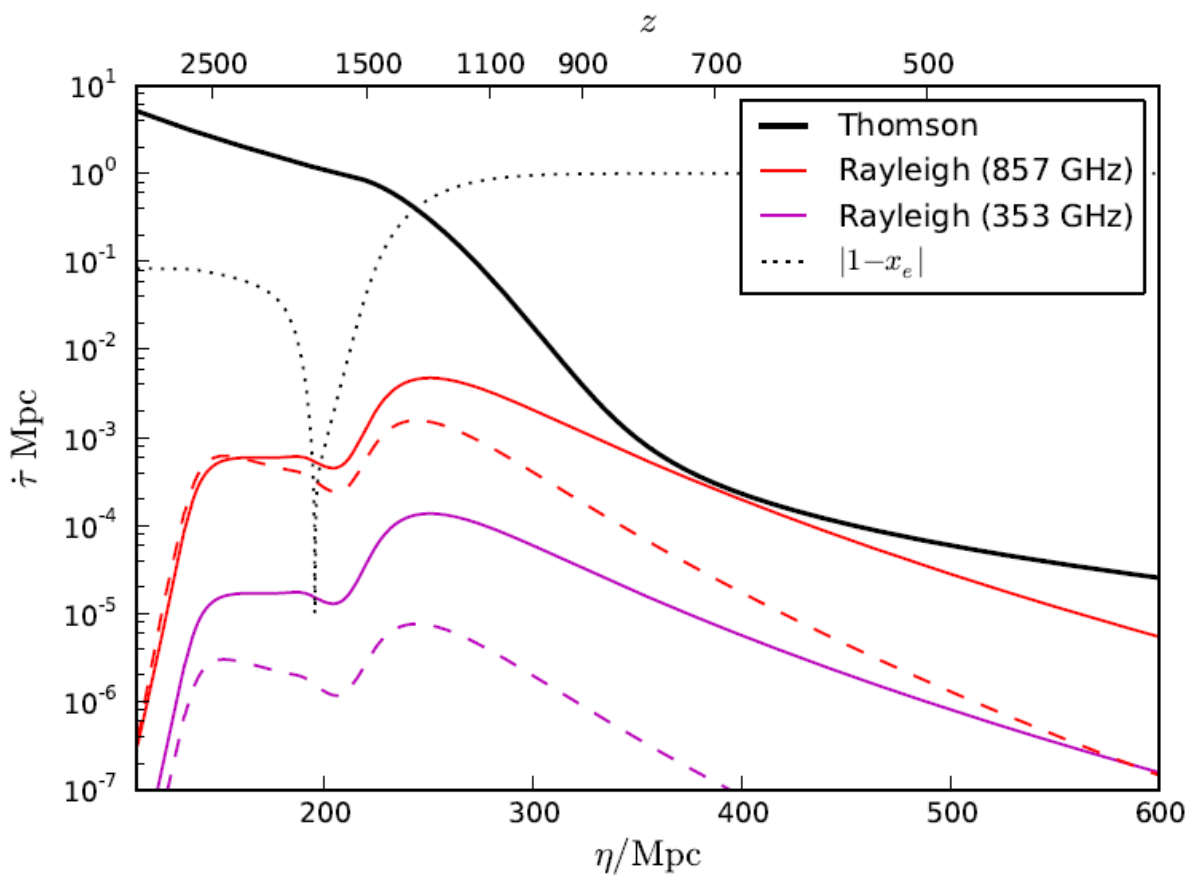
Only negligible compared to Thomson for $n_H \left(\frac{(1+z)\nu_{\rm obs}}{3\times10^6{\rm GHz}}\right)^4 \ll n_e$

Scattering rate

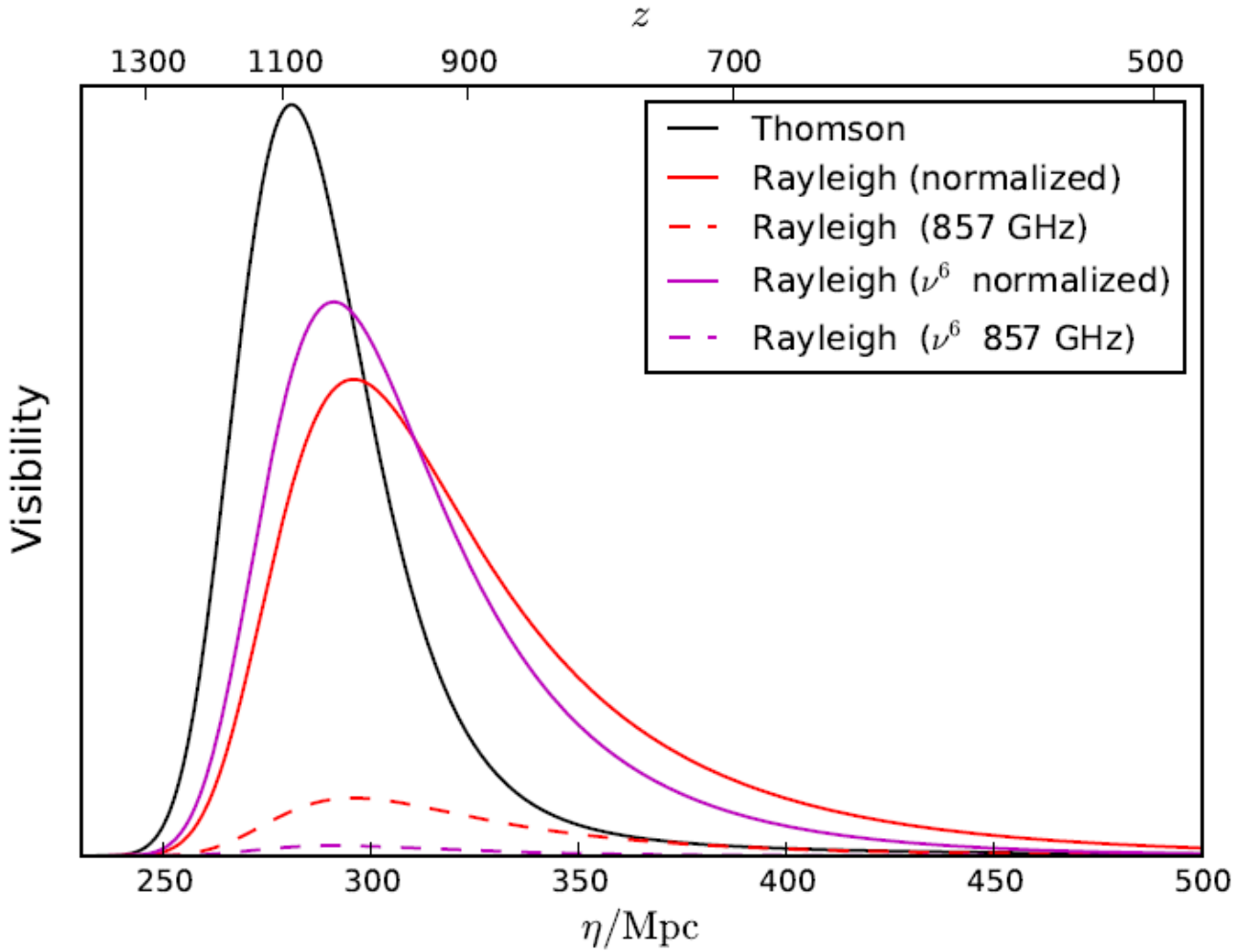
Total cross section ≈ $\Gamma(\nu) = n_e \sigma_T + \sigma_R(\nu)\left[n_H + R_{He} n_{He}\right]$

$(R_{He} \approx 0.1)$

$\dot{\tau} = \Gamma/(1+z)$



Visibility

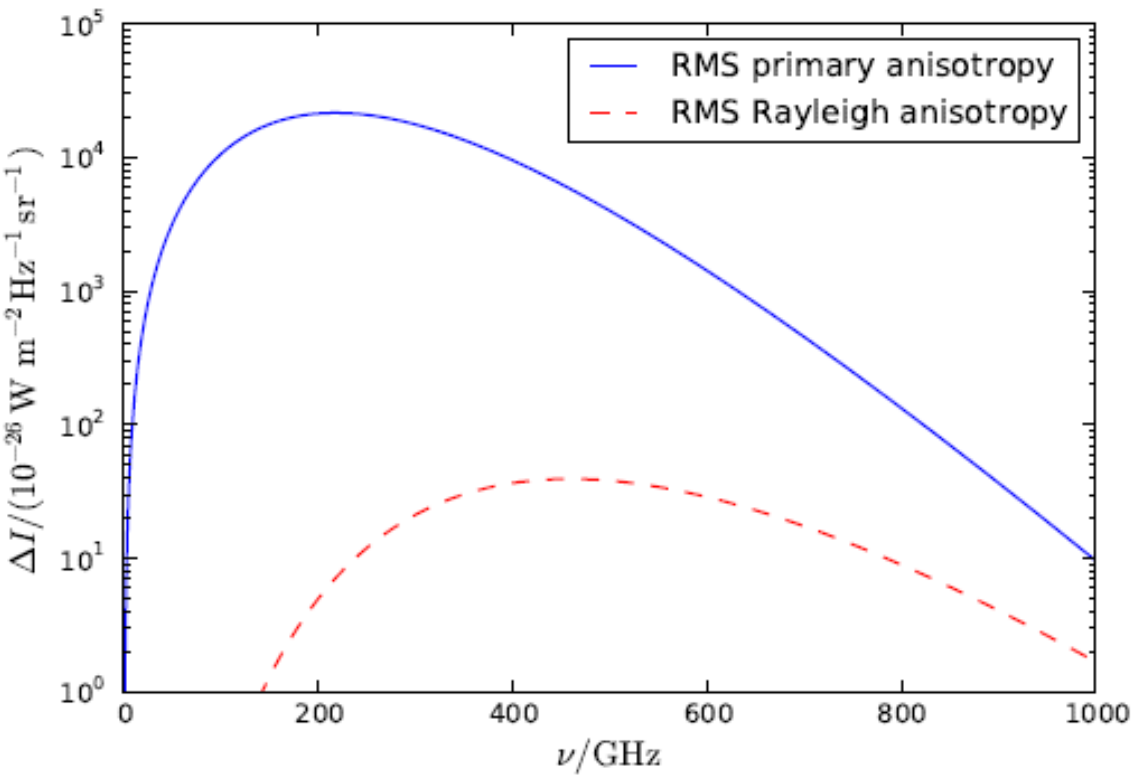


In principle Rayleigh scattering could do tomography of last scattering and beyond

Expected signal as function of frequency

Zero order: uniform blackbody not affected by Rayleigh scattering (elastic scattering, photons conserved)

1st order: anisotropies modified, no longer frequency independent



May be detectable signal at $200\mathrm{GHz} \leq \nu \leq 800\mathrm{GHz}$

Effects of Rayleigh scattering

- Moves total visibility to lower redshift:
  larger sound horizon, so shift in peak scales

- Total visibility function broader:
  additional scattering to lower redshifts ⇒ more Silk damping

- More total baryon-photon coupling:
  frequency independent longer tight coupling ($< 0.04\%$ - neglect)

Yu, Spergel, Ostriker, astro-ph/0103149

Rayleigh signal

Total (frequency i) = primary blackbody + Rayleigh signal

$$X^i_{lm} \approx X_{lm} + \left(\frac{\nu_i}{\nu_0}\right)^4 \Delta X_{4,lm} + \left(\frac{\nu_i}{\nu_0}\right)^6 \Delta X_{6,lm}$$

$X, Y \in \{T, E, B\}$

Gaussian sky at multiple frequencies: sufficient statistics are $C_l^{X^i Y^j} = \langle X^{i*}_{lm} Y^j_{lm} \rangle$

$$C_l^{X^i Y^j} \approx C_l^{XY} + \left(\frac{1}{\nu_0}\right)^4 \left[\nu_j^4 C_l^{X \Delta Y_4^j} + \nu_i^4 C_l^{\Delta X_4^i Y}\right] + \left(\frac{1}{\nu_0}\right)^6 \left[\nu_j^6 C_l^{X \Delta Y_6^j} + \nu_i^6 C_l^{\Delta X_6^i Y}\right] + \left(\frac{\nu_i \nu_j}{\nu_0^2}\right)^4 C_l^{\Delta X_4^i \Delta Y_4^j} + \ldots$$

Large scales

Temperature

Rayleigh signal only generated by sub-horizon scattering
(no Rayleigh monopole background to distort by anisotropic photon redshifting)

$$\frac{\Delta T_0}{T}(\hat{\mathbf{n}}) = \frac{\Delta_\gamma(\eta_*)}{4} + \underbrace{\Psi(\eta_*) - \Psi_0}_{\text{Sachs-Wolfe}} + \underbrace{\hat{\mathbf{n}} \cdot (\mathbf{v}_o - \mathbf{v})}_{\text{Doppler}} + \underbrace{\int_{\eta_*}^{\eta_0} d\eta(\Psi' + \Phi')}_{\text{ISW}}$$

Temperature perturbation at recombination
(Newtonian Gauge)

Polarization

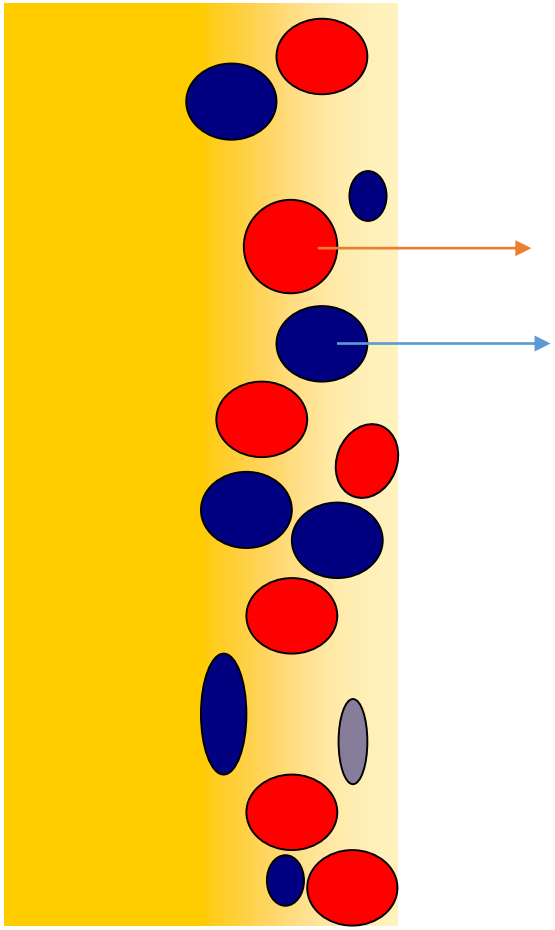
Generated by scattering of the blackbody quadrupole:
similar to primary, but larger sound horizon

$$E_l^{\pm}(\eta_0) = \frac{l(l-1)}{l+1} \int^{\eta_0} d\eta S n_e \sigma_T e^{-\tau} \left[\frac{1}{\chi} \frac{dj_l(\chi)}{d\chi} + \frac{j_l(\chi)}{\chi^2} \right] \zeta^{\pm}$$

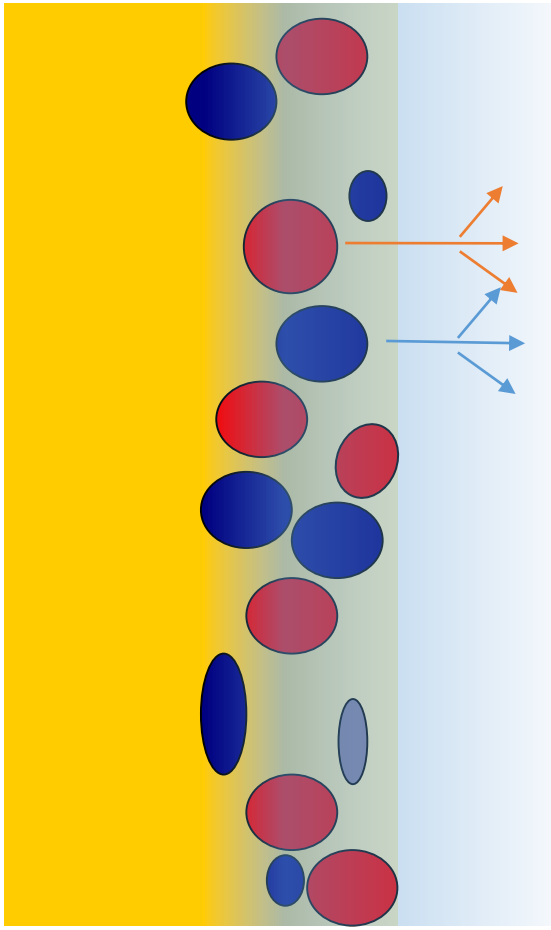
$$B_l^{\pm}(\eta_0) = -\frac{l(l-1)}{l+1} \int^{\eta_0} d\eta S n_e \sigma_T e^{-\tau} \frac{j_l(\chi)}{\chi} \zeta^{\mp}$$

Small scales

Primary signal



Primary + Rayleigh signal



Rayleigh difference signal: photons scattered in to line of sight - scattered out

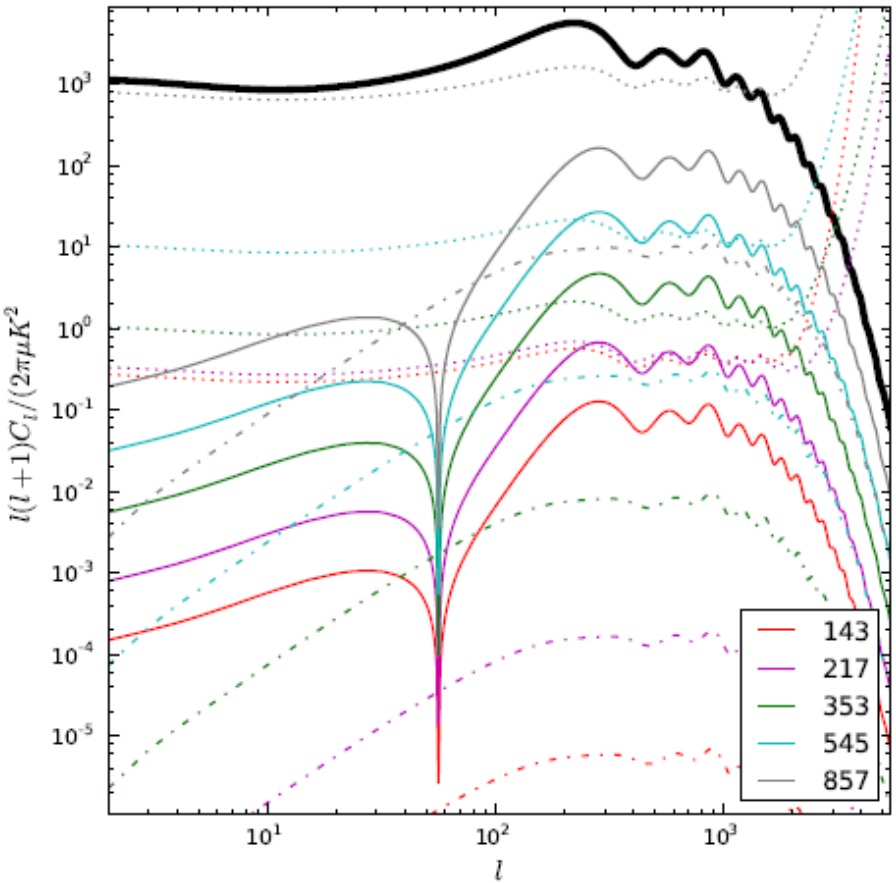
$$\sim \tau_R \, \Delta T$$

Hot spots are red, cold spots are blue

Polarization is scattered and is red too

Rayleigh temperature power spectrum

$$(\text{Primary+Rayleigh})^2 = \text{Primary}^2 + 2\ \text{Primary} \times \text{Rayleigh} + \text{Rayleigh}^2$$



Solid: Rayleigh × Primary
Dot-dashed: Rayleigh × Rayleigh

Dots: naïve Planck sensitivity to the cross per $\Delta l/l$ = 10 bin

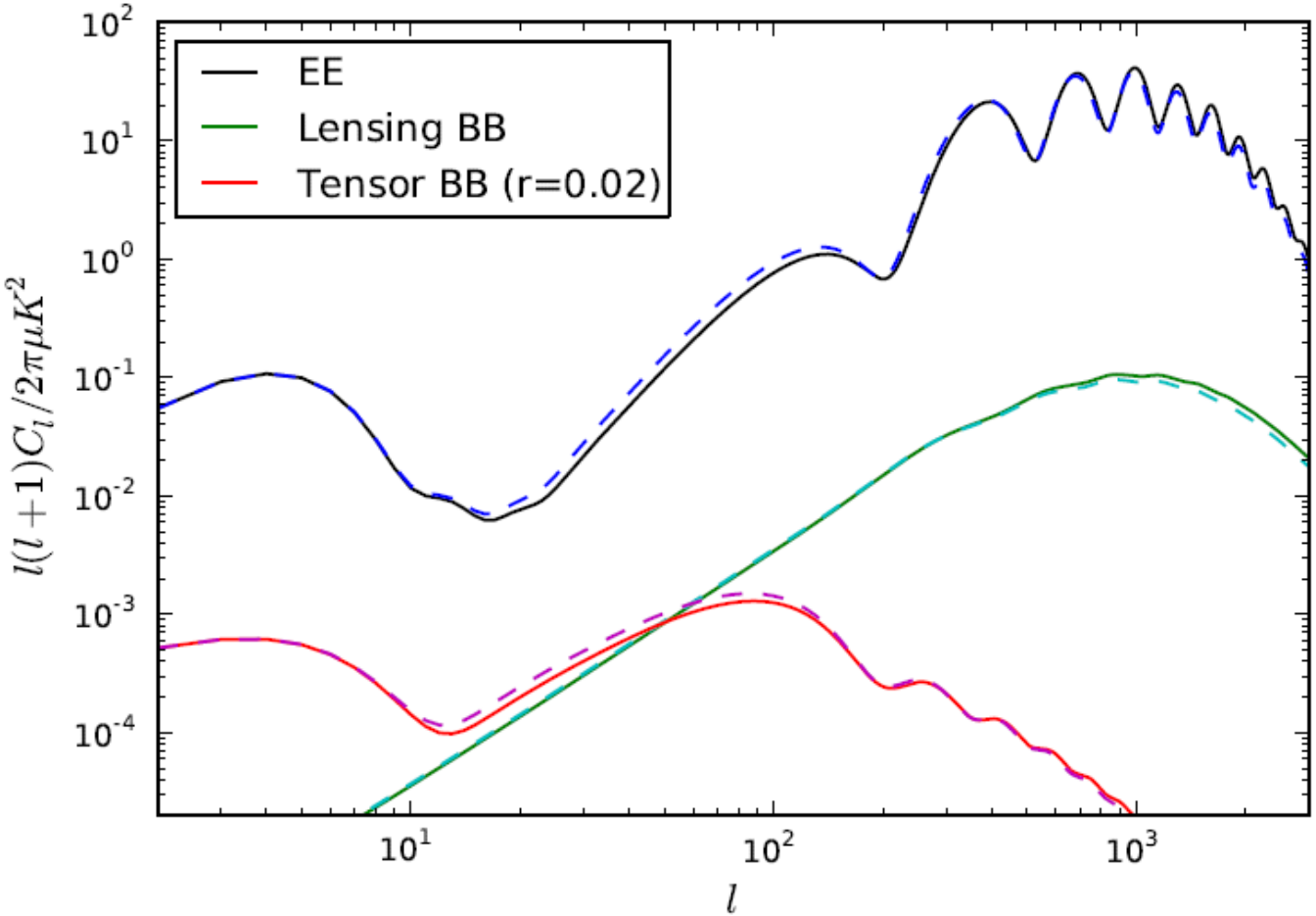
Small-scale signal is highly correlated to primary

Can hope to isolate using
Low frequency × High frequency

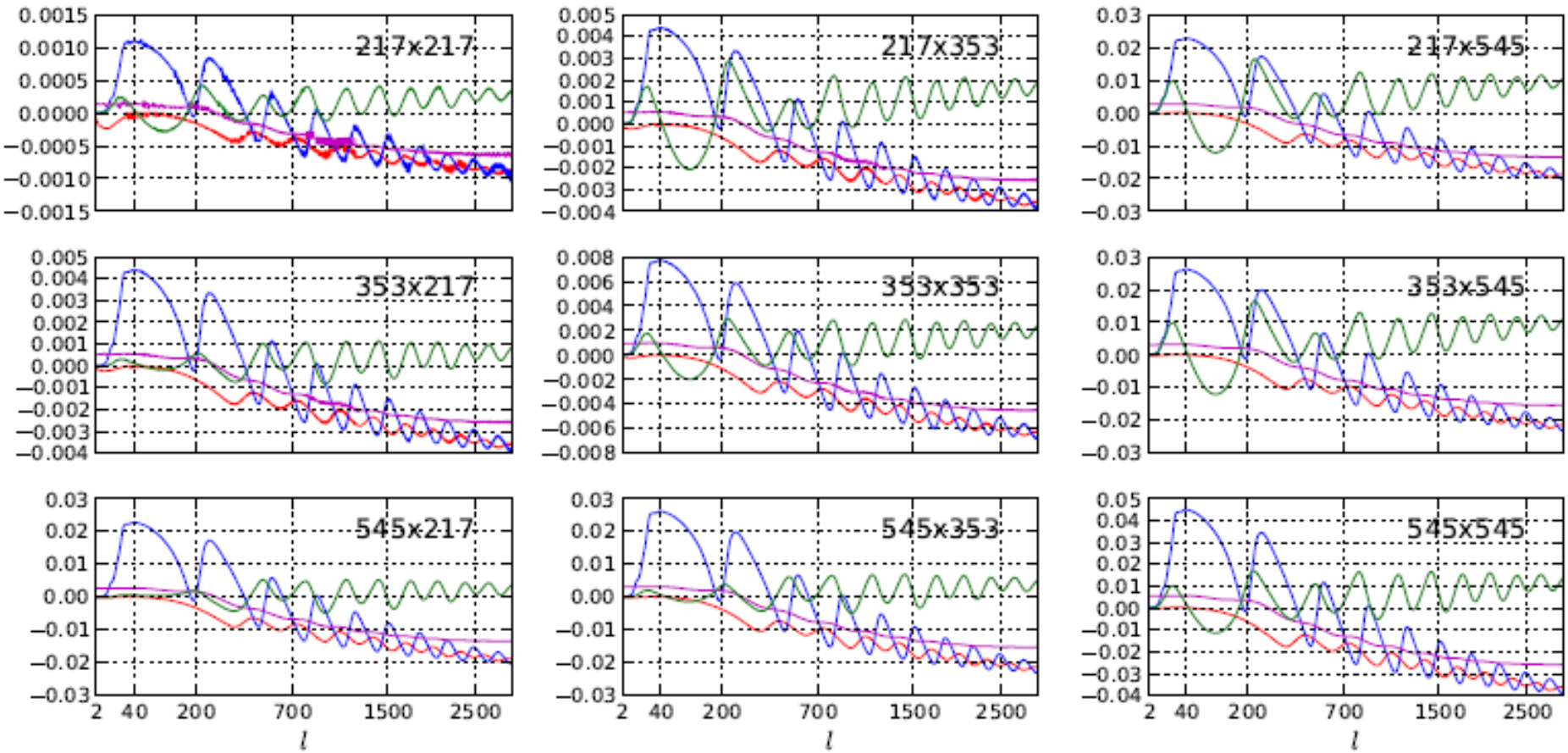
Note: not limited by cosmic variance of primary anisotropy
– multi-tracer probe of same underlying perturbation realization

Rayleigh polarization power spectra

Solid: primary Dashed: primary + Rayleigh (857GHz)



Fractional differences at realistic frequencies



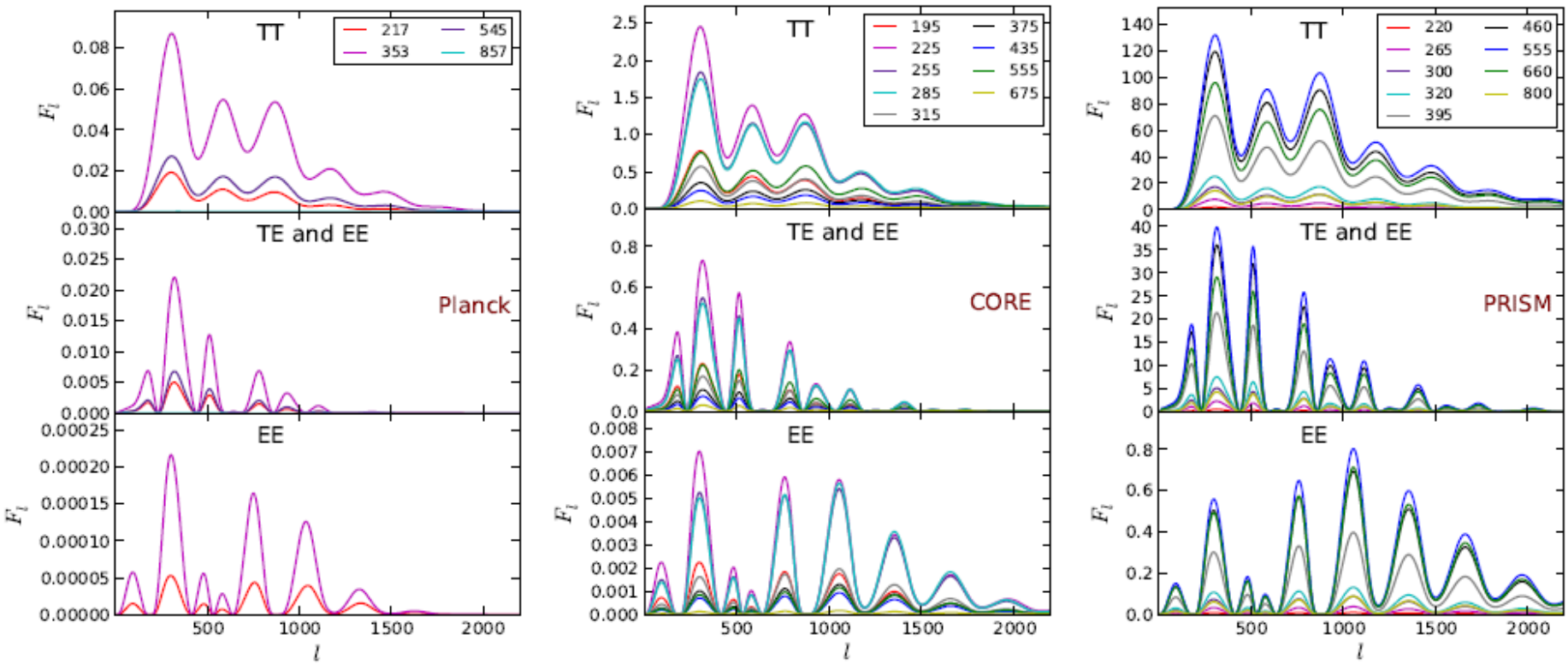
TT, EE, BB: $\frac{\Delta C_l}{C_l}$

TE: $\frac{\Delta C_l^{TE}}{\sqrt{C_l^{EE} C_l^{TT}}}$

Detectability

Blue sky thinking ≡ ignore foregrounds

S/N per l for primary × Rayleigh cross-spectra (single channel)



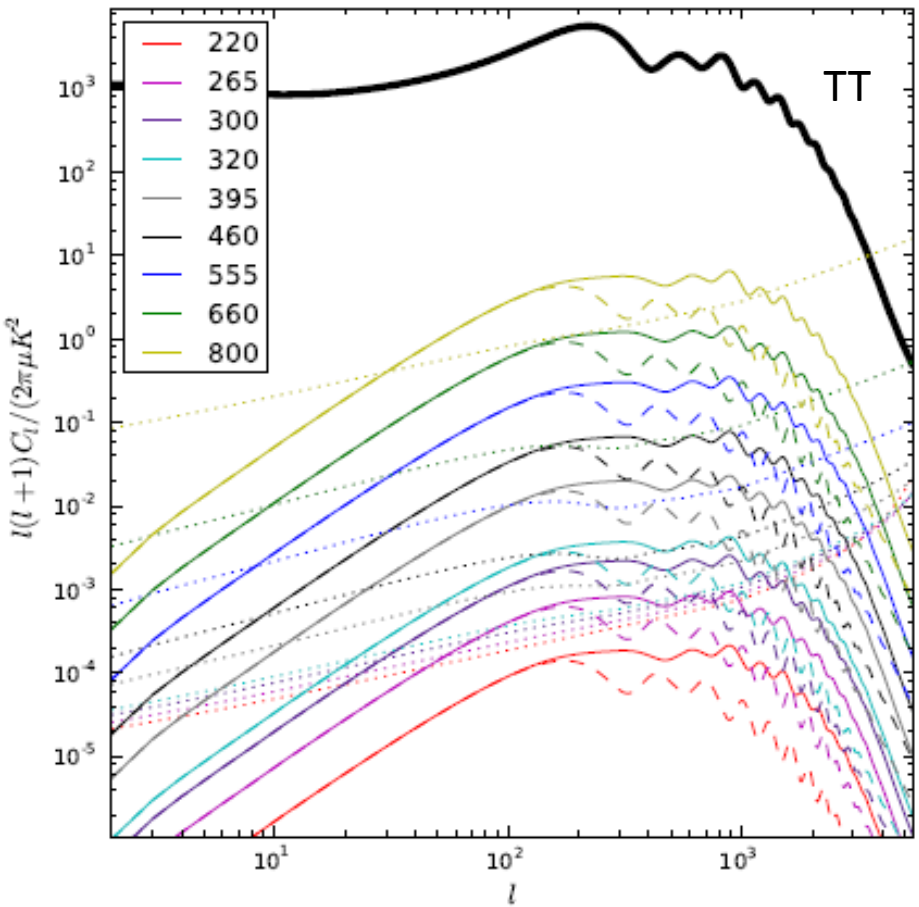
May be just detectable by Planck; strong signal in future CMB missions

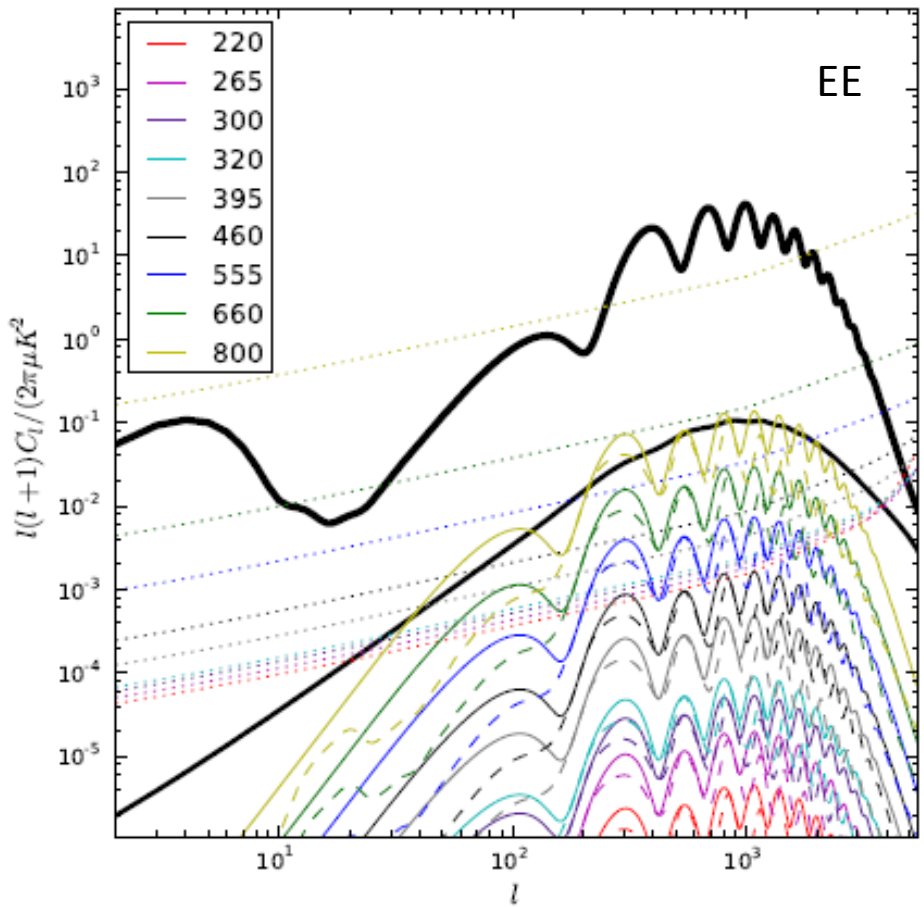
e.g. PRISM: Rayleigh amplitude measured to 0.4%, EE detected at 20σ (several channels)

Measure new primordial modes?

In principle could double number of modes compared to T+E!

BUT: signal highly correlated to primary on small scales; need the uncorrelated part



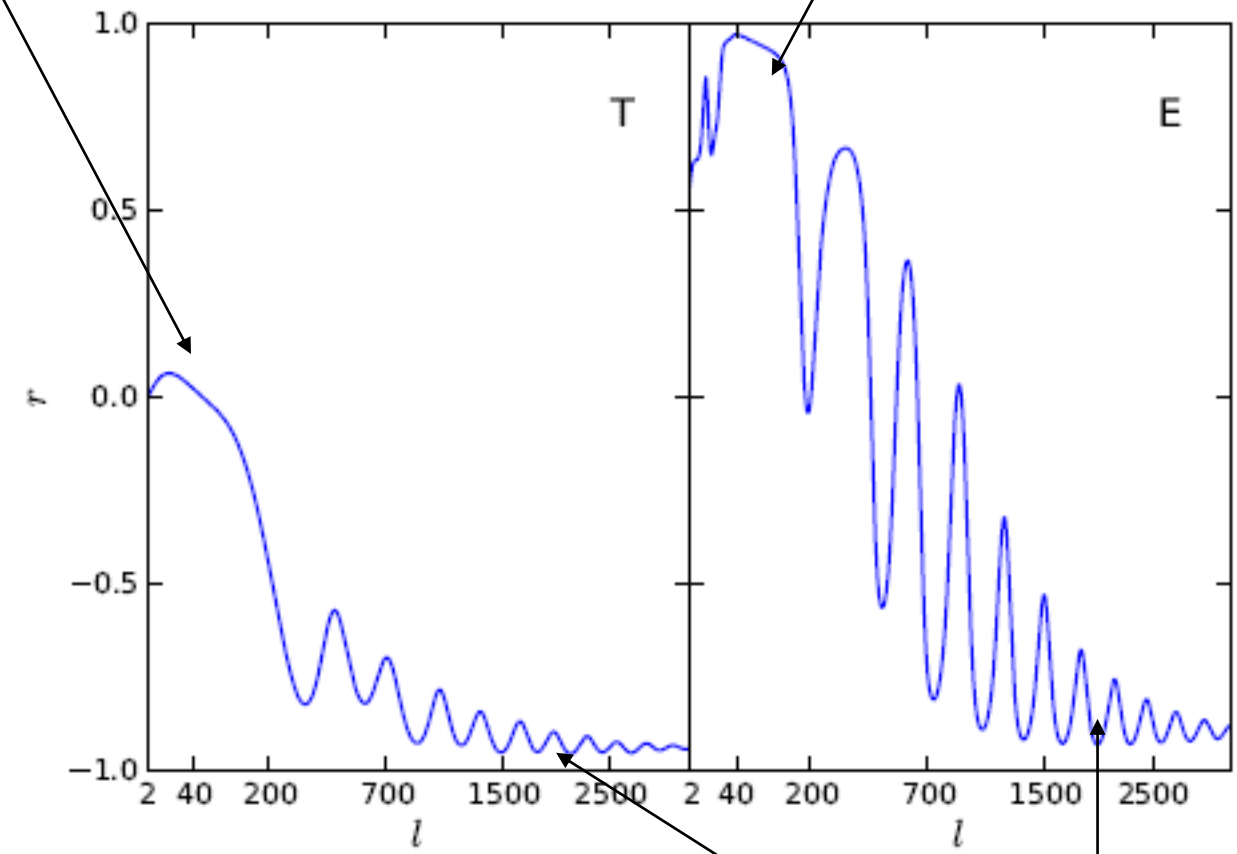


Solid: Rayleigh× Rayleigh total; Dashed: uncorrelated part; Dots: error per $\frac{\Delta l}{l} = 10$ bin a from PRISM

Rayleigh-Primary correlation coefficient

Scattering from same quadrupole

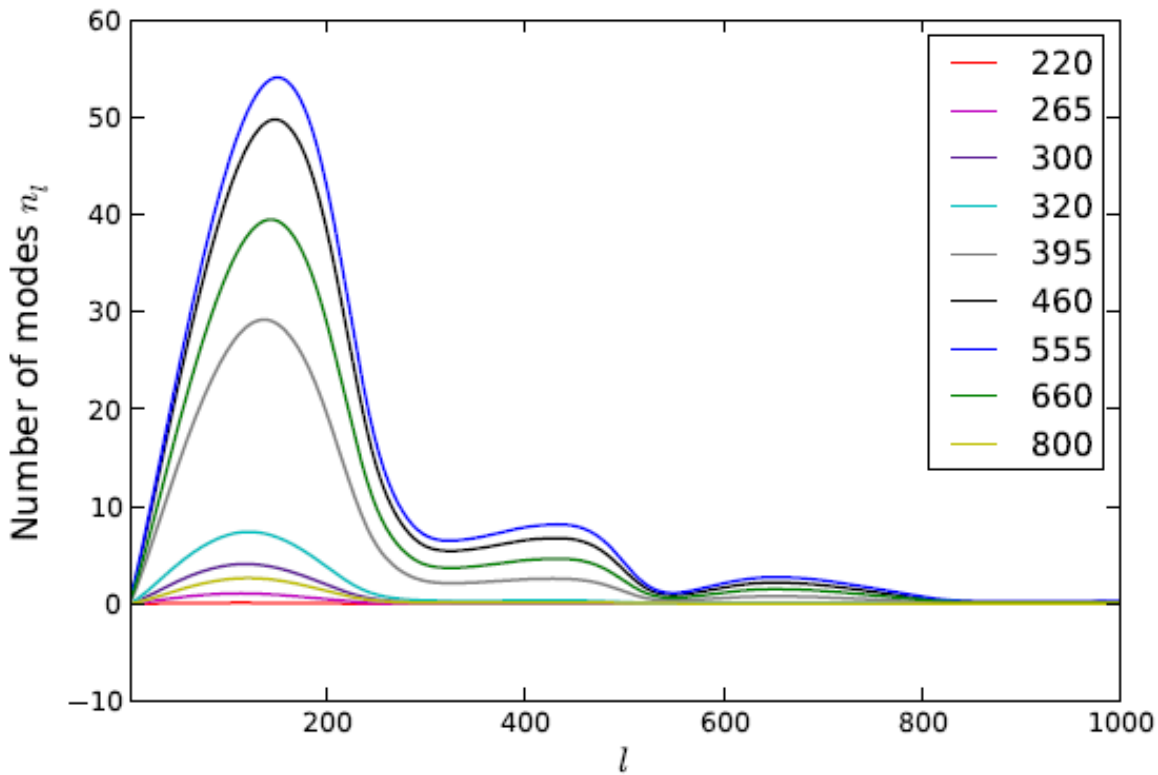
Rayleigh only from sub-horizon (mainly Doppler)



Damping of the primary

Number of new modes with PRISM

Define $n_l \equiv (2l+1) f_{\rm sky} \mathrm{Tr}\left[\left([C_l + N_l]^{-1} C_l\right)^2\right]$



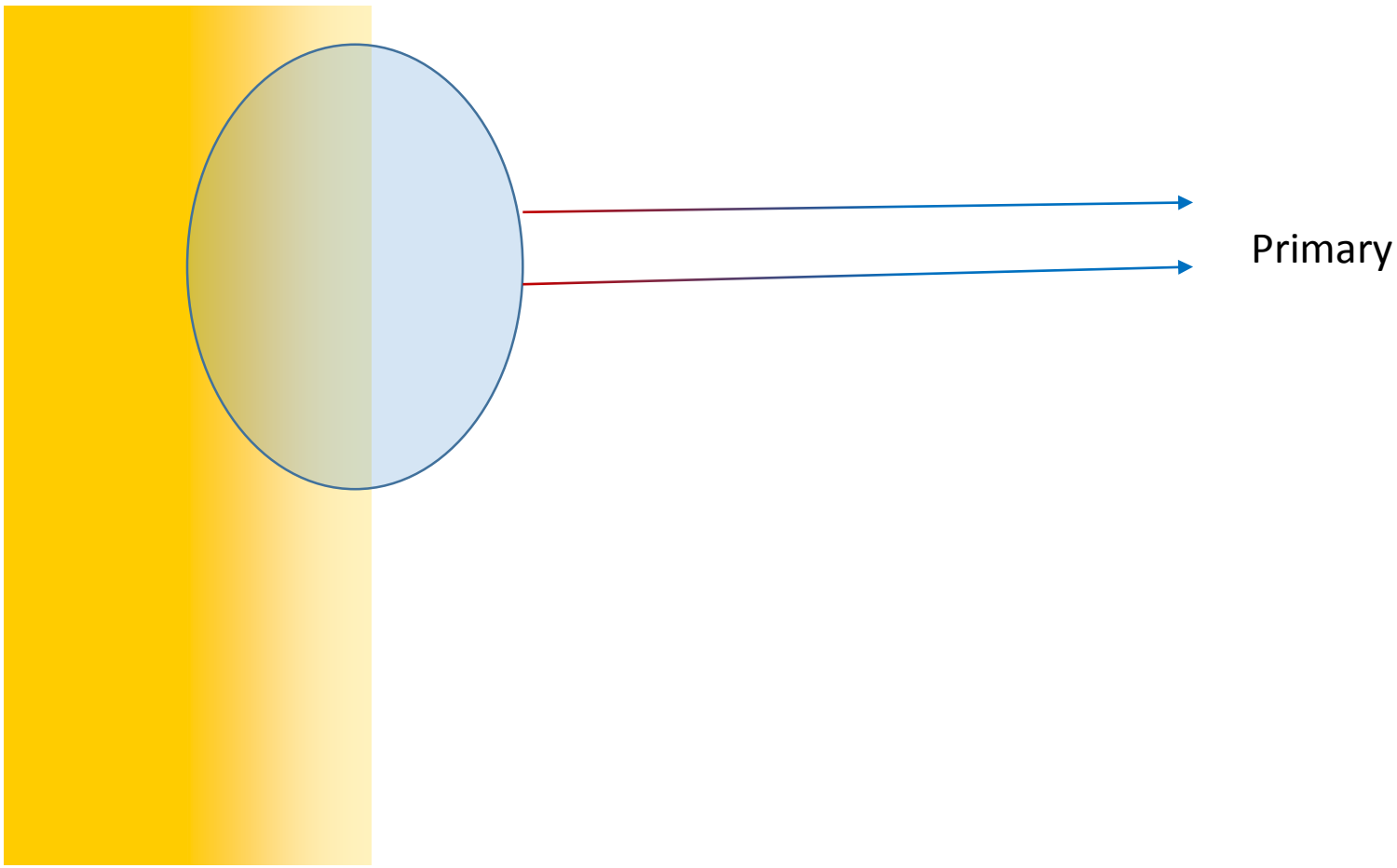
New modes almost all in the $l \leq 500$ temperature signal: total $\approx 10\,000$ extra modes

More horizon-scale information (disentangle Doppler and Sachs-Wolfe terms?)

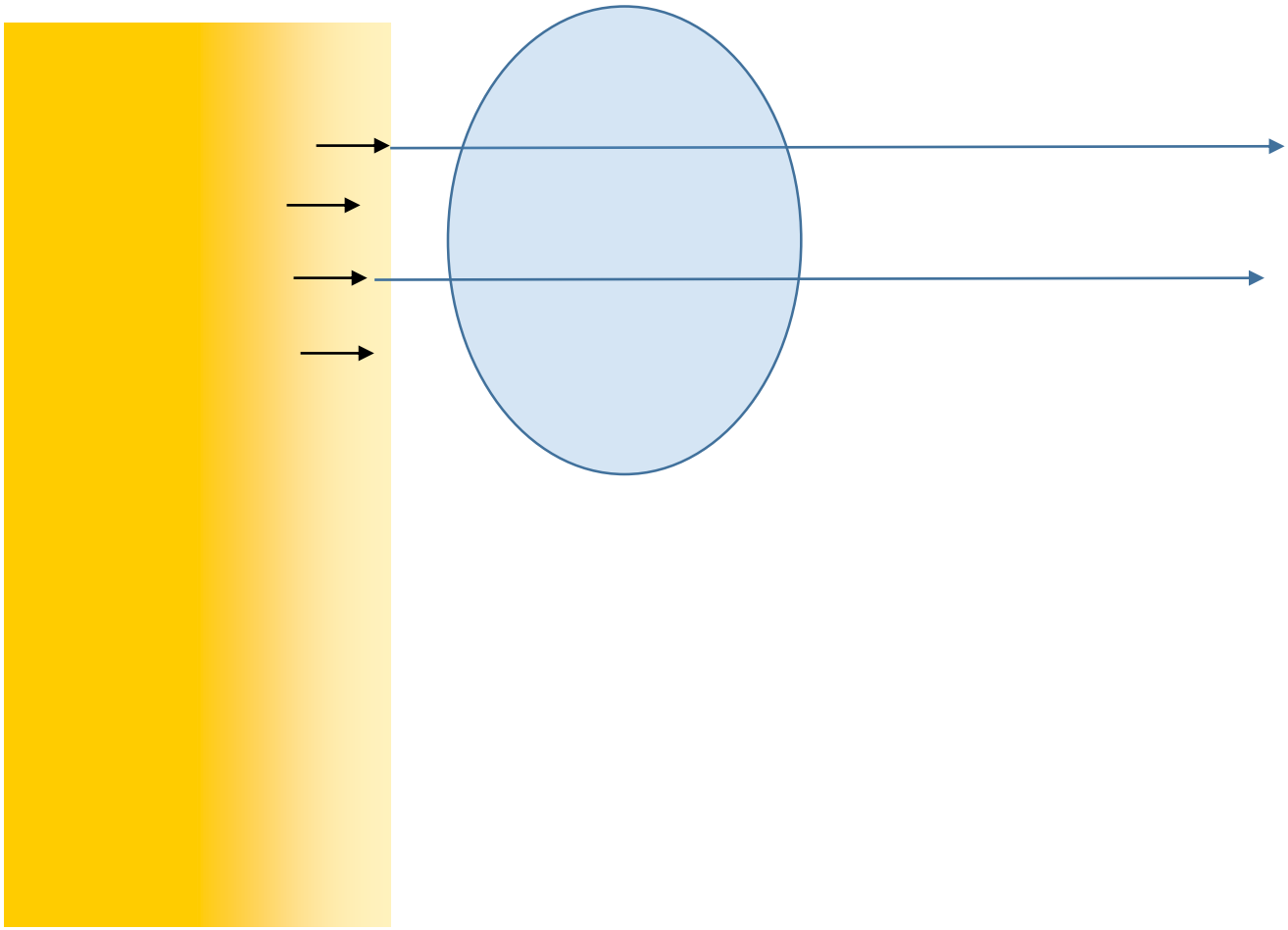
Would need much higher sensitivity to get more modes from polarization/high l

Large scale $\frac{\Delta T}{T} + \Phi + \mathrm{ISW}$

(anisotropic redshifting to constant temperature recombination surface)

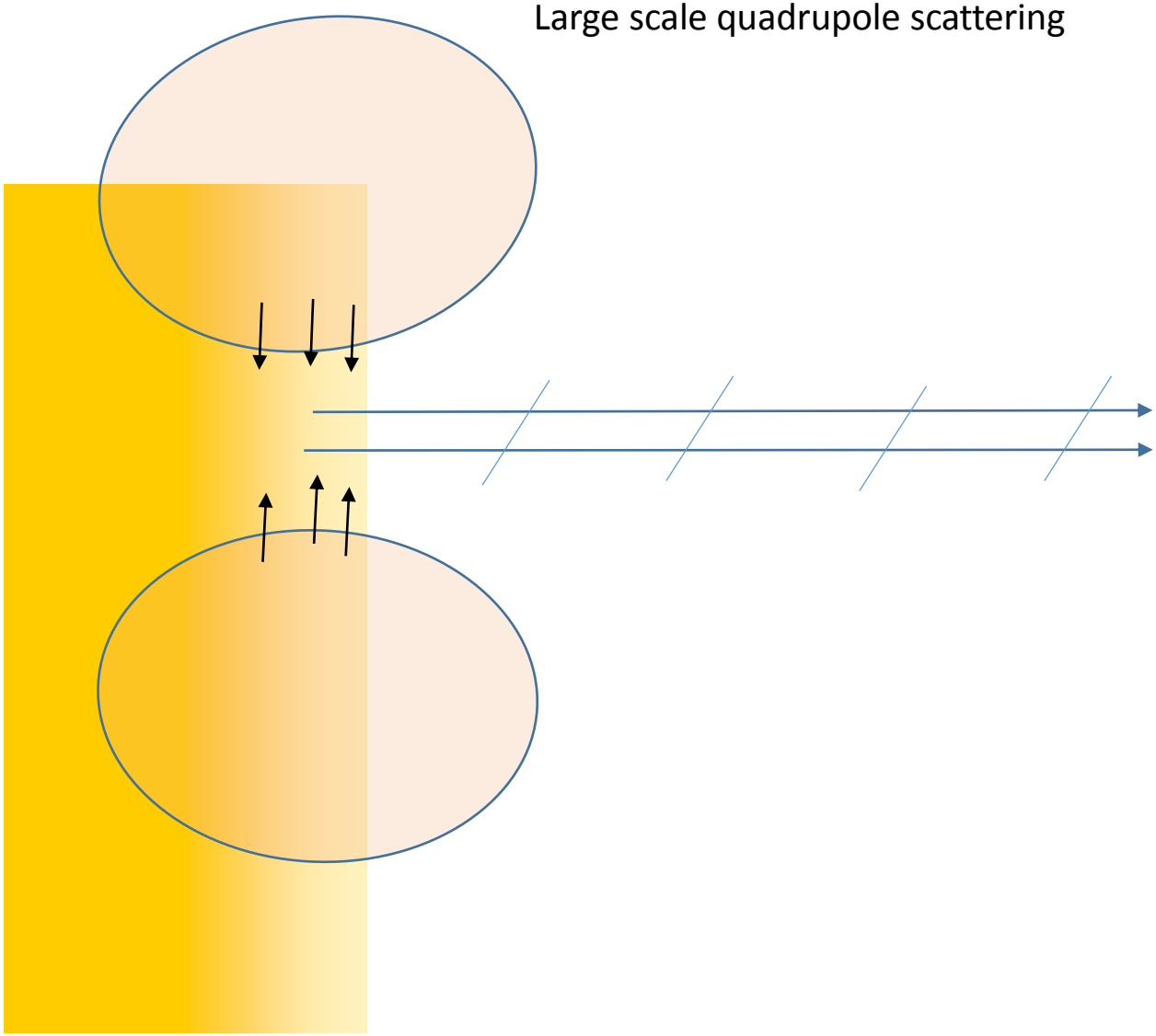


Large scale $\hat{n} \cdot v_b$: Doppler



Doppler
(Rayleigh, Primary)

Large scale quadrupole scattering



Polarization
(Rayleigh, Primary)

Three different perturbation modes being probed

Conclusions

- Significant Rayleigh signal at $\nu \geq 200$ GHz; several percent on T, E at $\nu \geq 500$GHz
- Strongly correlated to primary signal on small scales (mostly damping)
  – robust detection via cross-correlation?
- Multi-tracer probe of last-scattering
  - limited by noise/foregrounds, not cosmic variance
- Boosts large-scale polarization (except B modes from lensing)
- Must be modelled for consistency
- Powerful consistency check on recombination physics/expansion
- May be able to provide additional primordial information
  - mostly large-scale modes at recombination?

Question:

How well can foregrounds be removed/how large is uncorrelated foreground noise? (CIB, dust..)