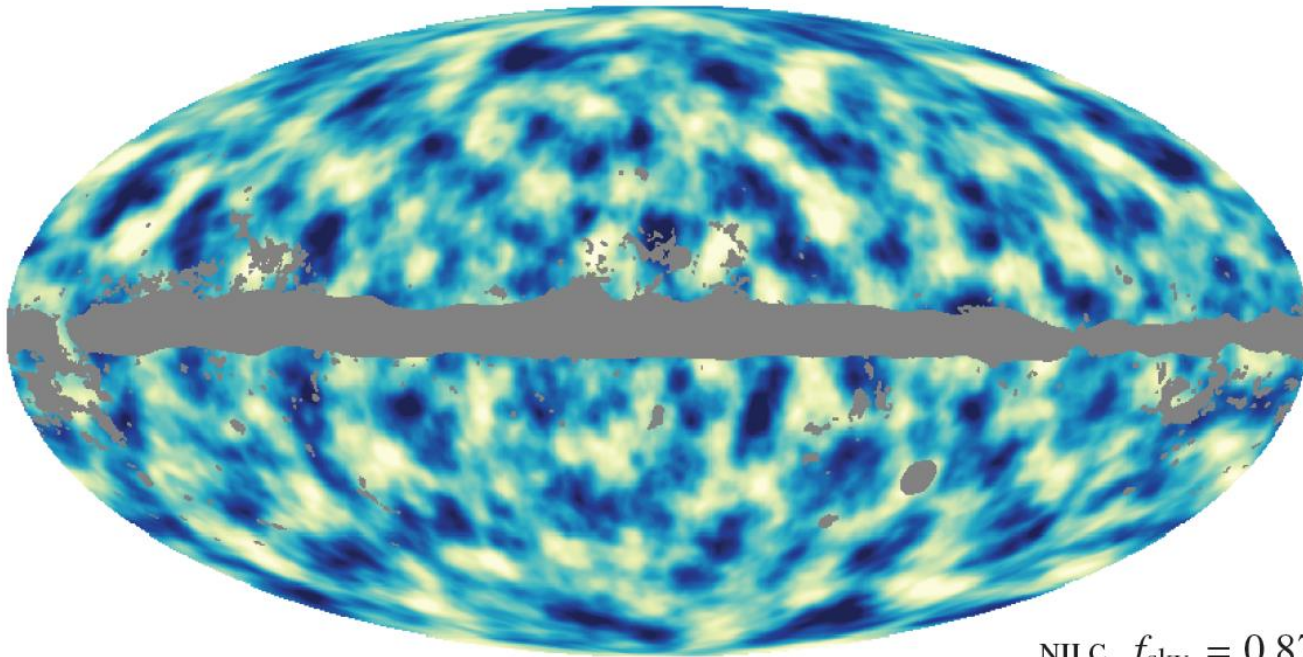


Lensing from Planck

Antony Lewis

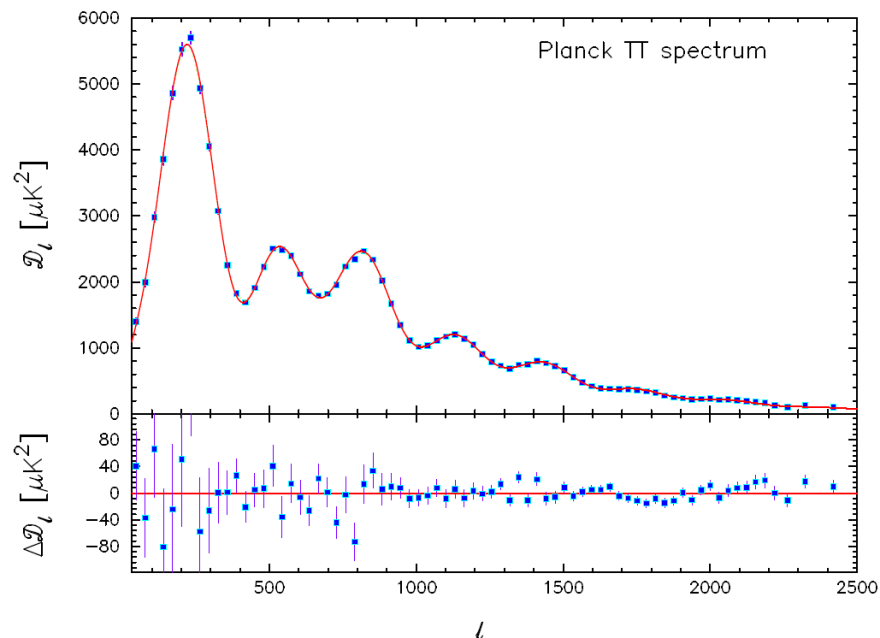
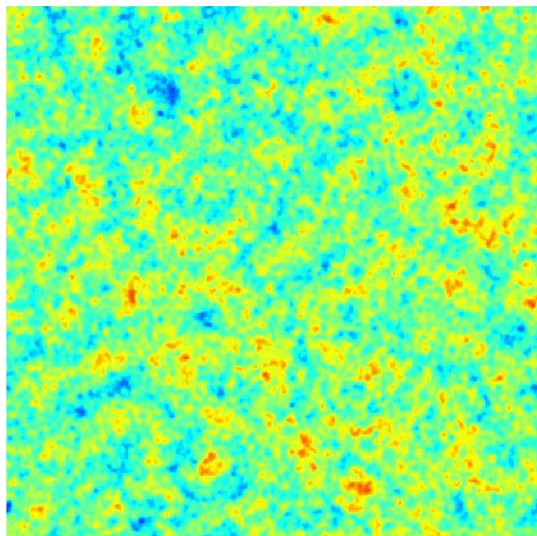
On behalf of the Planck Collaboration
(several slides credit Duncan Hanson)



NILC, $f_{\text{sky}} = 0.87$

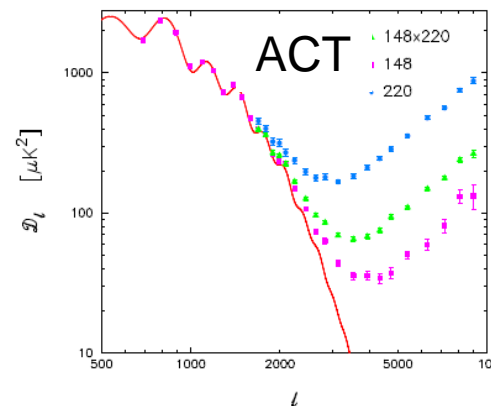
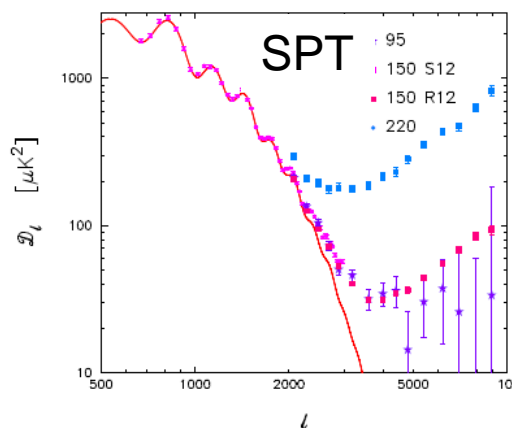
Observed CMB temperature power spectrum

Perturbations accurately linear and Gaussian at last-scattering
 - statistics completely described by the power spectrum C_l

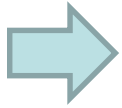


TT well-measured by Planck ($l < 2500$) and smaller scales by ACT and SPT ($l > 500$)

+ large foregrounds at $l \gg 2000$



Detailed measurement of 6 power spectrum acoustic peaks



Accurate measurement of cosmological parameters?

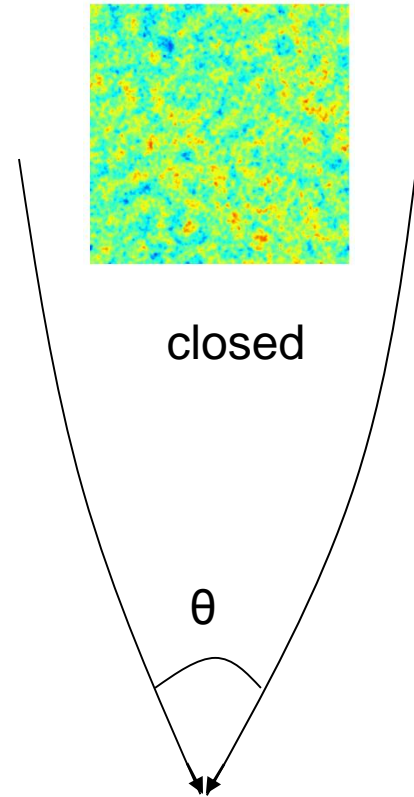
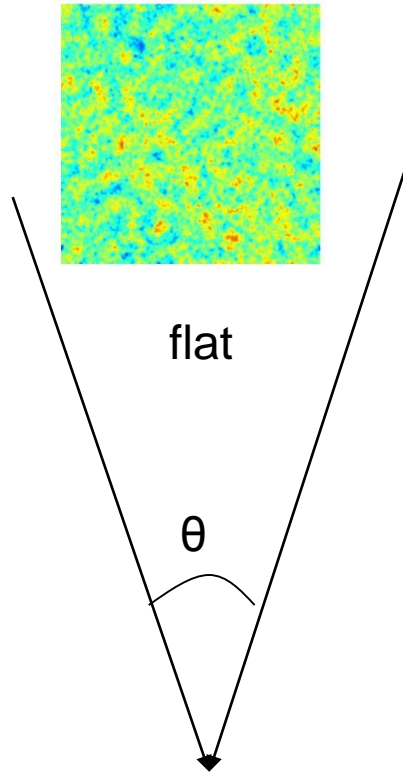
YES: some particular parameters measured very accurately

0.1% accurate measurement of the acoustic scale:

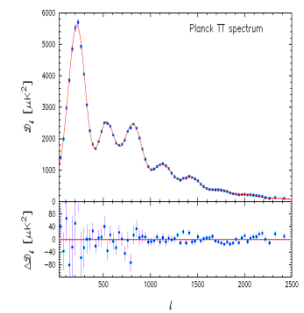
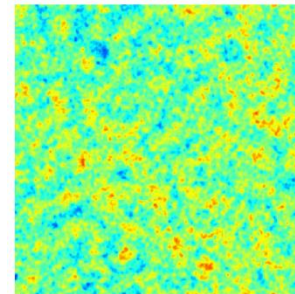
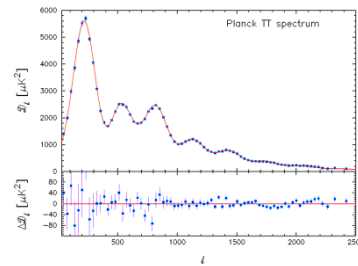
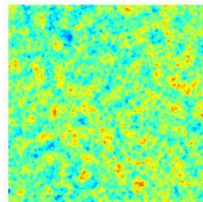
$$\theta_* = (1.04148 \pm 0.00066) \times 10^{-2} = 0.596724^\circ \pm 0.00038^\circ.$$

But need full cosmological model to relate to underlying physical parameters..

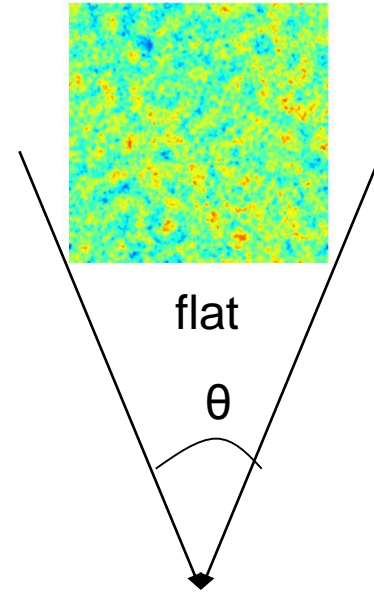
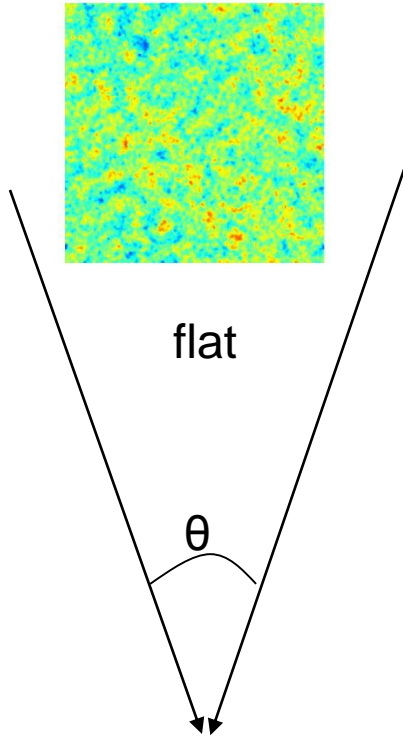
e.g. Geometry: curvature



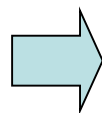
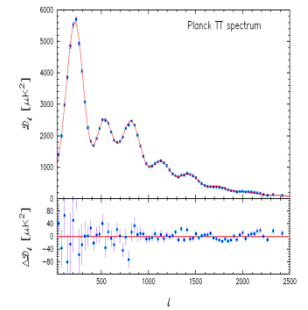
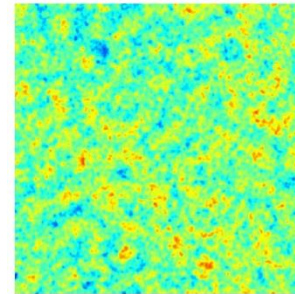
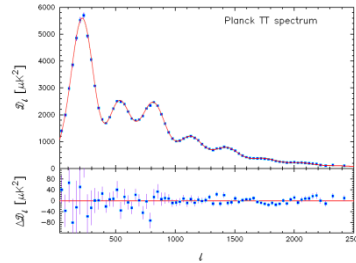
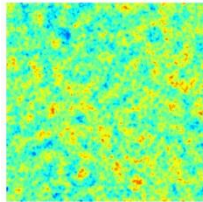
We see:



or is it just closer??



We see:



Degeneracies between parameters

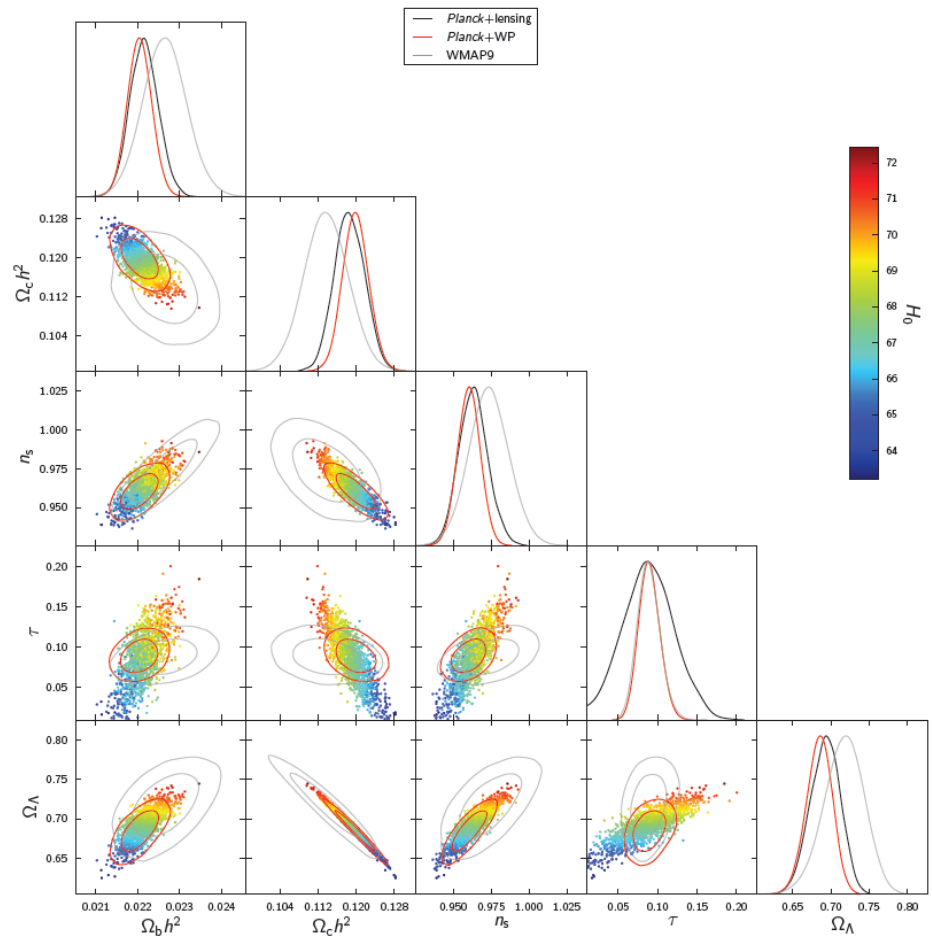
1. Assume a model

LCDM baseline model:

Flat, dark matter, cosmological constant, neutrinos, photons: six free parameters.

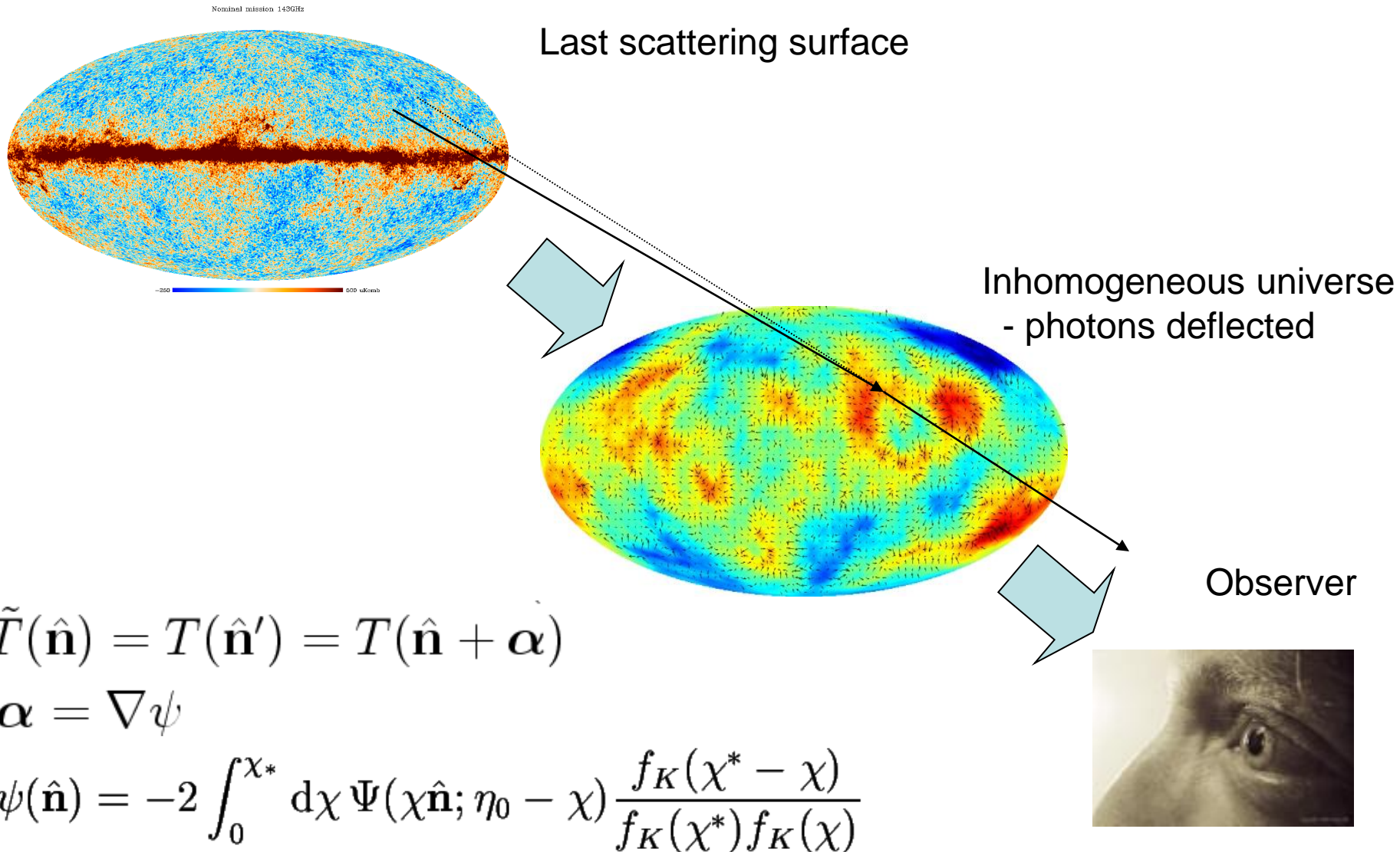
Assume 3 neutrinos, minimal-mass hierarchy with $\sum m_\nu = 0.06\text{eV}$.

Parameter	<i>Planck</i>	
	Best fit	68% limits
$\Omega_b h^2$	0.022068	0.02207 ± 0.00033
$\Omega_c h^2$	0.12029	0.1196 ± 0.0031
$100\theta_{\text{MC}}$	1.04122	1.04132 ± 0.00068
τ	0.0925	0.097 ± 0.038
n_s	0.9624	0.9616 ± 0.0094
$\ln(10^{10} A_s)$	3.098	3.103 ± 0.072
Ω_Λ	0.6825	0.686 ± 0.020
Ω_m	0.3175	0.314 ± 0.020
σ_8	0.8344	0.834 ± 0.027
z_{re}	11.35	$11.4^{+4.0}_{-2.8}$
H_0	67.11	67.4 ± 1.4

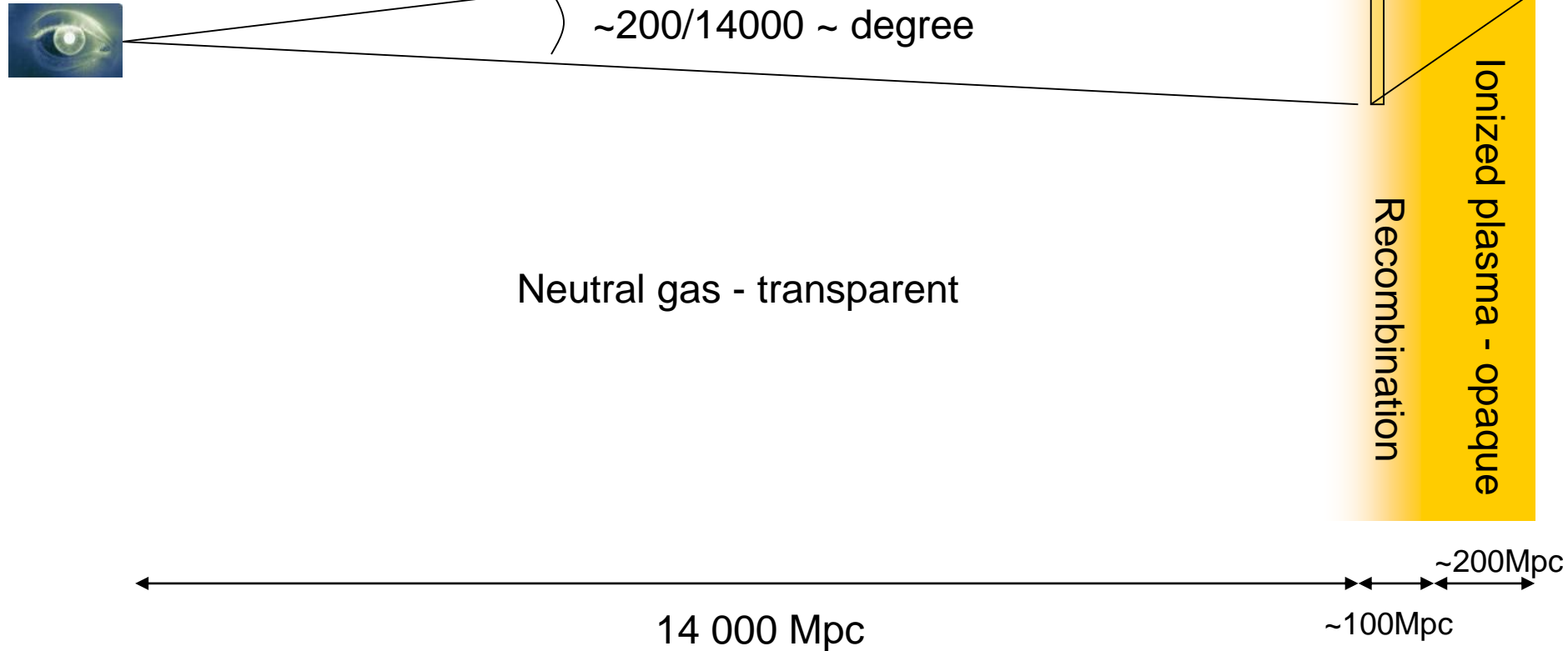


2. Use additional data or non-Gaussianities to break degeneracies

Weak lensing of the CMB perturbations

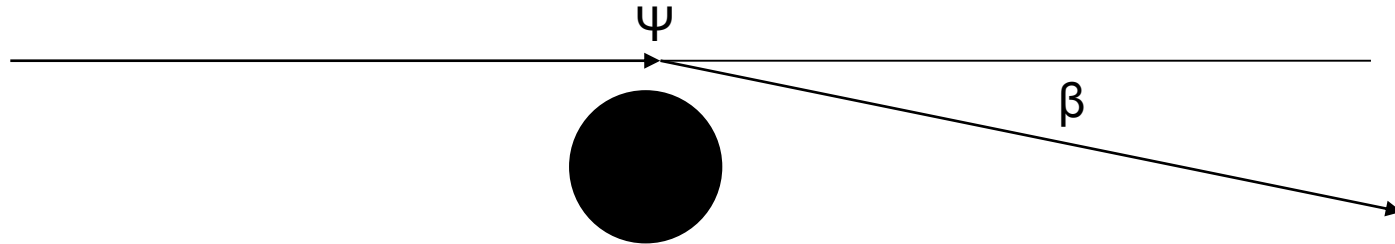


Not to scale!
All distances are comoving



Good approximation: CMB is single source plane at $\sim 14\,000\text{ Mpc}$
Angular diameter distance well measured by angle of acoustic peaks

Lensing order of magnitude



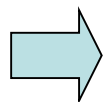
General Relativity: $\beta = 4 \Psi$ ($\beta \ll 1$)

Potentials linear and approx Gaussian: $\Psi \sim 2 \times 10^{-5} \Rightarrow \beta \sim 10^{-4}$

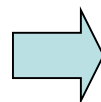
Potentials scale-invariant on large scales, decay on scales smaller than matter-power spectrum turnover:

\Rightarrow most abundant efficient lenses have size \sim peak of matter power spectrum $\sim 300\text{Mpc}$

Comoving distance to last scattering surface $\sim 14000 \text{ MPc}$



pass through ~ 50 lenses



assume uncorrelated

total deflection $\sim 50^{1/2} \times 10^{-4}$

~ 2 arcminutes

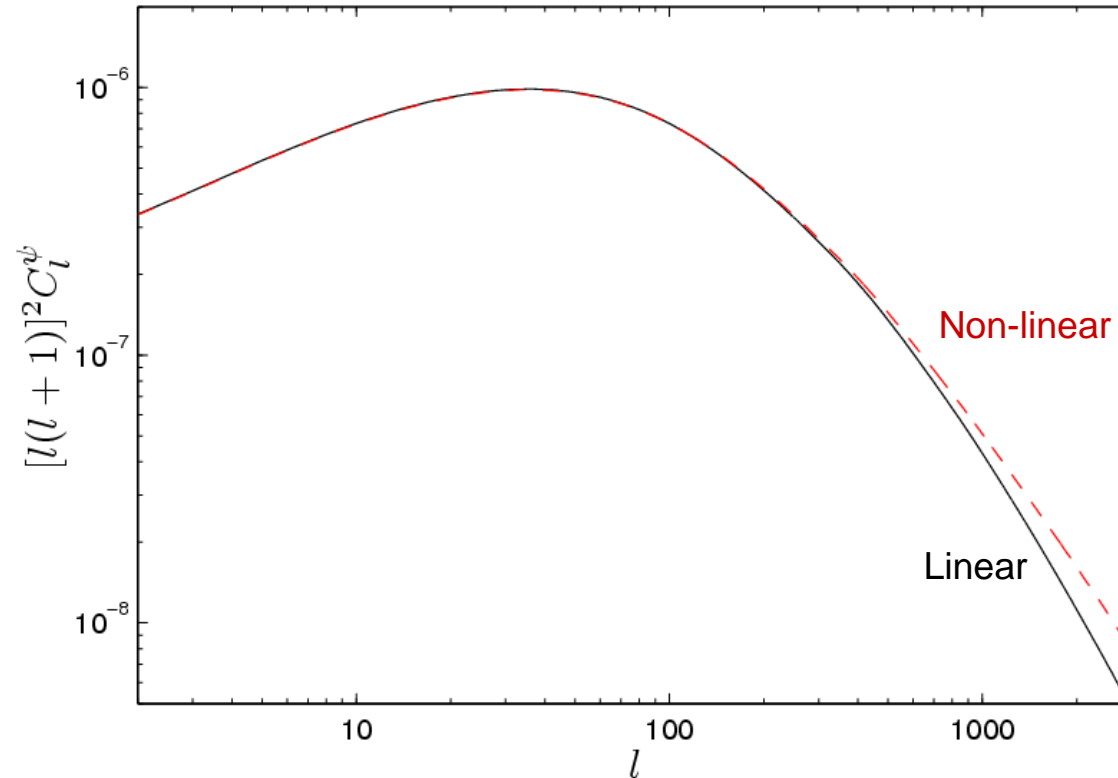
(neglects angular factors, correlation, etc.)

Why lensing is important

- 2arcmin deflections: $l \sim 3000$
 - On small scales CMB is very smooth so lensing dominates the linear signal at high l
- Deflection angles coherent over $300/(14000/2) \sim 2^\circ$
 - comparable to CMB scales
 - expect 2arcmin/60arcmin $\sim 3\%$ effect on main CMB acoustic peaks
- Non-linear: observed CMB is non-Gaussian
 - more information
 - potential confusion with primordial non-Gaussian signals
- Does not preserve E/B decomposition of polarization: e.g. $E \rightarrow B$
 - Confusion for primordial B modes (“r-modes”)
 - No primordial B \Rightarrow B modes clean probe of lensing

Potentials nearly linear \Rightarrow lensing potential nearly Gaussian

(also central limit theorem on small less-linear scales – lots of small lenses)



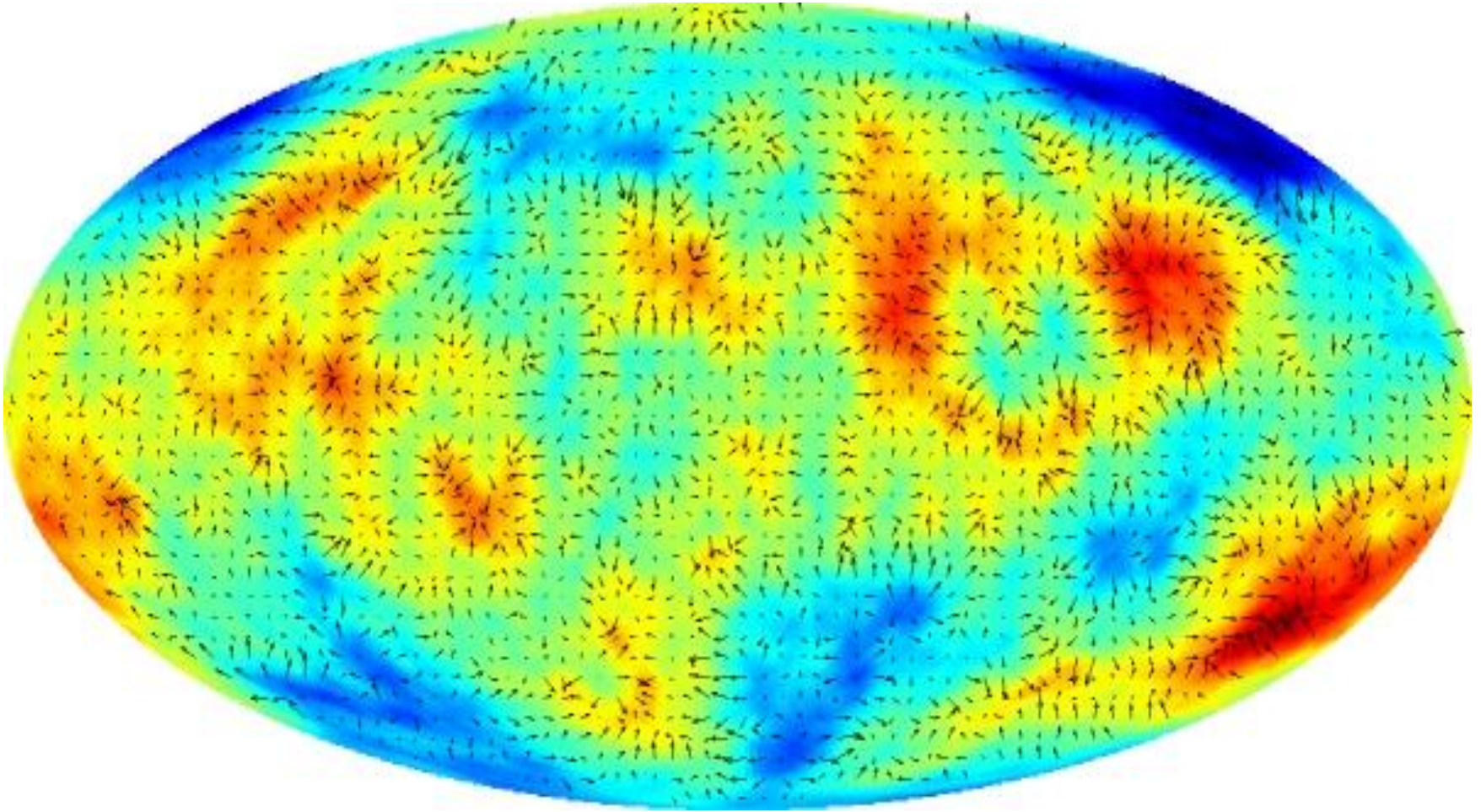
Note: In this talk
 $\psi = \phi$ in many places

Deflections $O(10^{-3})$, but coherent on degree scales

Probes matter distribution at roughly $0.5 < z < 6$ depending on l

Non-linear structure growth effects important but not a major headache

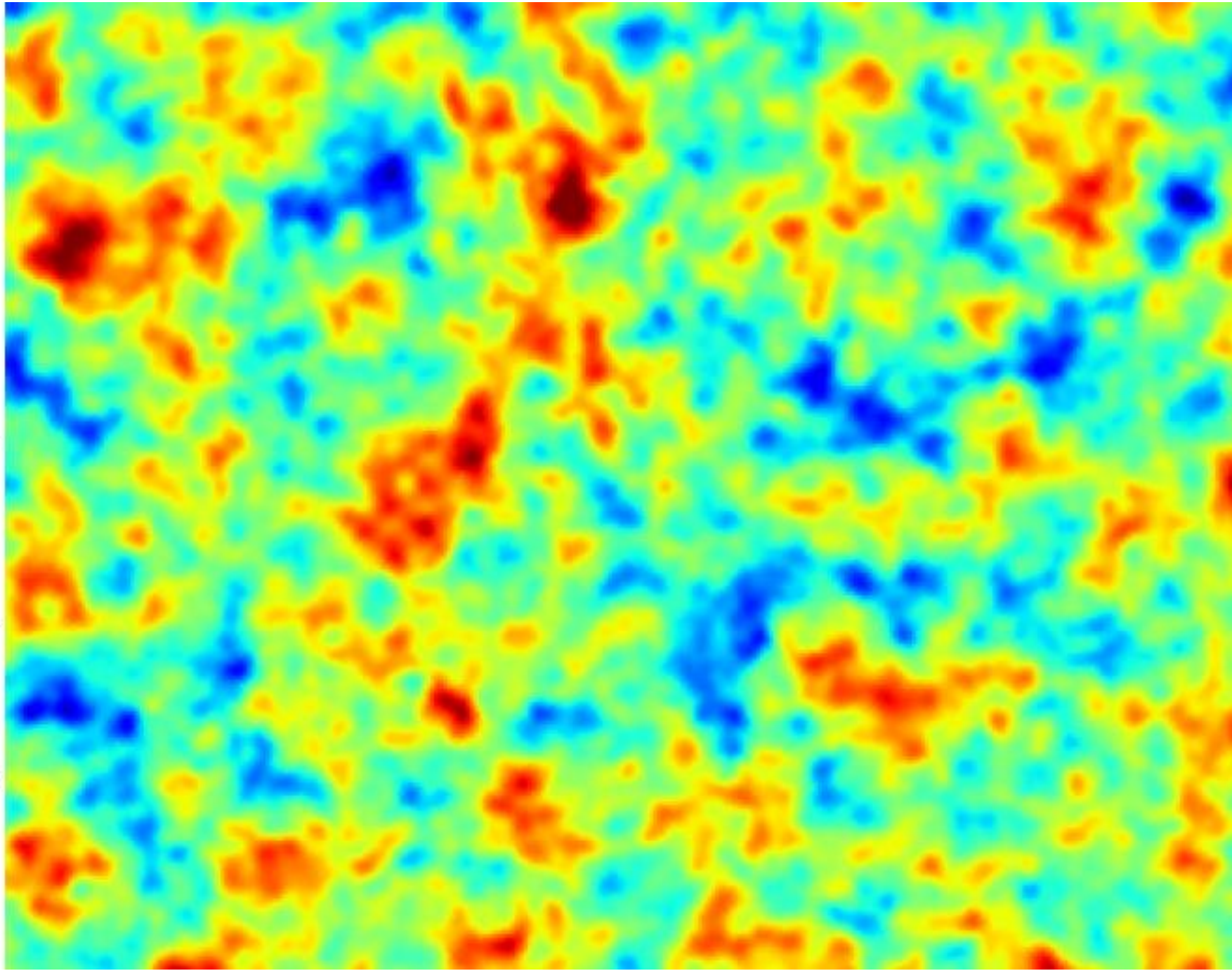
Simulated full sky lensing potential and (enlarged) deflection angle fields



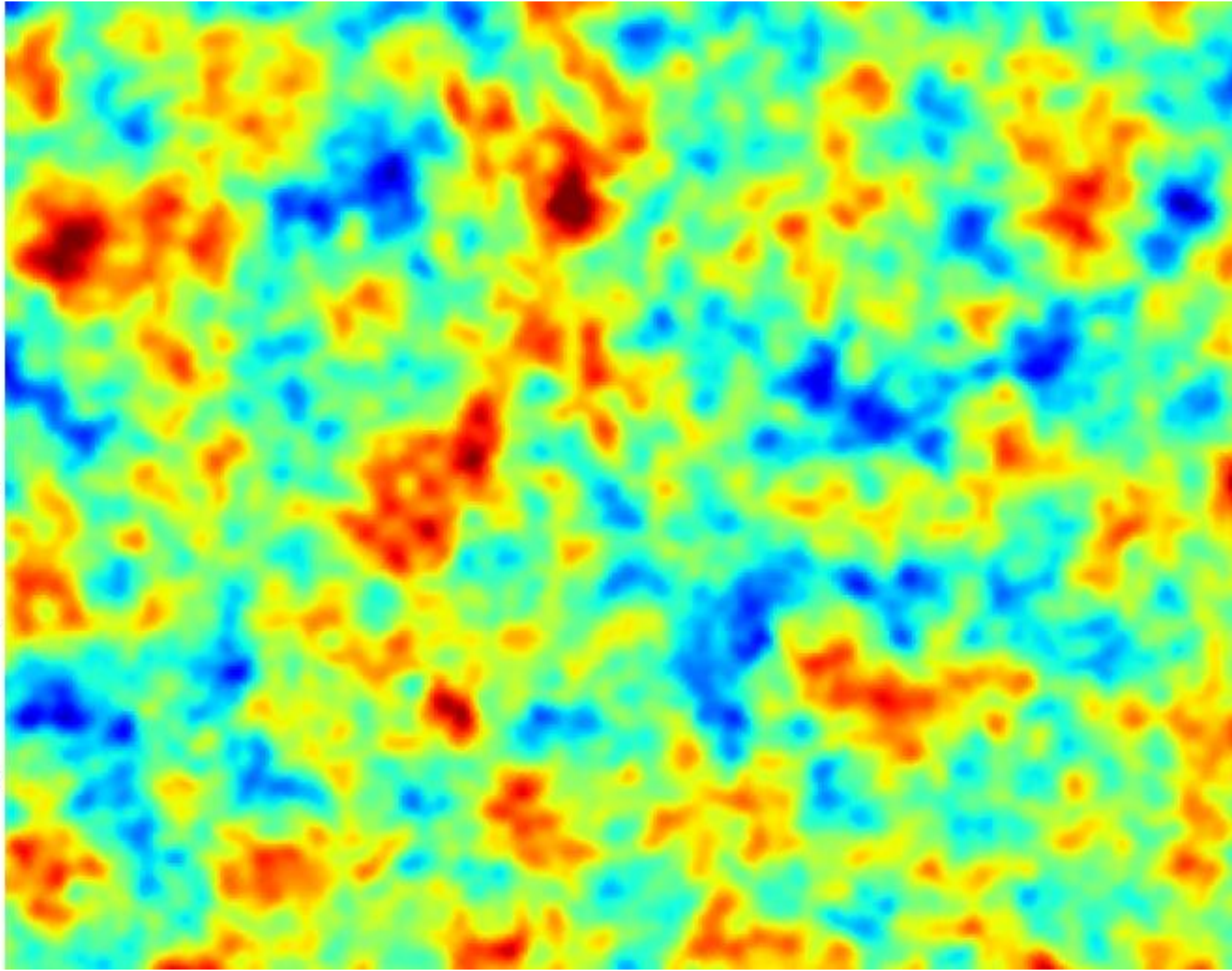
Easily simulated assuming Gaussian fields

- just re-map points using Gaussian realisations of CMB and potential

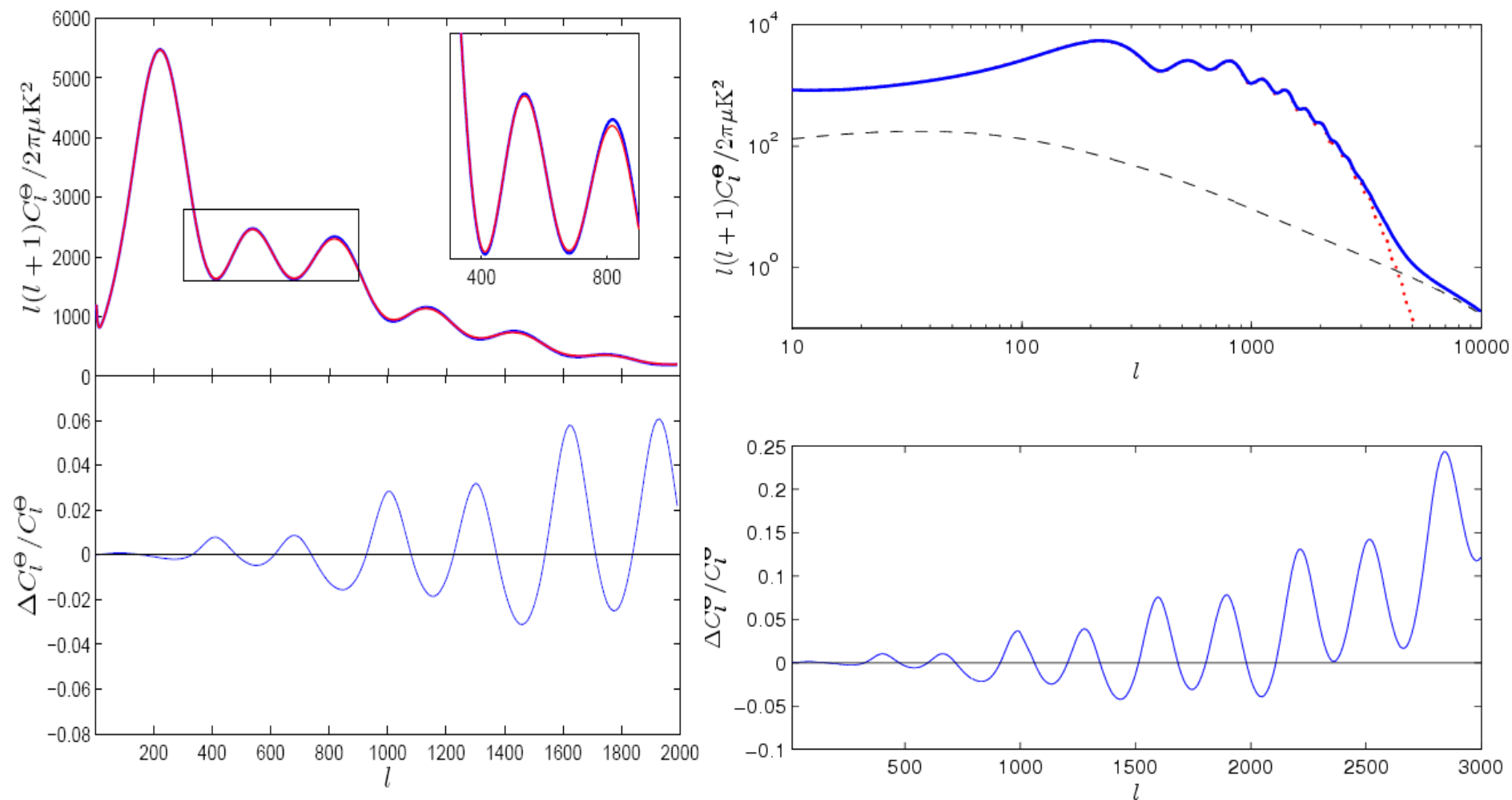
CMB Temperature (Unlensed)



CMB Temperature (Lensed)



Lensing effect on CMB temperature power spectrum

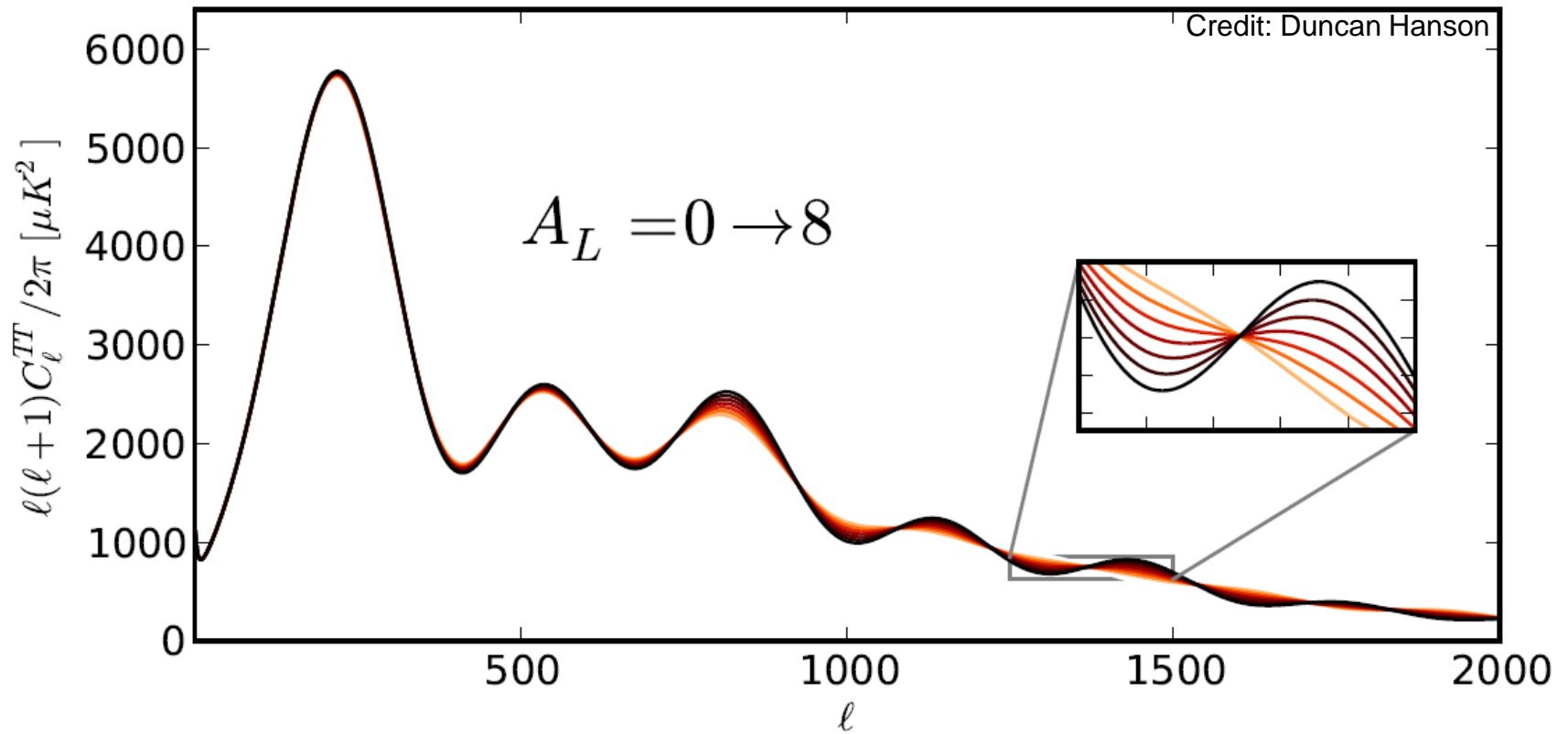


Important, but accurately modelled

Like a convolution with the deflection power spectrum: mainly from peak (L~60) scales

Consistency check, is amount of smoothing at the expected level: $A_L = 1$?

A_L defined so that lensing smoothing calculated using $A_L C_l^{\psi\psi}$ rather than physical $C_l^{\psi\psi}$



- Can we detect preference for $A_L > 0$ in the data?
- Marginalizing over A_L is also a way of “removing” lensing information from C_l

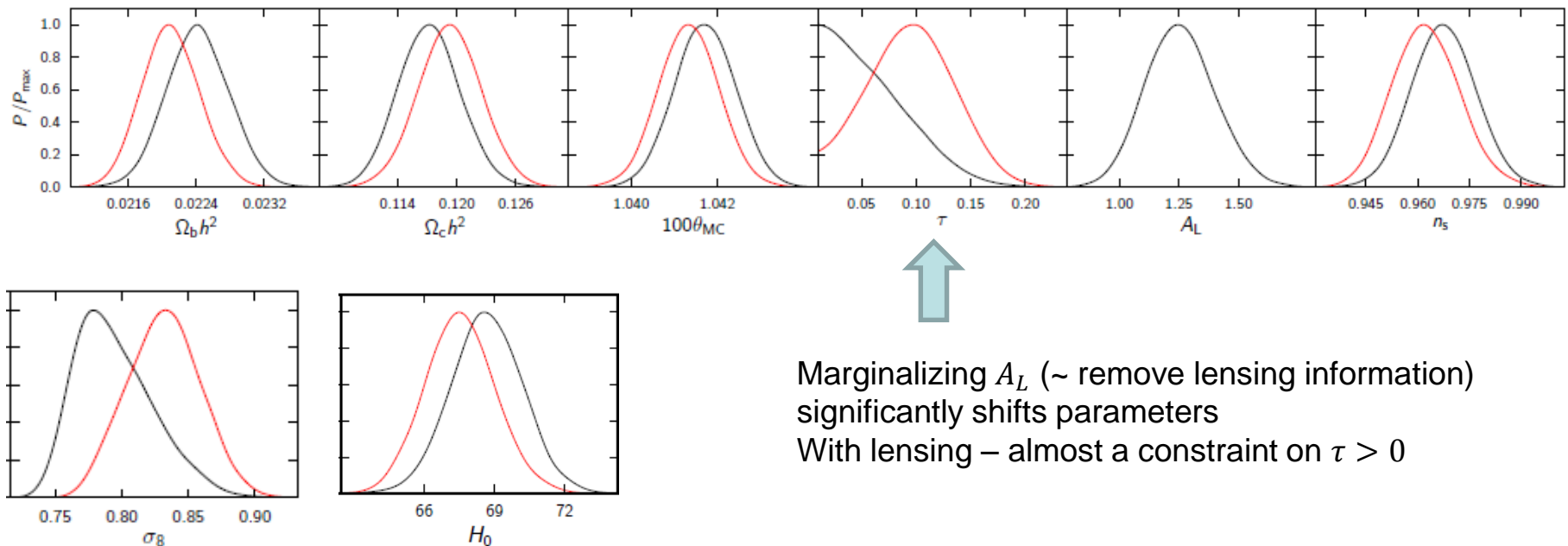
ΛCDM parameter constraints from Planck alone (no WMAP polarization)

Lensing in the power spectrum
is detected at high significance:

$$A_L = 1.28^{+0.29}_{-0.26} \text{ (} 2\sigma \text{)}$$

(actually a bit high??)

$A_L = 1$ (physical, main result) A_L varying (non-physical)



Marginalizing A_L (~ remove lensing information)
significantly shifts parameters
With lensing – almost a constraint on $\tau > 0$

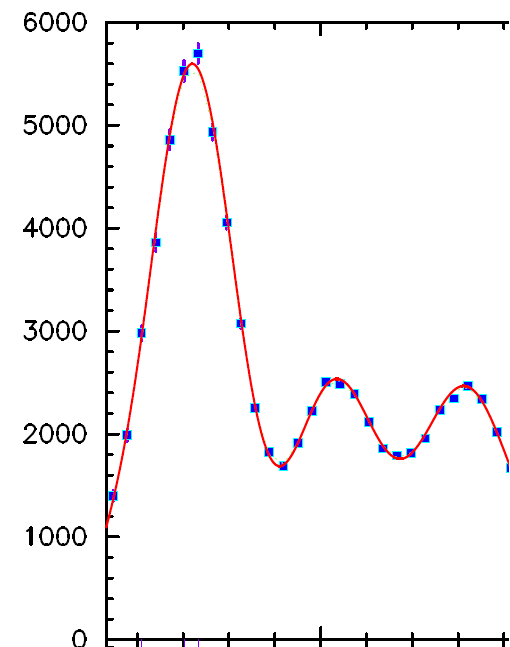
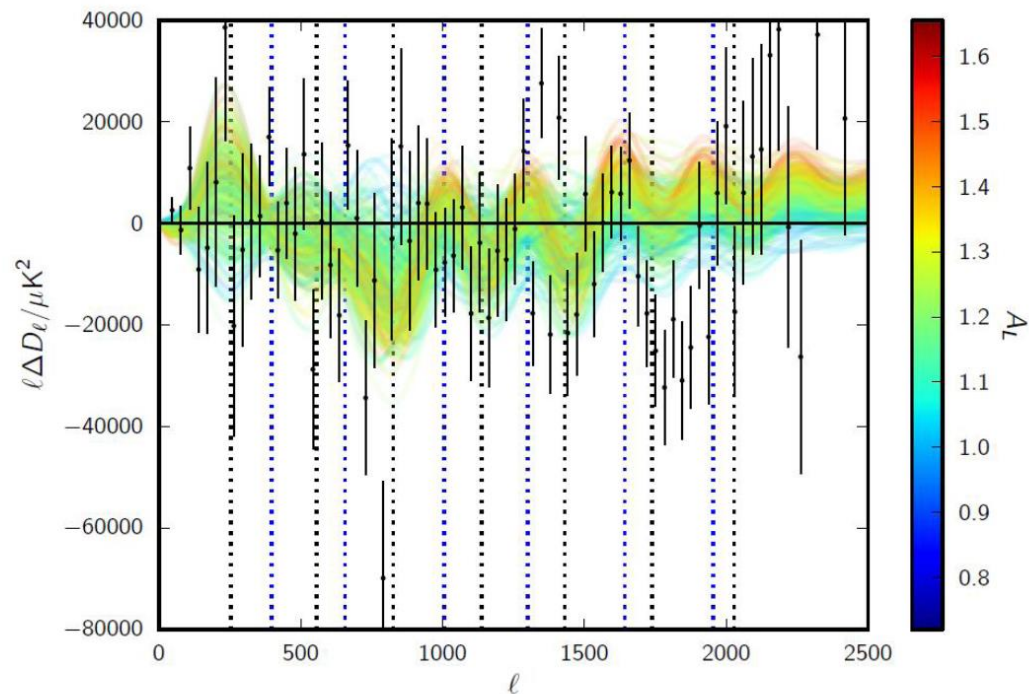
Oddity: power spectrum data seems to like high A_L

Preference for high A_L ?

Varies with data cuts, can be $> 3\sigma$ discrepant with $A_L = 1$!

Parameter	CamSpec	Plik	$l_{\max} = 2000$	$l_{\min} = 1200$	no 217×217
	95% limits	95% limits	95% limits	95% limits	95% limits
A_L	$1.23^{+0.22}_{-0.21}$	$1.26^{+0.26}_{-0.25}$	$1.38^{+0.26}_{-0.25}$	$1.31^{+0.24}_{-0.23}$	$1.30^{+0.24}_{-0.22}$

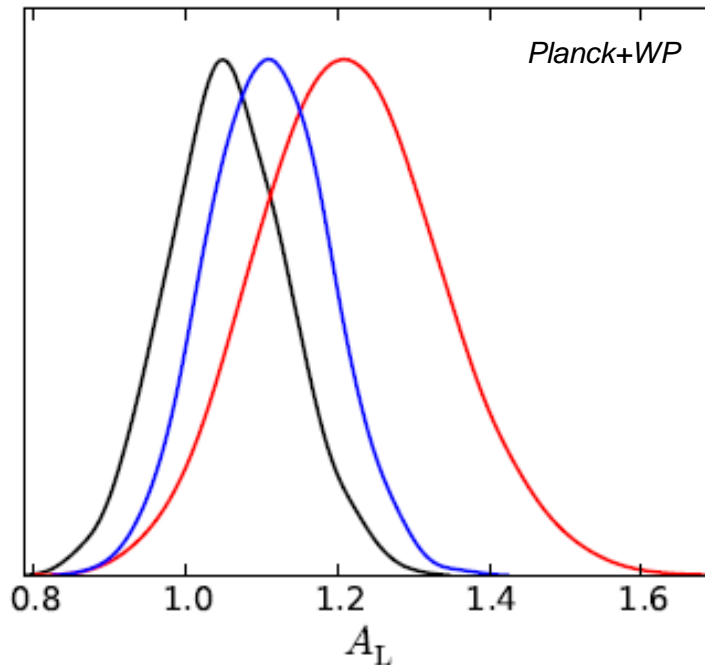
Planck+WP Alens diff to LCDM planck best fit



Why does Planck favour high (non-physical) A_L ?

- Actual preference for more peak smoothing
- Parameter degeneracies (higher A_L allows higher n_s , better fit at low l)

(note lower $\Omega_c h^2$ or $A_s \Rightarrow$ less lensing \Rightarrow higher A_L for same physical smoothing effect)



Vary A_L and 6 other LCDM parameters
(smoothing+degeneracies)

Vary A_L , all other cosmological parameters
fixed to LCDM best fit with $A_L = 1$
(really measuring smoothing wrt best fit)

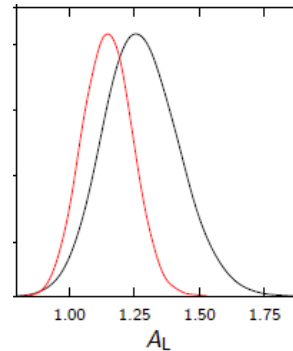
Vary A_L , all other cosmological parameters
fixed to LCDM best fit when low- l replaced by τ prior
(measuring smoothing wrt best fit which is not
pulled by low low- l power: higher $\Omega_c h^2$ and hence lensing in reference model)

Similarly, could define alternative A'_L to scale amplitude of lensing power w.r.t. fixed value (rather than ratio to physical in each model). E.g.

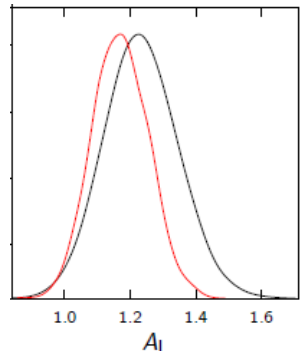
$$A'_L = \frac{\max(l^4 C_l^{\phi\phi})}{8.7 \times 10^{-7}} \quad (\text{ratio to LCDM Planck+WP best fit lensing power})$$

$$A_L = \frac{C_l^{\phi\phi}}{C_l^{\phi\phi}(\text{physical})}$$

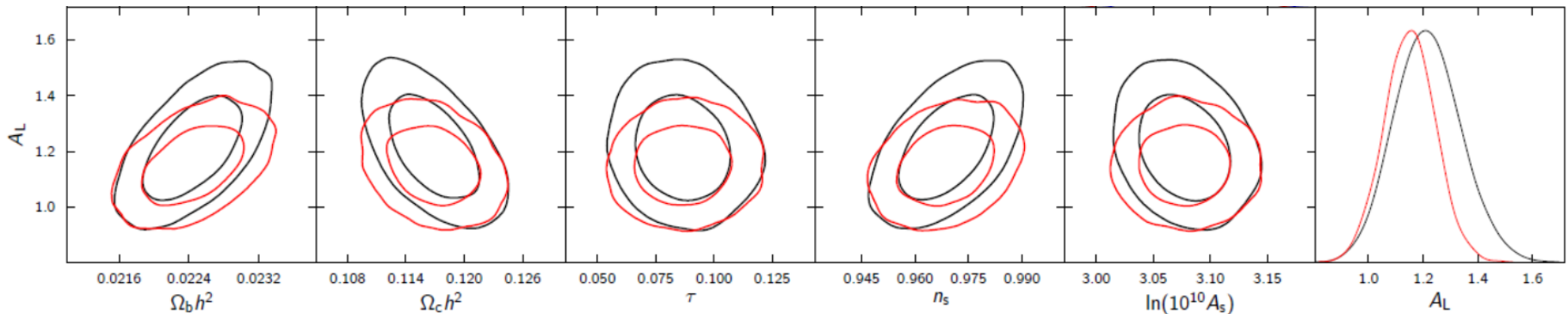
Planck only



Planck+WP+highL



Planck+WP



- High A_L preference is partly driven by degeneracies
- But also some preference for more lensing smoothing
- High A_L models do have significantly better fit
(may be hint that LCDM is not good fit, systematic, or lensing amplitude is wrong)

Really need to measure the lensing amplitude more directly...

Non-Gaussianity/statistical anisotropy reconstructing the lensing field

Marginalized over (unobservable) lensing field:

$$T \sim \int P(T, \psi) d\psi$$

- Non-Gaussian statistically isotropic temperature distribution

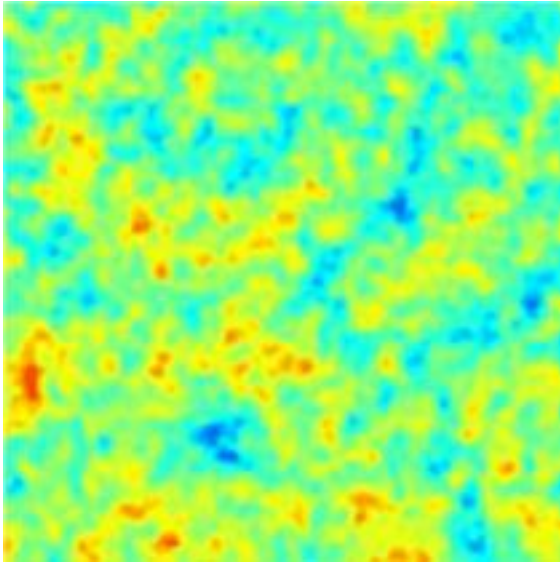
For a given lensing field :

$$T \sim P(T|\psi)$$

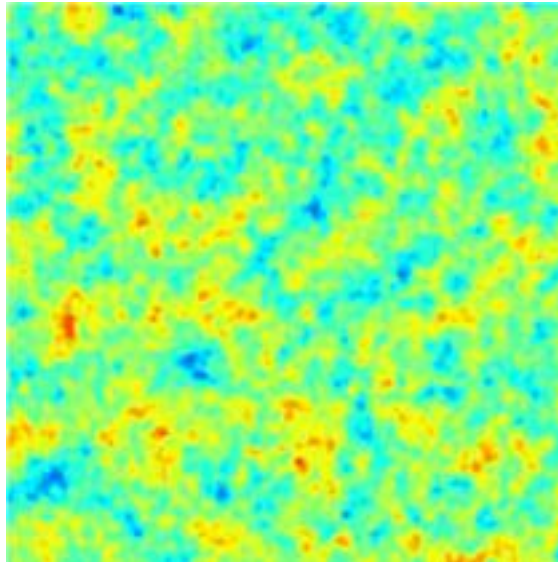
- Anisotropic Gaussian temperature distribution

Fractional magnification \sim convergence $\kappa = -\nabla \cdot \alpha/2$

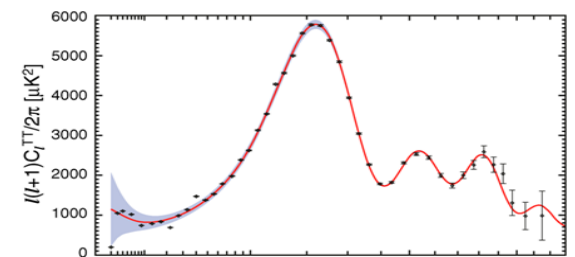
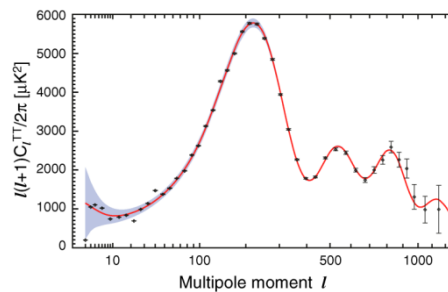
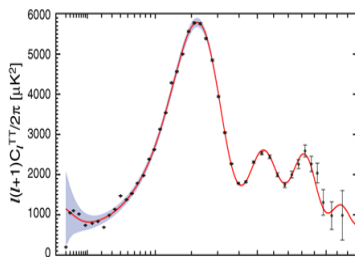
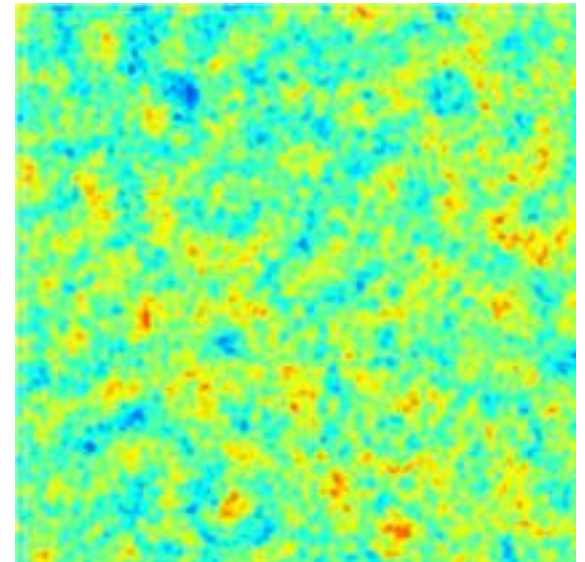
Magnified



Unlensed



Demagnified

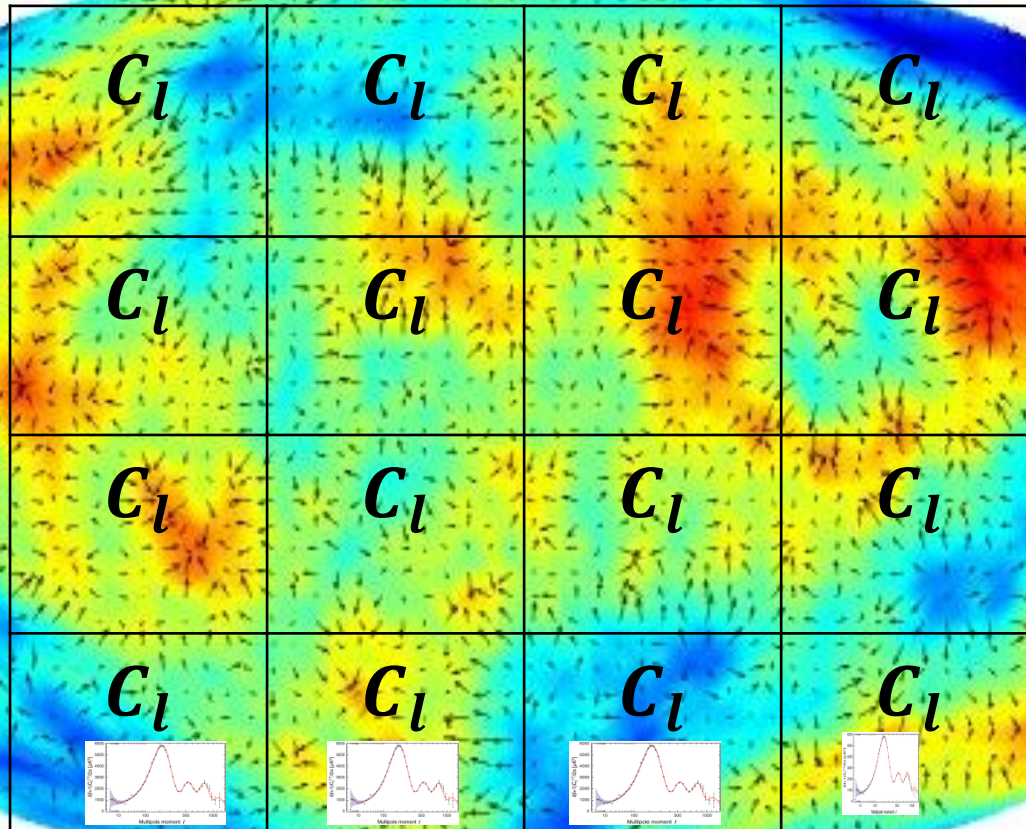


+ shear modulation

$$\langle \tilde{T}(l_2) \tilde{T}(l_3) \rangle = C_{l_2}^{TT} \delta(l_2 + l_3) \left[1 + \kappa \frac{d \ln(l_2^2 C_{l_2}^{TT})}{d \ln l_2} + \hat{l}_2^T \gamma \hat{l}_2 \frac{d \ln C_{l_2}^{TT}}{d \ln l_2} \right]$$

Lensing reconstruction

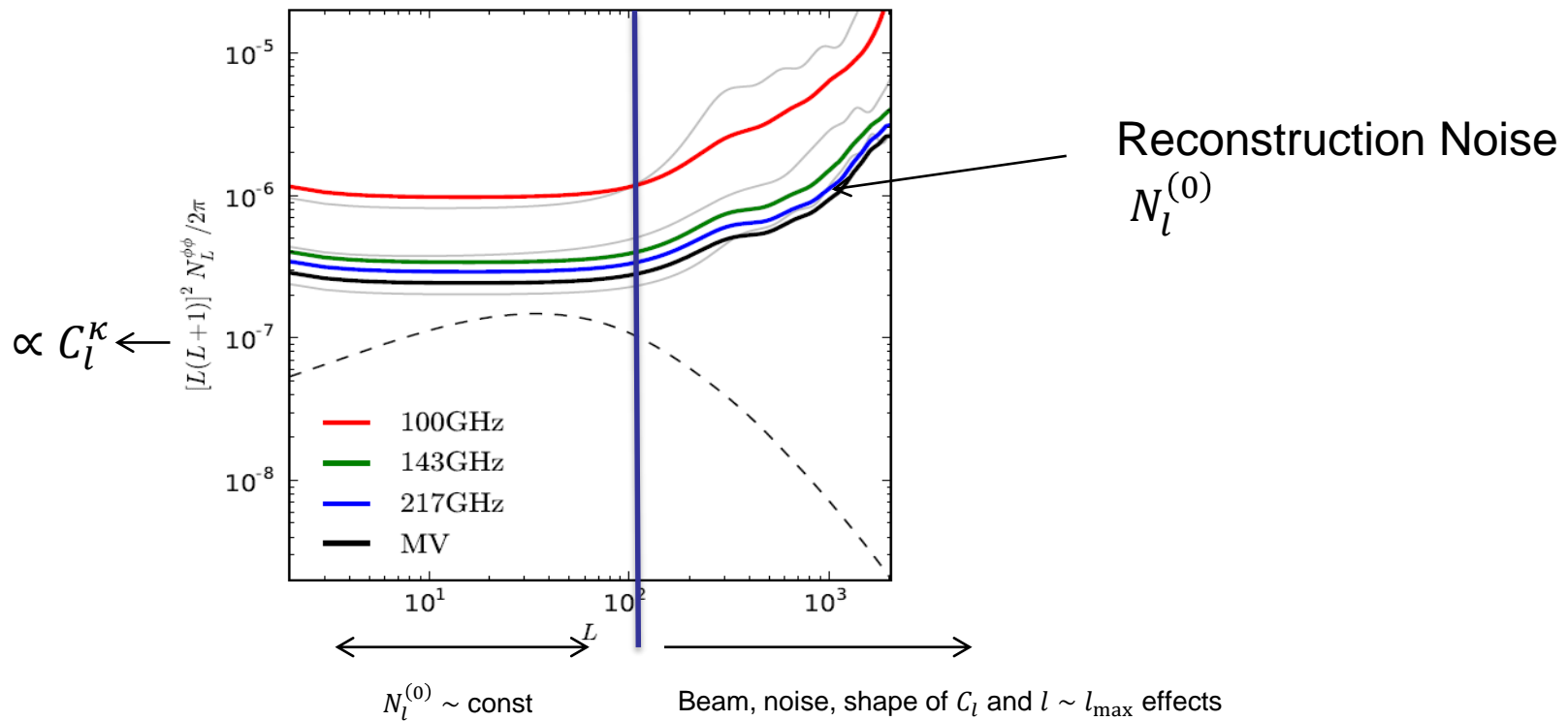
-concept



Variance in each C_l measurement $\propto 1/N_{\text{modes}}$

$N_{\text{modes}} \propto l_{\text{max}}^2$ - dominated by smallest scales

- \Rightarrow measurement of angular scale ($\Rightarrow \kappa$) in each box nearly independent
- \Rightarrow Uncorrelated variance on estimate of magnification κ in each box
- \Rightarrow Nearly white 'reconstruction noise' $N_l^{(0)}$ on κ , with $N_l^{(0)} \propto 1/l_{\text{max}}^2$



Lensing reconstruction information mostly in the *smallest scales* observed

- Need high resolution and sensitivity
- Almost totally insensitive to large-scale T (so only *small-scale* foregrounds an issue)
 - Use separate frequencies and check consistency
 - Combine (Minimum Variance – MV) for best estimate
 - Also cross-check with foreground cleaned maps

Lensing reconstruction

- Maths and algorithm sketch

For a *given* (fixed) lensing field, $T \sim P(T|X)$:

X here is lensing potential, deflection angle, or κ

Flat sky approximation: modes correlated for $\mathbf{k}_2 \neq \mathbf{k}_3$

First-order series expansion in the lensing field:

$$\langle \tilde{T}(\mathbf{k}_2) \tilde{T}(\mathbf{k}_3) \rangle_{P(\tilde{T}|X)} \approx \int d\mathbf{K} X(\mathbf{K})^* \underbrace{\left\langle \frac{\delta}{\delta X(\mathbf{K})^*} \left(\tilde{T}(\mathbf{k}_2) \tilde{T}(\mathbf{k}_3) \right) \right\rangle}_{\mathcal{A}(K, k_2, k_3) \delta(K + k_2 + k_3)} \approx \mathcal{A}(K, k_2, k_3) X(\mathbf{K})^*|_{\mathbf{K} = -\mathbf{k}_2 - \mathbf{k}_3}$$

function easy to calculate for $X(\mathbf{K}) = 0$

$$A(L, l_1, l_2) \sim (\mathbf{l}_1 \cdot \mathbf{L} \tilde{C}_{l_1} + \mathbf{l}_2 \cdot \mathbf{L} \tilde{C}_{l_2})$$

Can reconstruct the modulation field X

Full sky analysis similar, summing modes with optimal weights gives

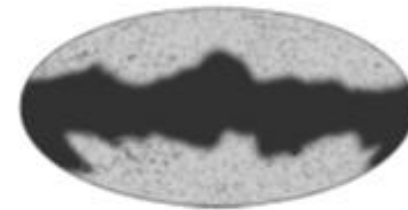
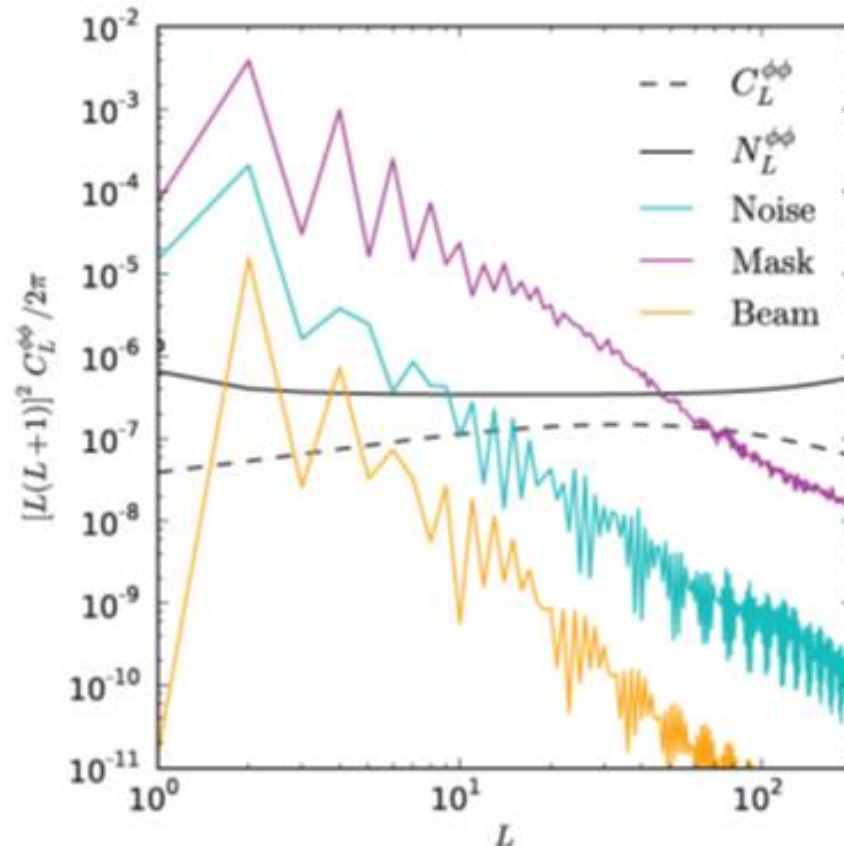
$$\hat{\psi}_{l_1 m_1}^* = N_{l_1}^{(0)} \sum_{l_2 l_3}^{l_1 \leq l_2 \leq l_3} \Delta_{l_1 l_2 l_3}^{-1} \mathcal{A}_{l_1 l_2 l_3}^{TT} \sum_{m_2 m_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \frac{\tilde{T}_{l_2 m_2} \tilde{T}_{l_3 m_3}}{\tilde{C}_{\text{tot } l_2}^{TT} \tilde{C}_{\text{tot } l_3}^{TT}}$$

On realistic cut sky can still construct “optimal” quadratic (QML) estimator $\hat{\phi}(K)$

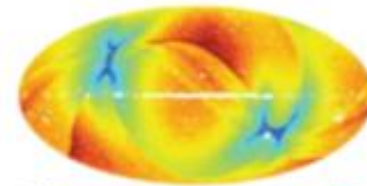
$$\hat{\phi}(L) \sim N_L^{(0)} [\sum_{\mathbf{k}_2, \mathbf{k}_3} A(L, k_2, k_3) \bar{T}(\mathbf{k}_2) \bar{T}(\mathbf{k}_3) - (\text{Monte carlo for zero signal})]$$

Filtered maps $\bar{T} = (S + N)^{-1}T$
weights for mask and noise anisotropy
(here approx. noise term as diagonal)

“Mean field” – accounts for other sources of anisotropy in the data

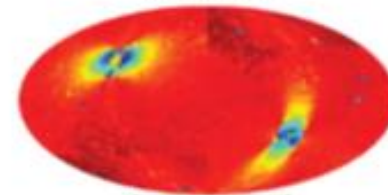


Mask



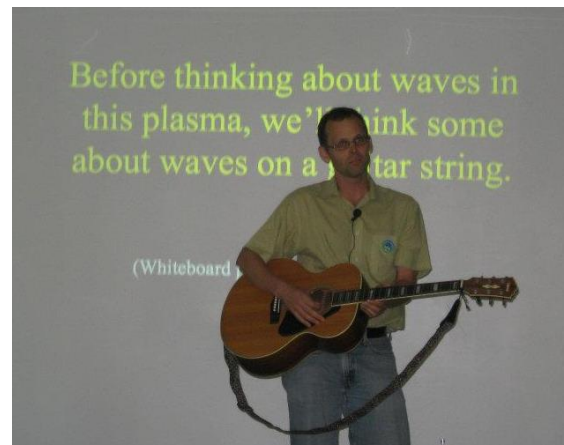
noise RMS

Ellipticity = 100 GHz



Beam ellipticity

- “Mean-field” corrections are very large at low- L . We fail some detailed consistency tests at $L < 10$ (though not very badly!).

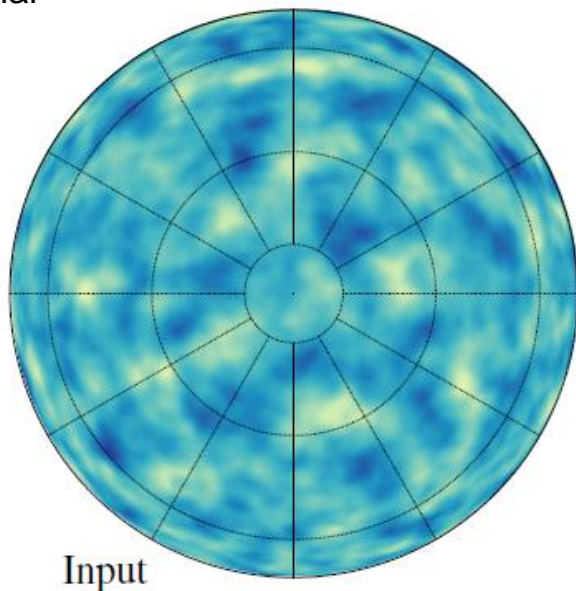


Can also re-write in as fast real-space estimator

$$\hat{\alpha}_{LM} \propto (\bar{T} \nabla W)_{LM} \quad W = S \bar{T} = S(S + N)^{-1} T \text{ is Wiener-filtered input T map}$$

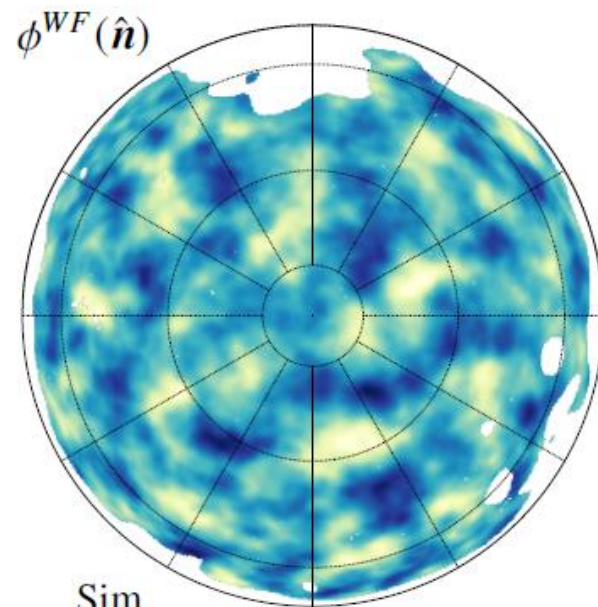
Planck simulation

True lensing potential



Input

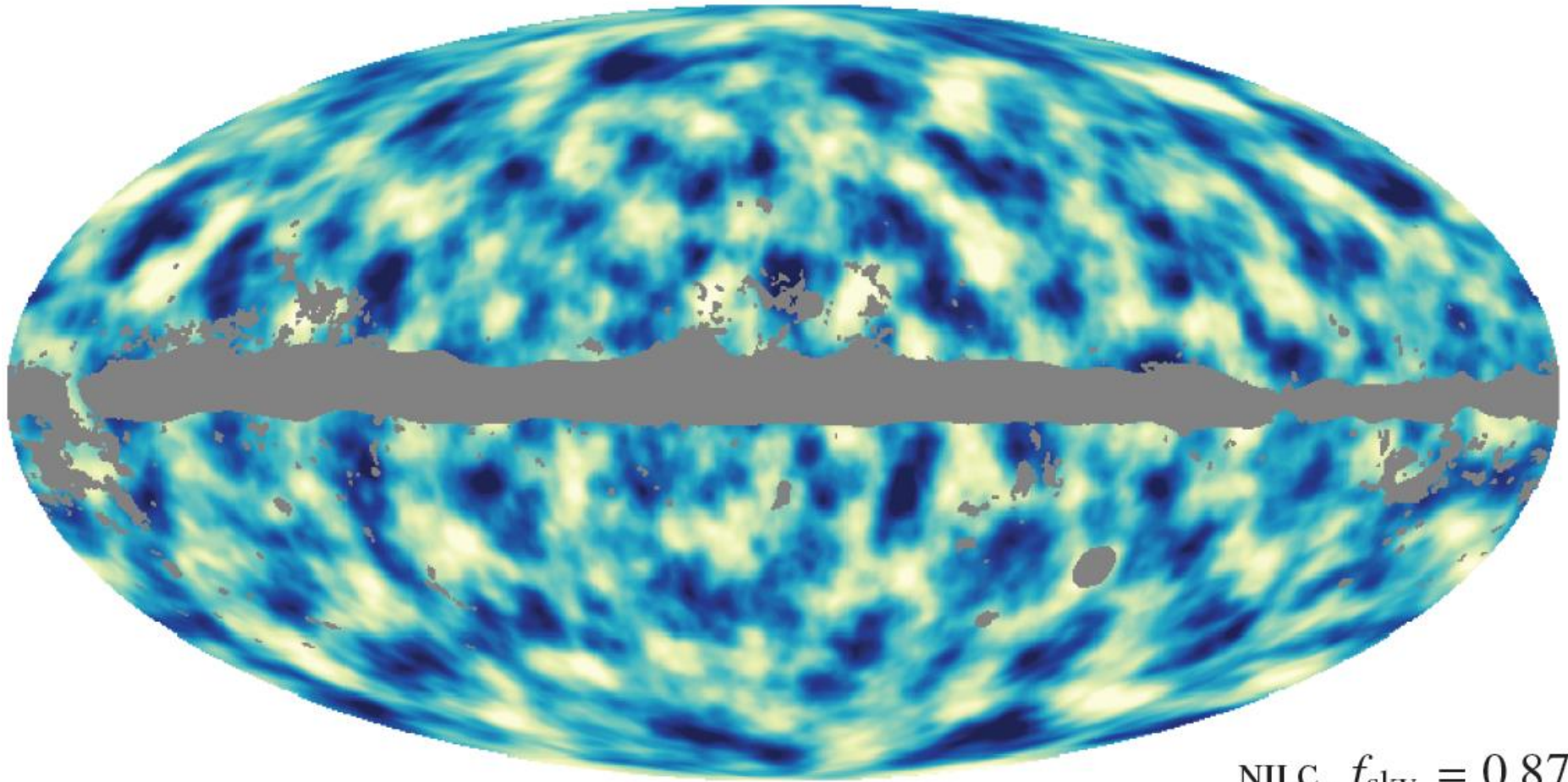
$S/(S+N)$ filtered reconstruction



Sim

= input + 'reconstruction noise'

Planck full-sky lensing potential reconstruction

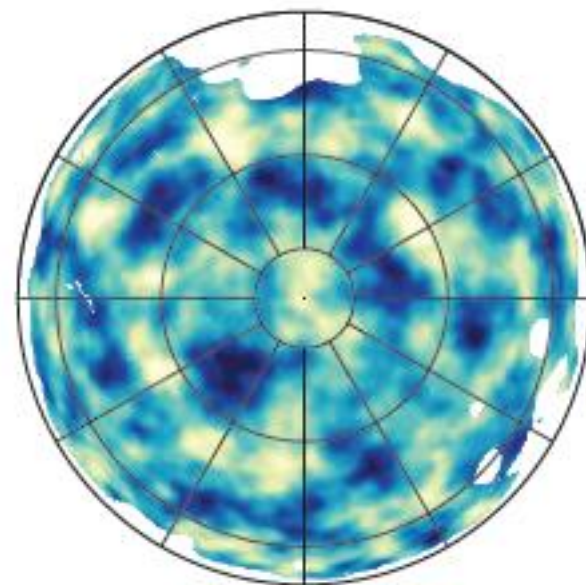
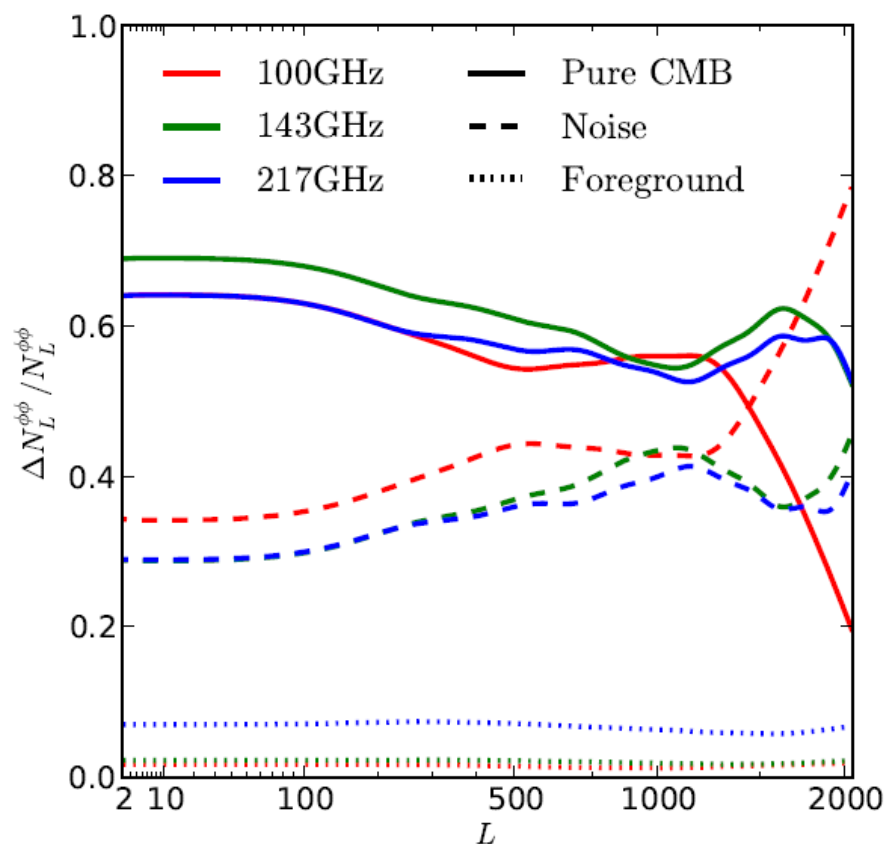


NILC, $f_{\text{sky}} = 0.87$

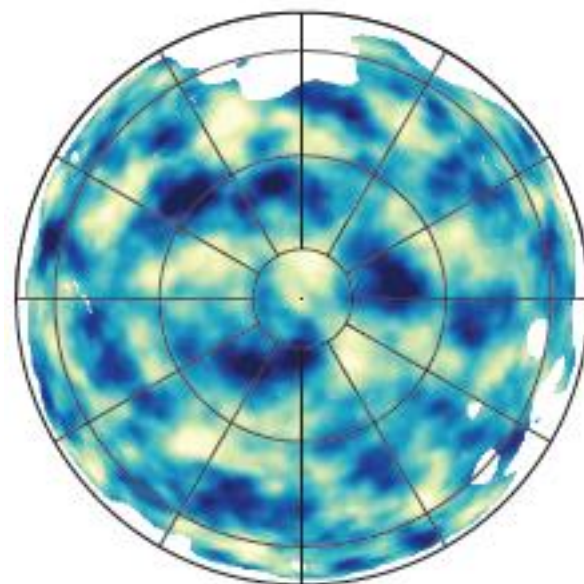
Note – about half signal, half noise,
not all structures are real: map is effectively Wiener filtered

Reconstruction noise budget

Lensing maps are reconstruction noise dominated, but maps from different channels are similar because mainly the same CMB cosmic variance.

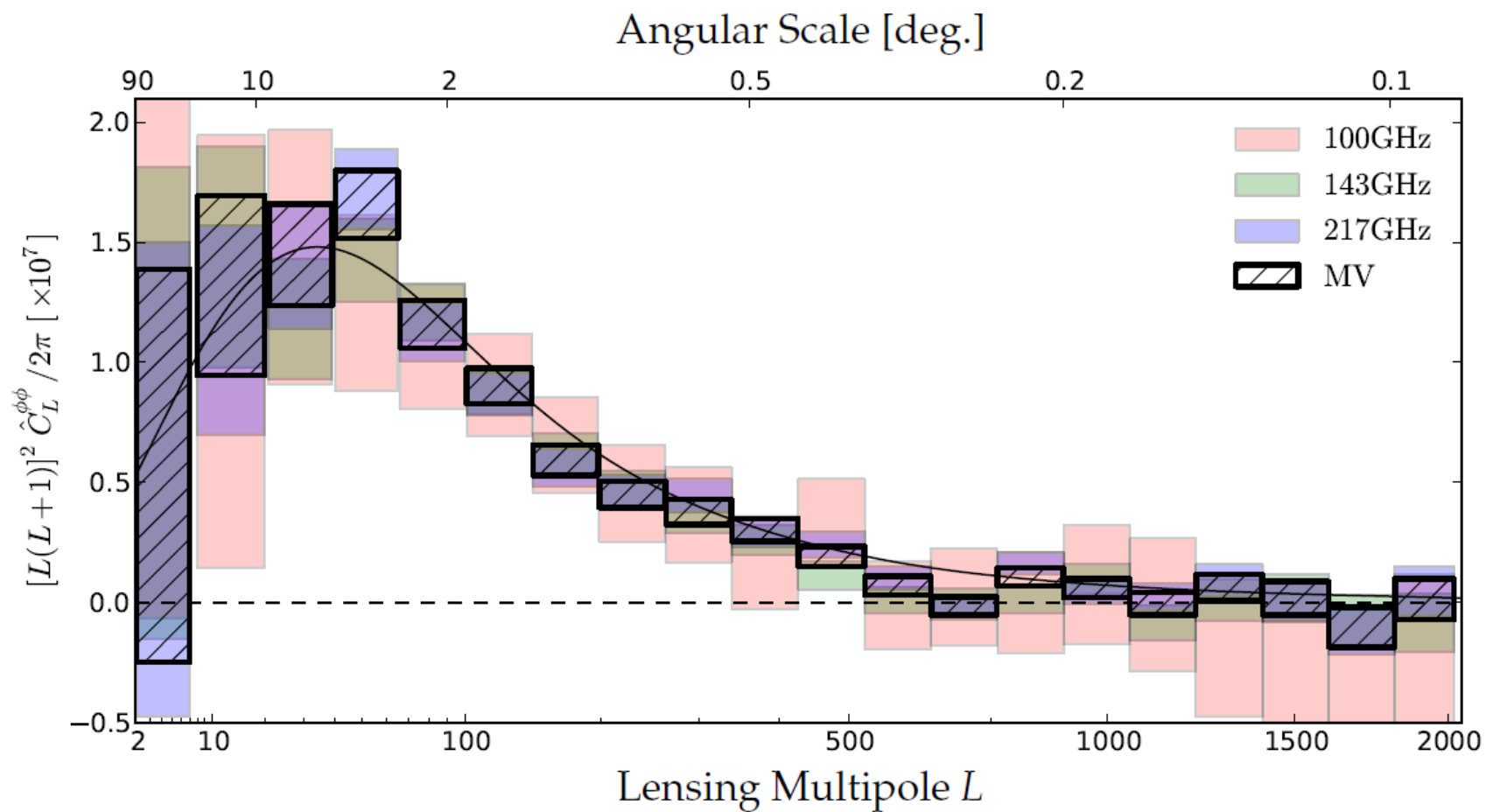


Galactic South - 143 GHz

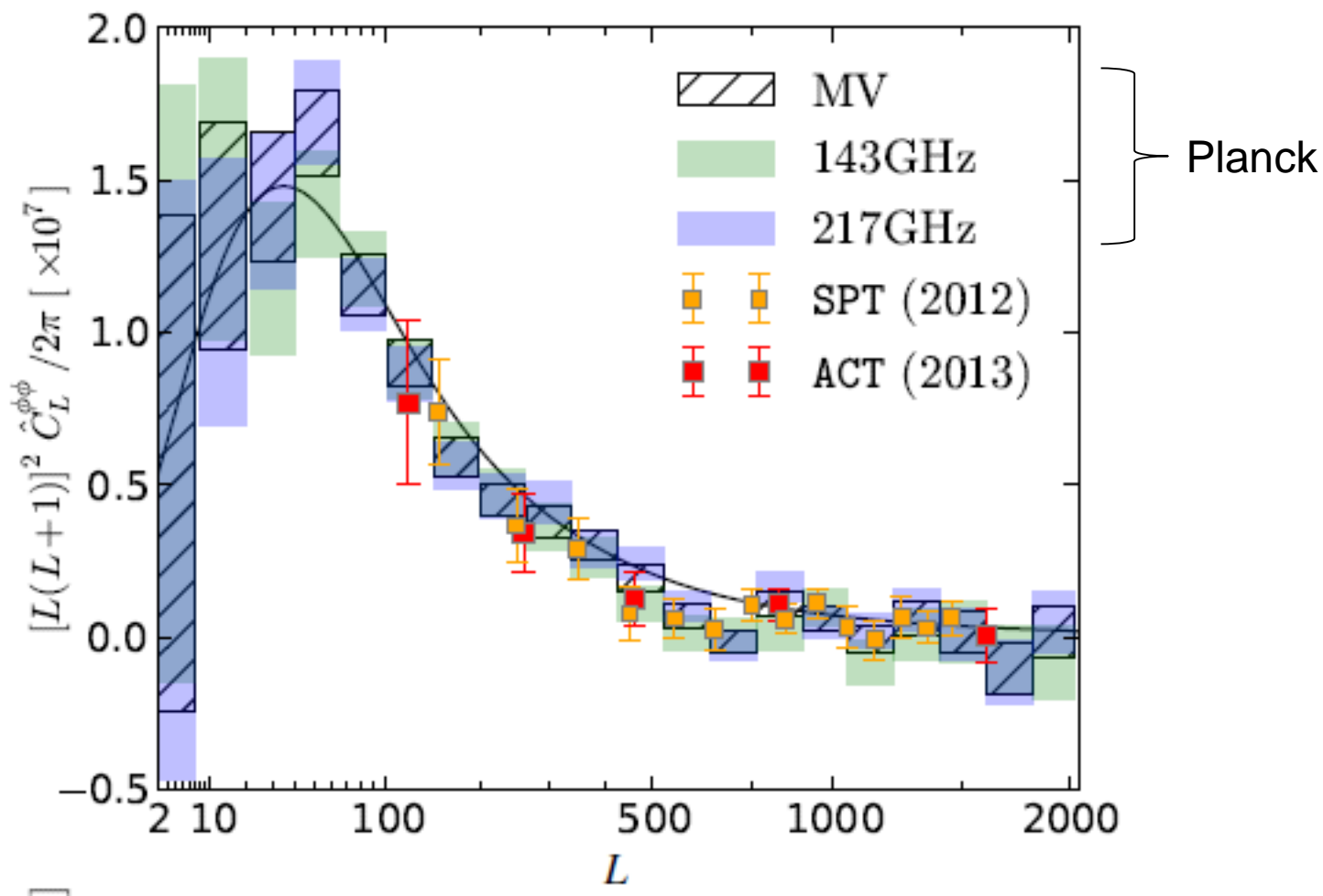


Galactic South - 217 GHz

Power spectrum of reconstruction $\Rightarrow C_l^{\psi\psi}$

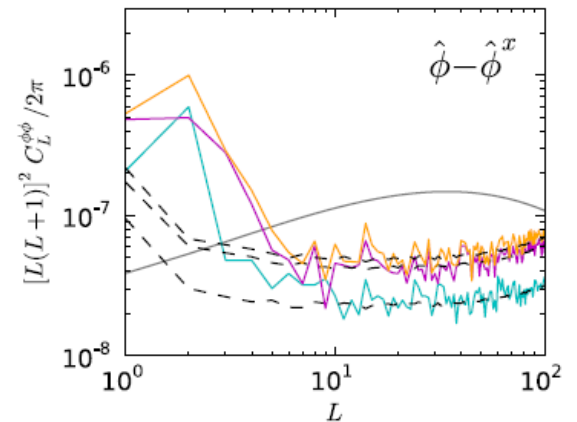
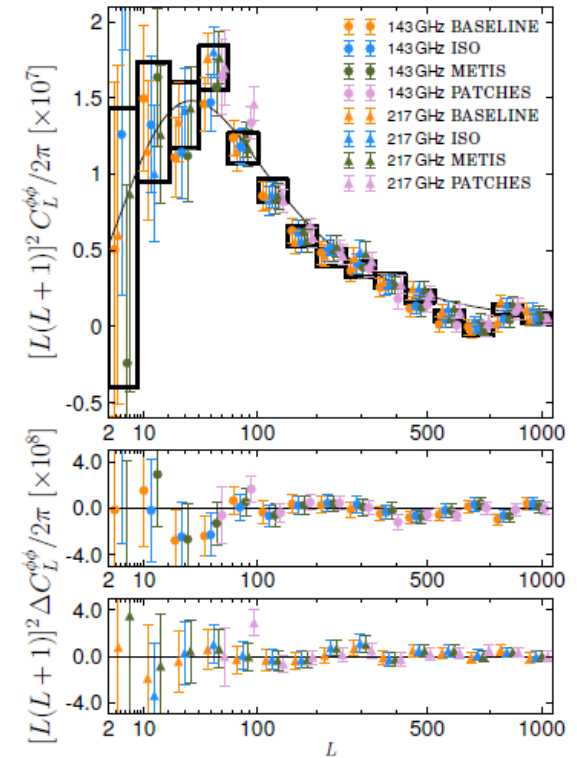
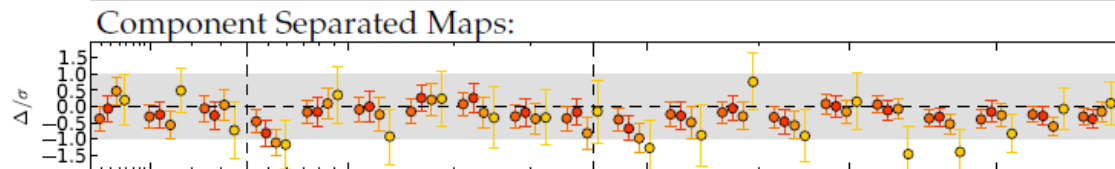
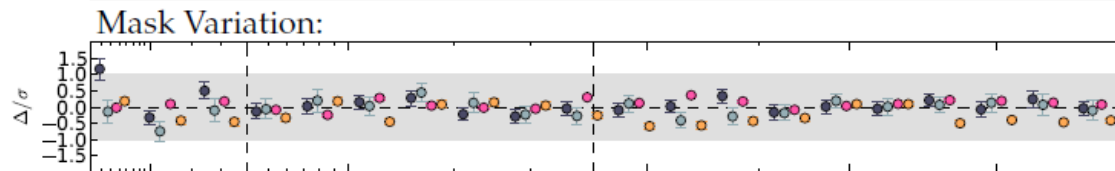
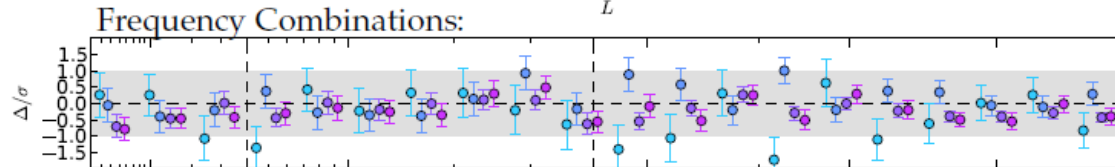
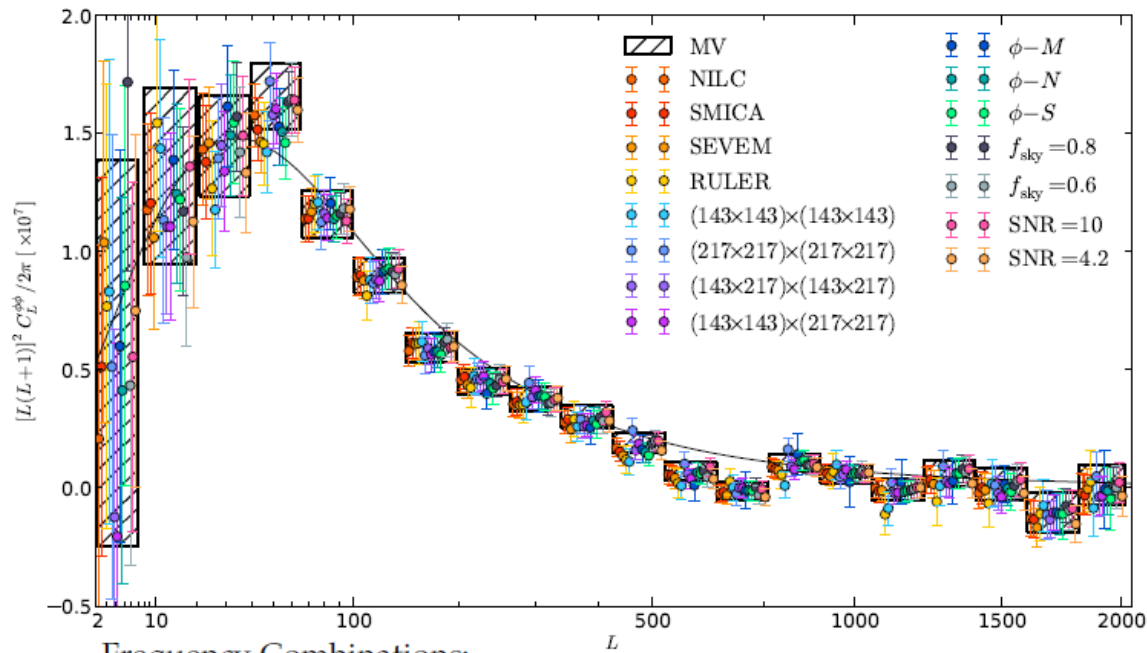


Comparison with ACT/SPT



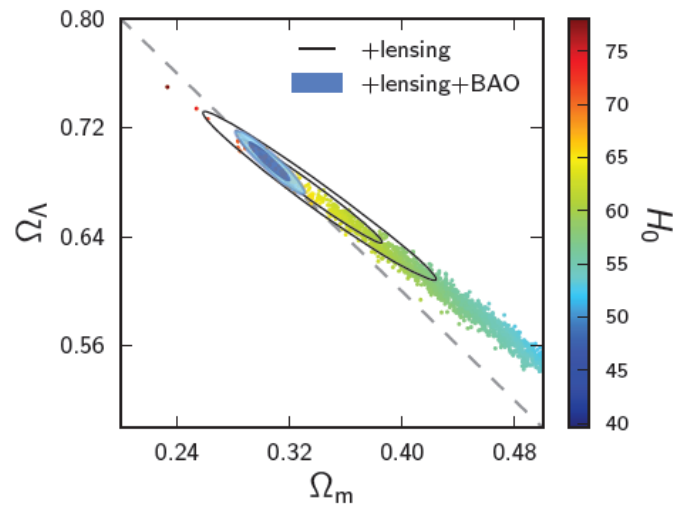
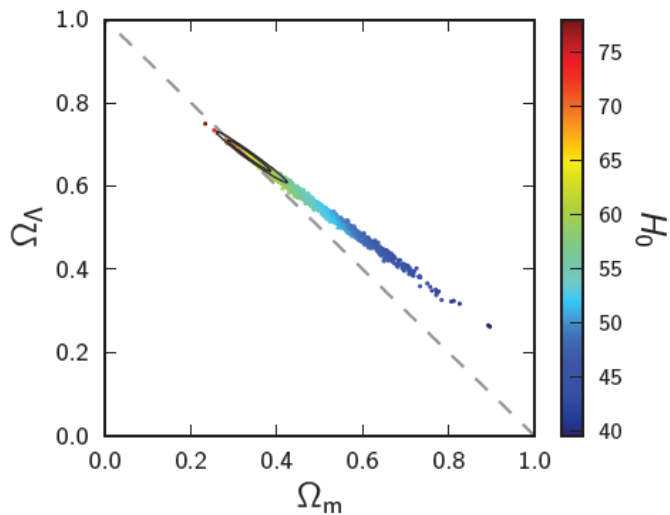
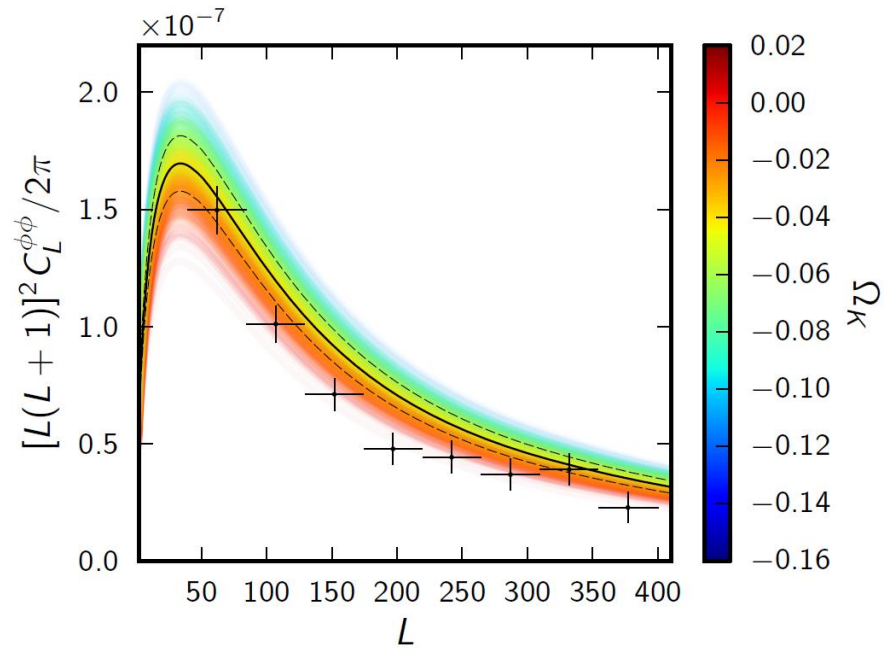
(new SPT data also coming “soon”)

CONSISTENCY TESTS

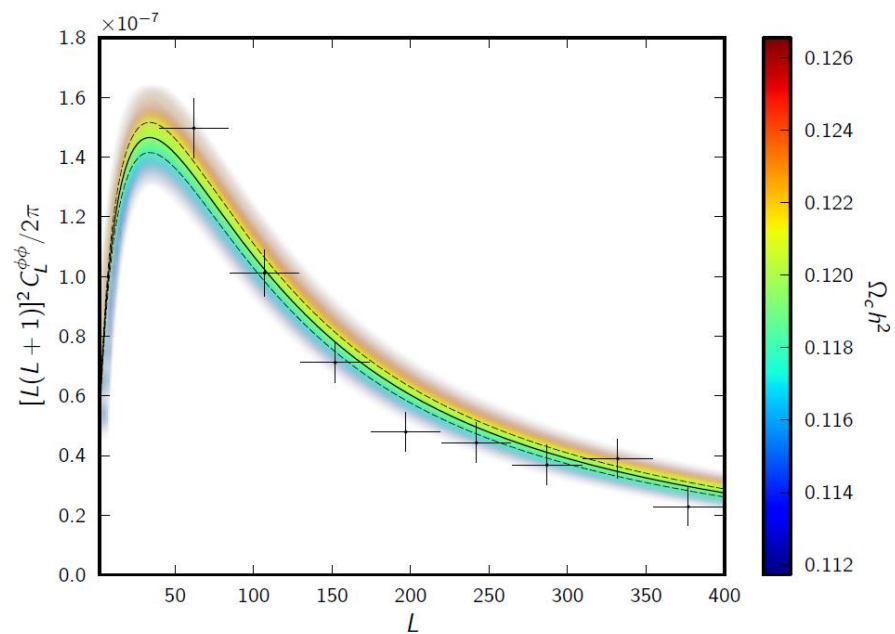


Extra information can help break parameter degeneracies

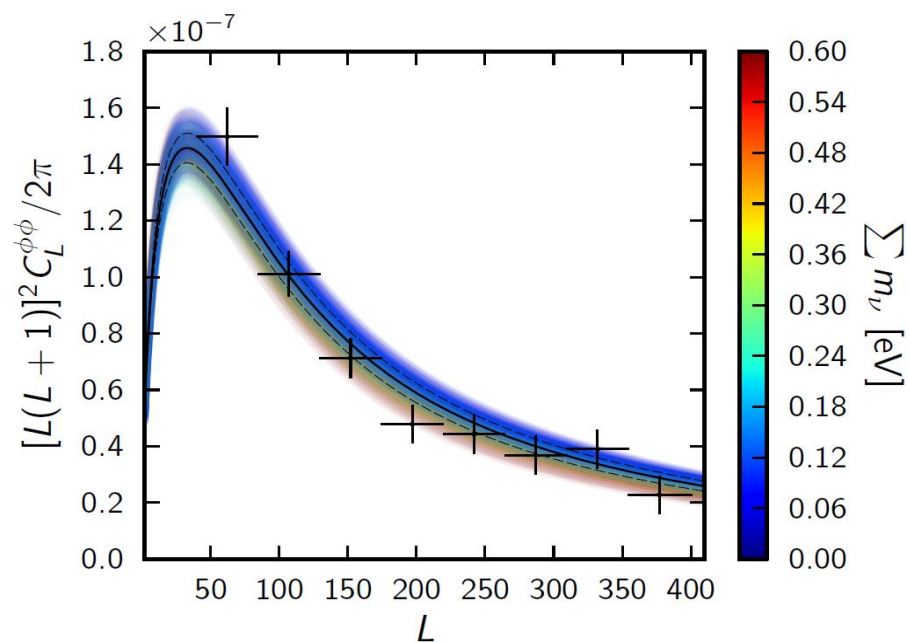
Colour: Planck TT constraint
Crosses: Planck lensing



LCDM matter density



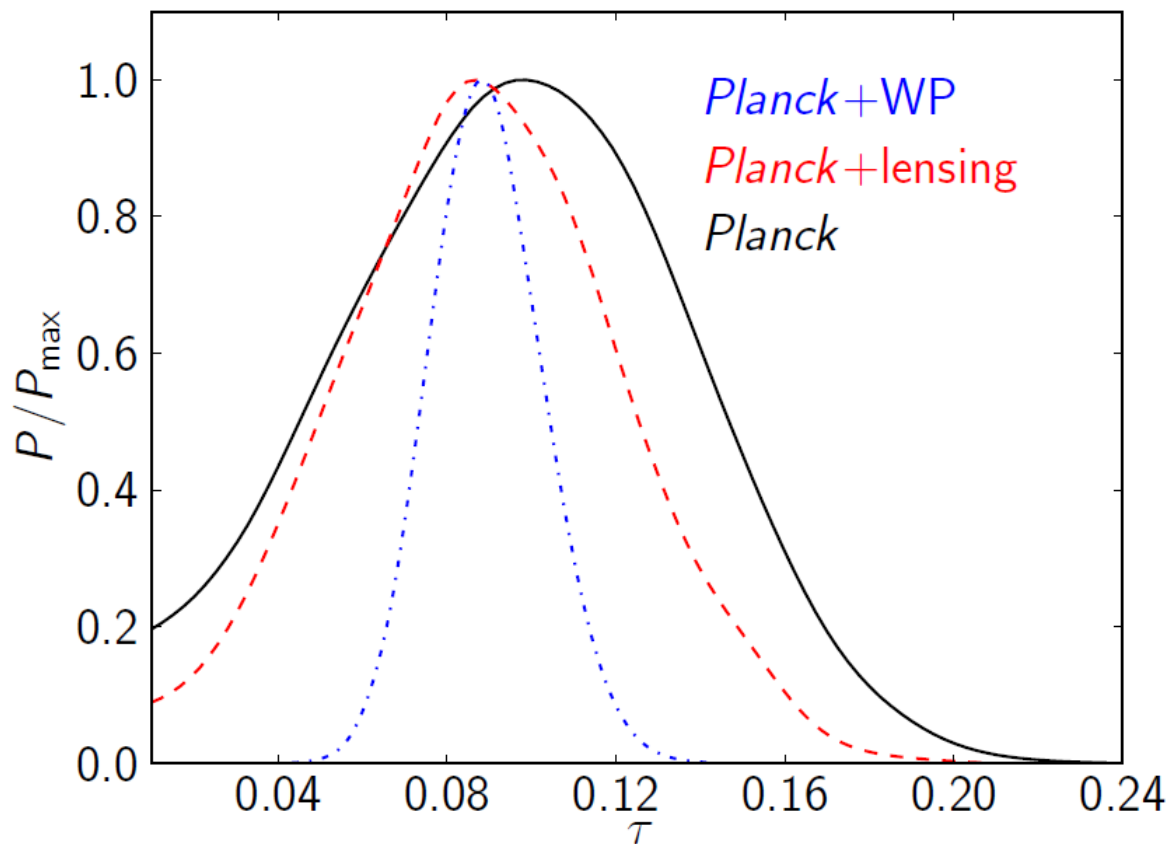
Neutrino mass



Some preference for *lower* lensing power on smaller scales compared to TT LCDM fits (lower $\Omega_c h^2$, higher m_ν)

Reminder: Lensing information enters main parameter results in two ways:

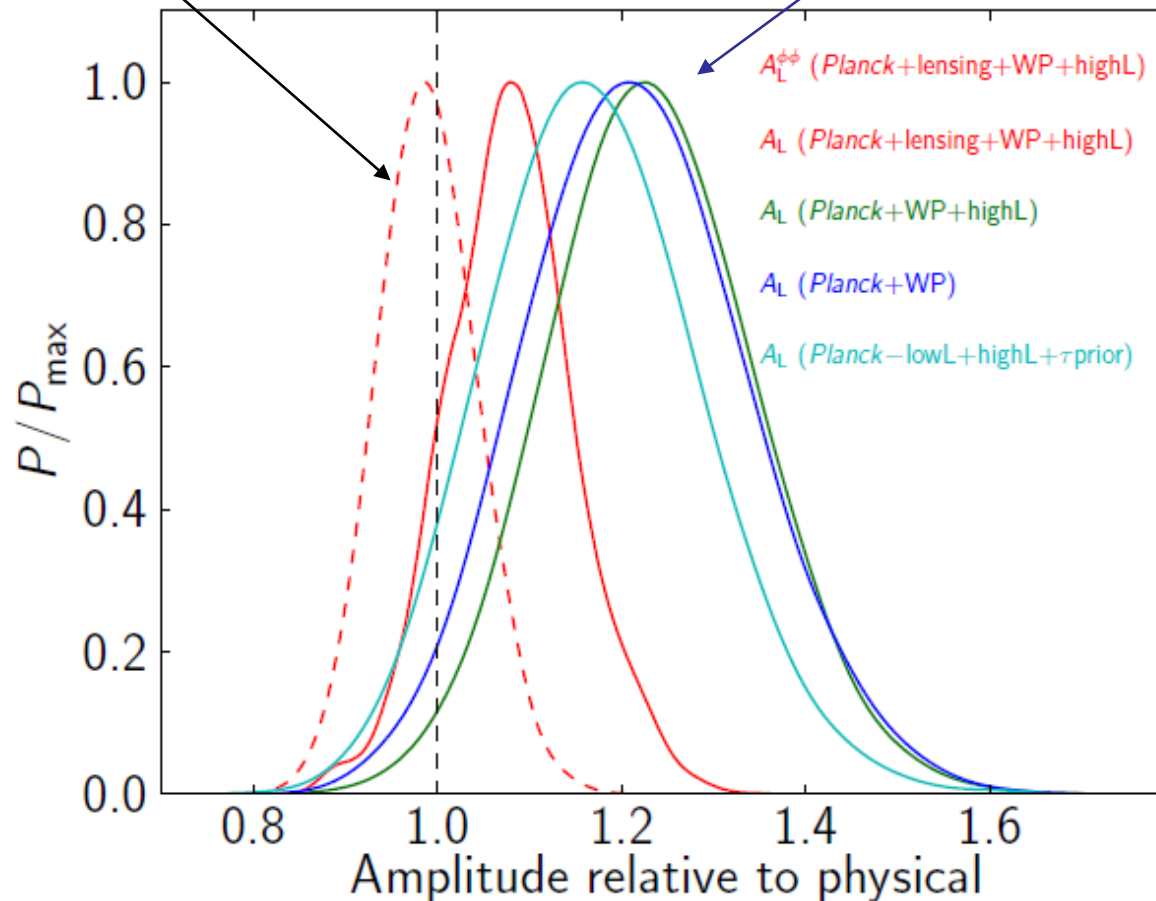
- lensed C_l (always included)
- lensing reconstruction $C_l^{\phi\phi}$ (optionally)



Independent τ constraint (*Planck+lensing*) consistent with WMAP (*Planck+WP*)

Lensing reconstruction
perfectly consistent with $A_L = 1$

Lensed power spectrum
pulls to higher A_L



Full reason for all high A_L results from power spectrum unclear but

No evidence that actual physical lensing effect is larger than expected

Hang on, aren't we double counting?

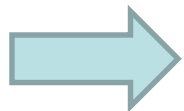
Lensed CMB TT signal and lensing reconstruction must be correlated.

But:

- C_l lensing sensitive to broad kernel in $C_L^{\phi\phi}$ a round peak at $L \sim 60$: many modes contribute, so cosmic variance is relatively small compared to reconstruction noise in the same range
 - *Correlation can be neglected because reconstruction is noise dominated*
- C_l lensing and reconstruction *noise* are also nearly uncorrelated since coming from different parts of the power spectrum (out of phase at low L):
 - C_l lensing is from smoothing of peak and dip heights
 - ϕ reconstruction is from *sideways changes* in C_l under magnification/shear:

$$\langle \tilde{T}(l_2) \tilde{T}(l_3) \rangle = C_{l_2}^{TT} \delta(l_2 + l_3) \left[1 + \kappa \frac{d \ln(l_2^2 C_{l_2}^{TT})}{d \ln l_2} + \hat{l}_2^T \gamma \hat{l}_2 \frac{d \ln C_{l_2}^{TT}}{d \ln l_2} \right]$$

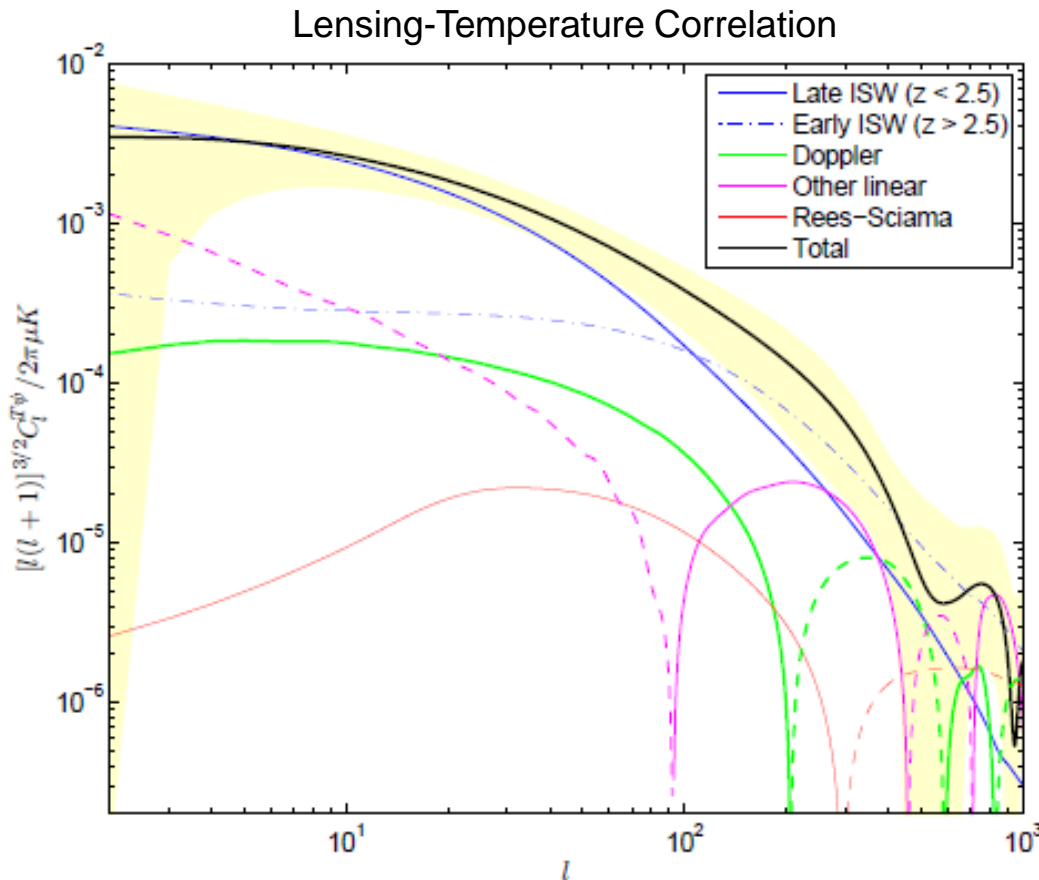
out of phase with peaks and dips



Good approximation to treat lensing reconstruction likelihood as independent

Trispectrum: quadratic estimator for $\psi \Rightarrow C_l^{\psi\psi} \sim \text{trispectrum}$

Bispectrum: quadratic estimator x temperature $\Rightarrow C_l^{T\psi}$



Large-scale bispectrum
from ISW-lensing correlation:

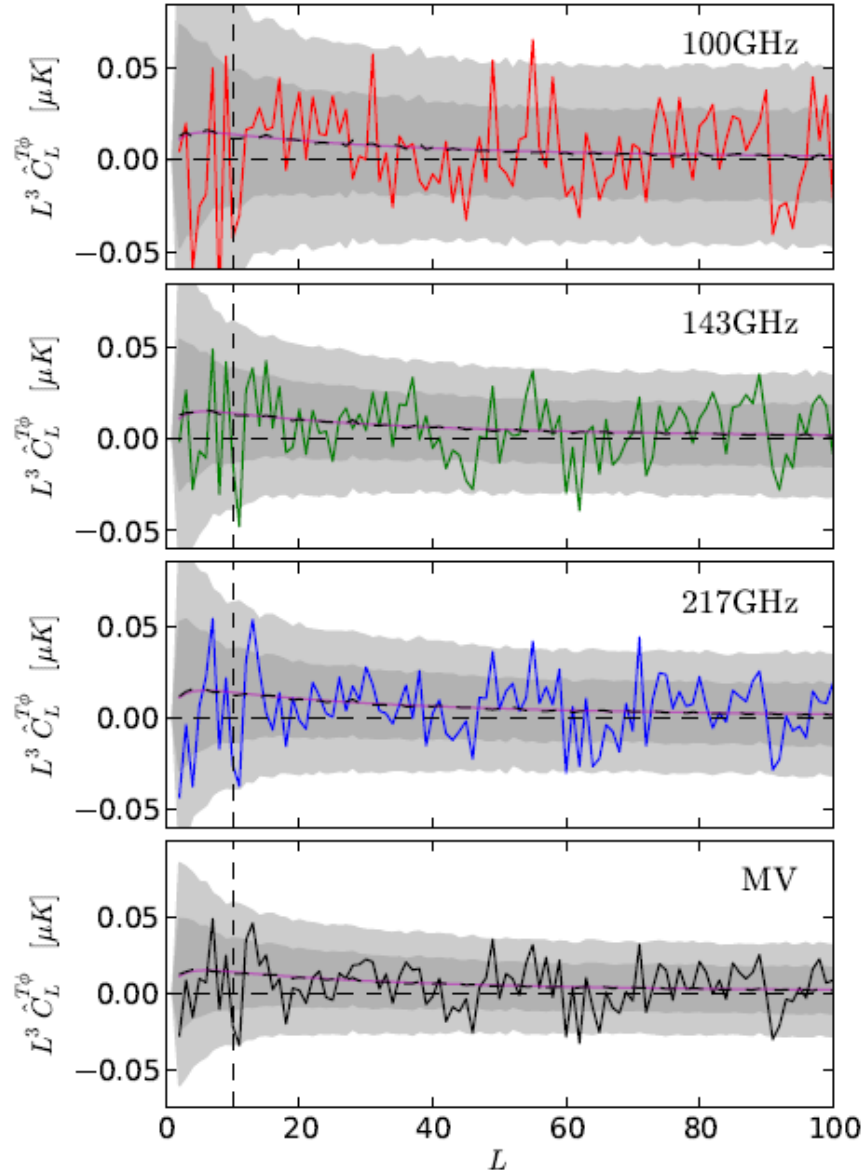
$\Rightarrow f_{NL} \sim 7$
(but very different squeezed shape)

- Must be accounted for in primordial non-Gaussianity results
- Up to 5 sigma detection possible: dark energy information

Lewis [arXiv:1204.5018](https://arxiv.org/abs/1204.5018)

(note Rees-Sciama contribution is small, numerical problem with much larger result of [Verde et al](#), [Mangilli et al.](#); see also [Junk et al. 2012](#))

Planck lensing bispectrum result



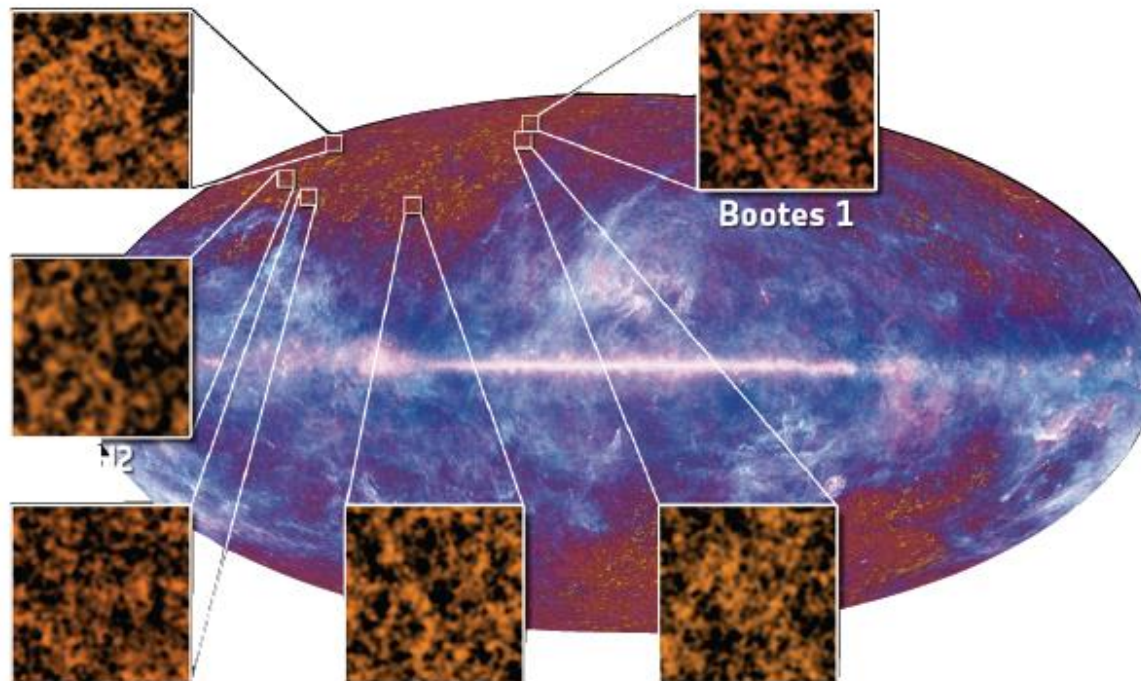
Large cosmic variance and reconstruction noise, but ‘detected’ at $\sim 2.5\sigma$

Table 2. Results for the amplitude of the ISW-lensing bispectrum from the SMICA, NILC, SEVEM, and C-R foreground-cleaned maps, for the KSW, binned, and modal (polynomial) estimators; error bars are 68% CL.

	SMICA	NILC	SEVEM	C-R
KSW	0.81 ± 0.31	0.85 ± 0.32	0.68 ± 0.32	0.75 ± 0.32
Binned	0.91 ± 0.37	1.03 ± 0.37	0.83 ± 0.39	0.80 ± 0.40
Modal	0.77 ± 0.37	0.93 ± 0.37	0.60 ± 0.37	0.68 ± 0.39

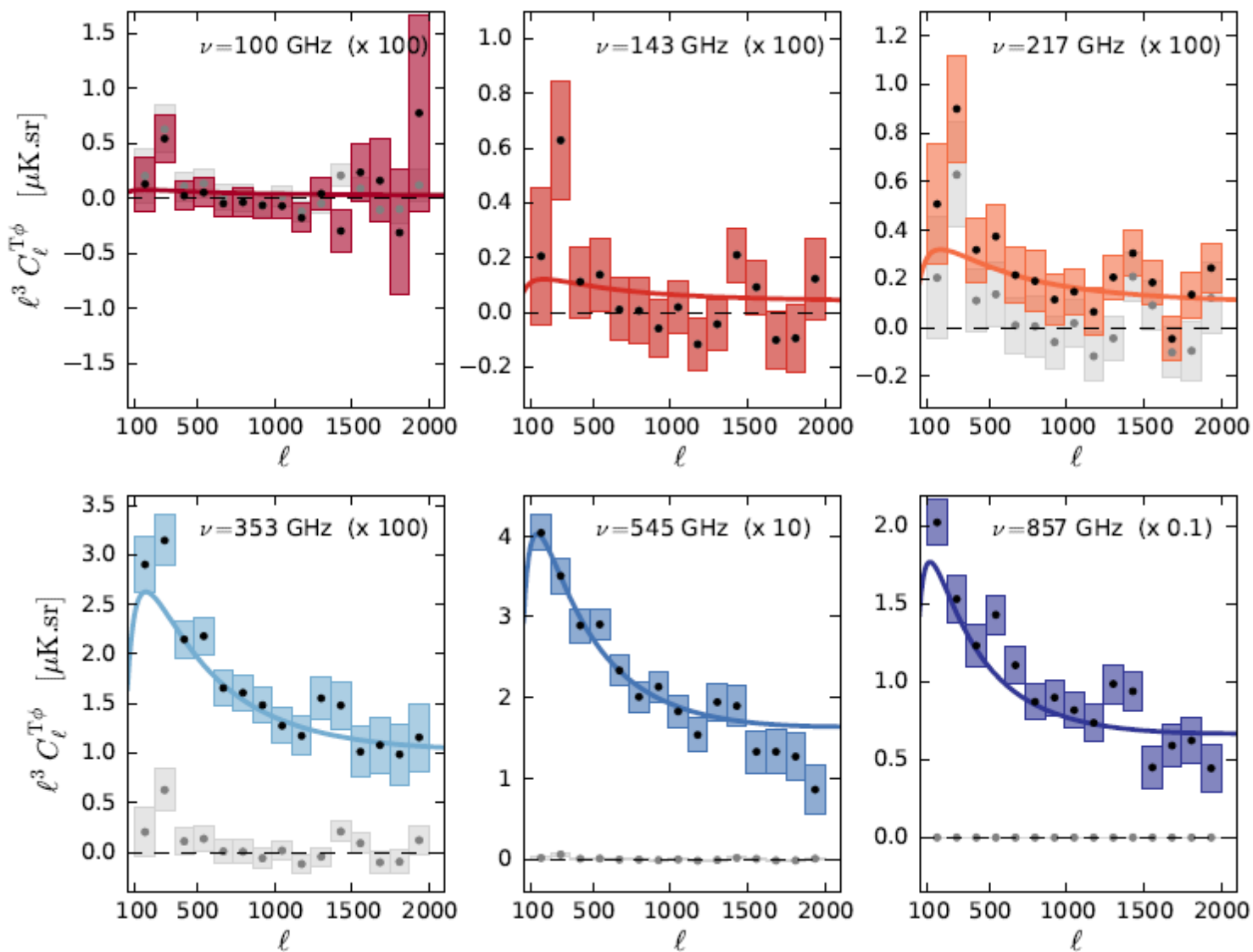
***Planck* 2013 results. XVIII.**

Gravitational lensing-infrared background correlation



The high-frequency Planck maps trace Cosmic Infrared Background (CIB) fluctuations, primarily sourced by star formation at high-redshift; strong correlations with lensing (Song et. al.).

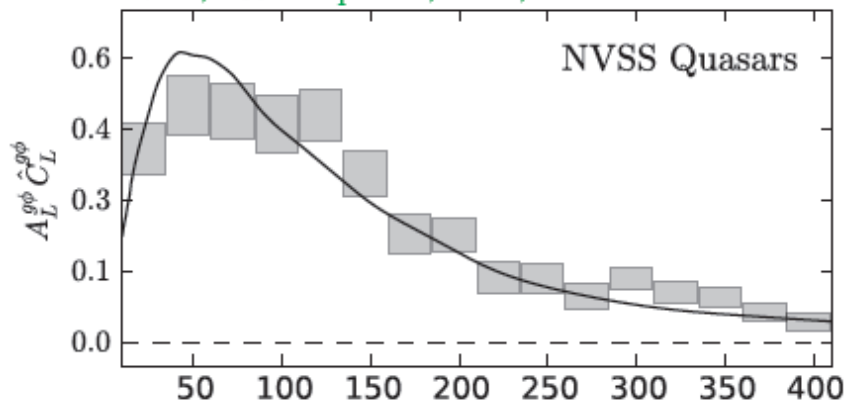
CIB-CMB lensing bispectrum detection



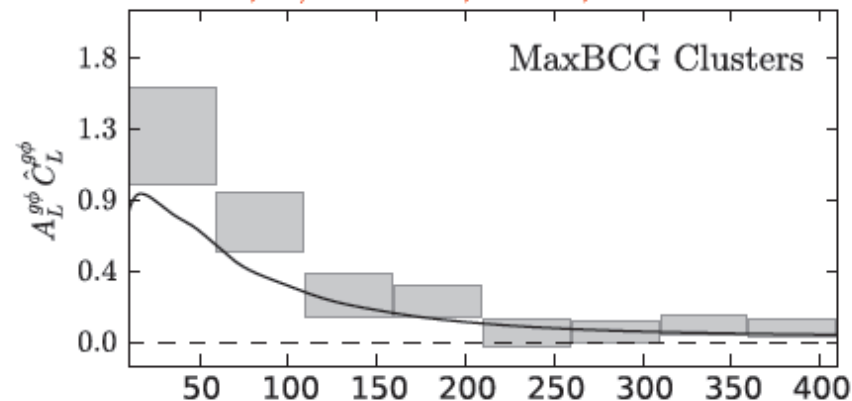
'42 σ at 545 GHz !

EXTERNAL CORRELATIONS

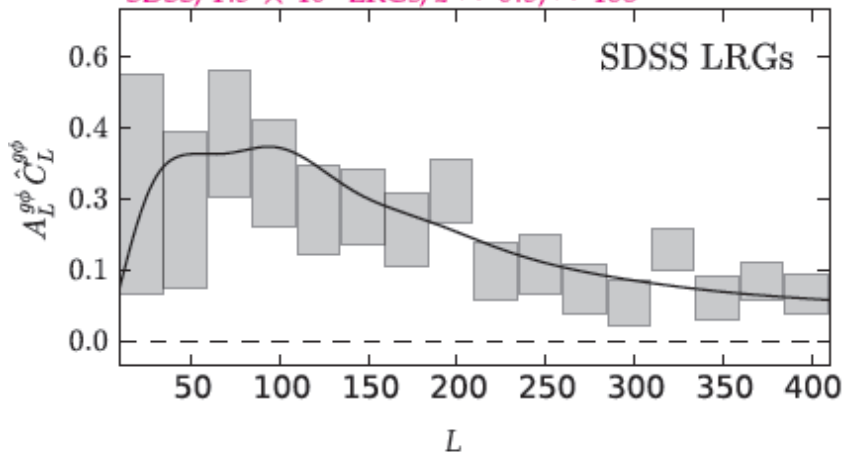
NVSS, 2×10^6 quasars, $z \sim 1$, $\sim 20\sigma$



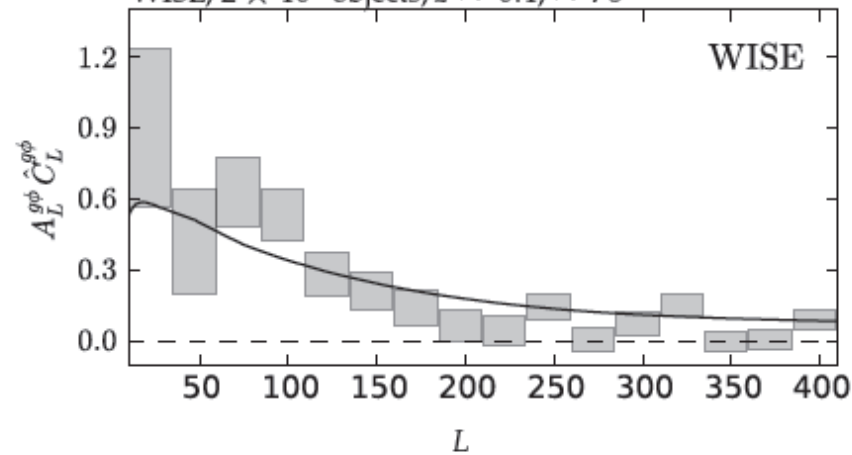
MaxBCG, 14,000 clusters, $z \sim 0.2$, $\sim 7\sigma$



SDSS, 1.5×10^6 LRGs, $z \sim 0.5$, $\sim 10\sigma$



WISE, 2×10^6 objects, $z \sim 0.1$, $\sim 7\sigma$



Conclusions

- Planck TT marks the beginning of the end for C_l^{TT}
 - cosmic variance limited to $L \sim 1600$
 - lensing smoothing detected, but possible issues with $A_L > 1$
- Planck lensing marks end of the beginning for $C_l^{\phi\phi}$
 - first nearly fully-sky maps of the lensing potential
 - detected at high significance, by itself and in correlation
 - reconstruction noise dominated
 - can break parameter degeneracies (but not yet competitive with e.g. BAO)
 - broadly consistent with expected amplitude, no preference for $A_L > 1$

Future: SPT TT (~months), Planck full mission (1 yr), ACTpol, SPTpol