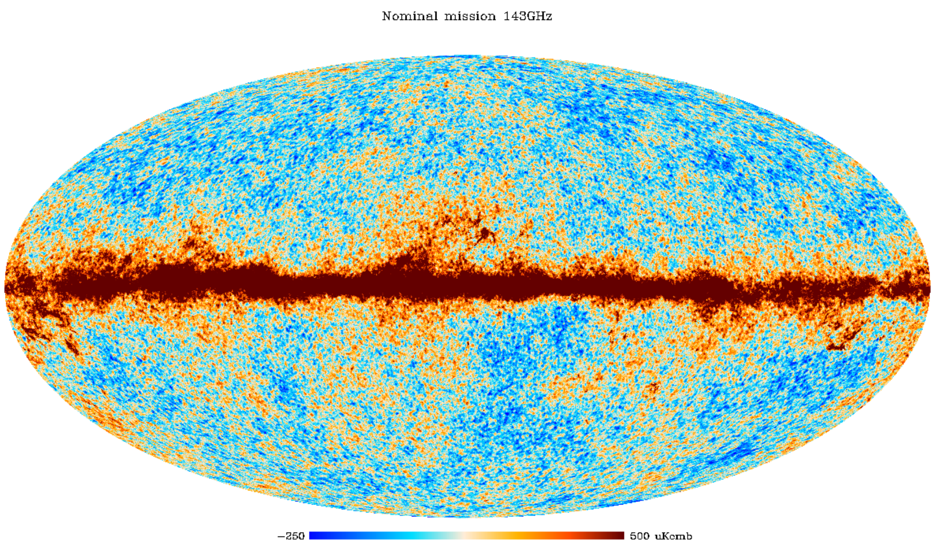
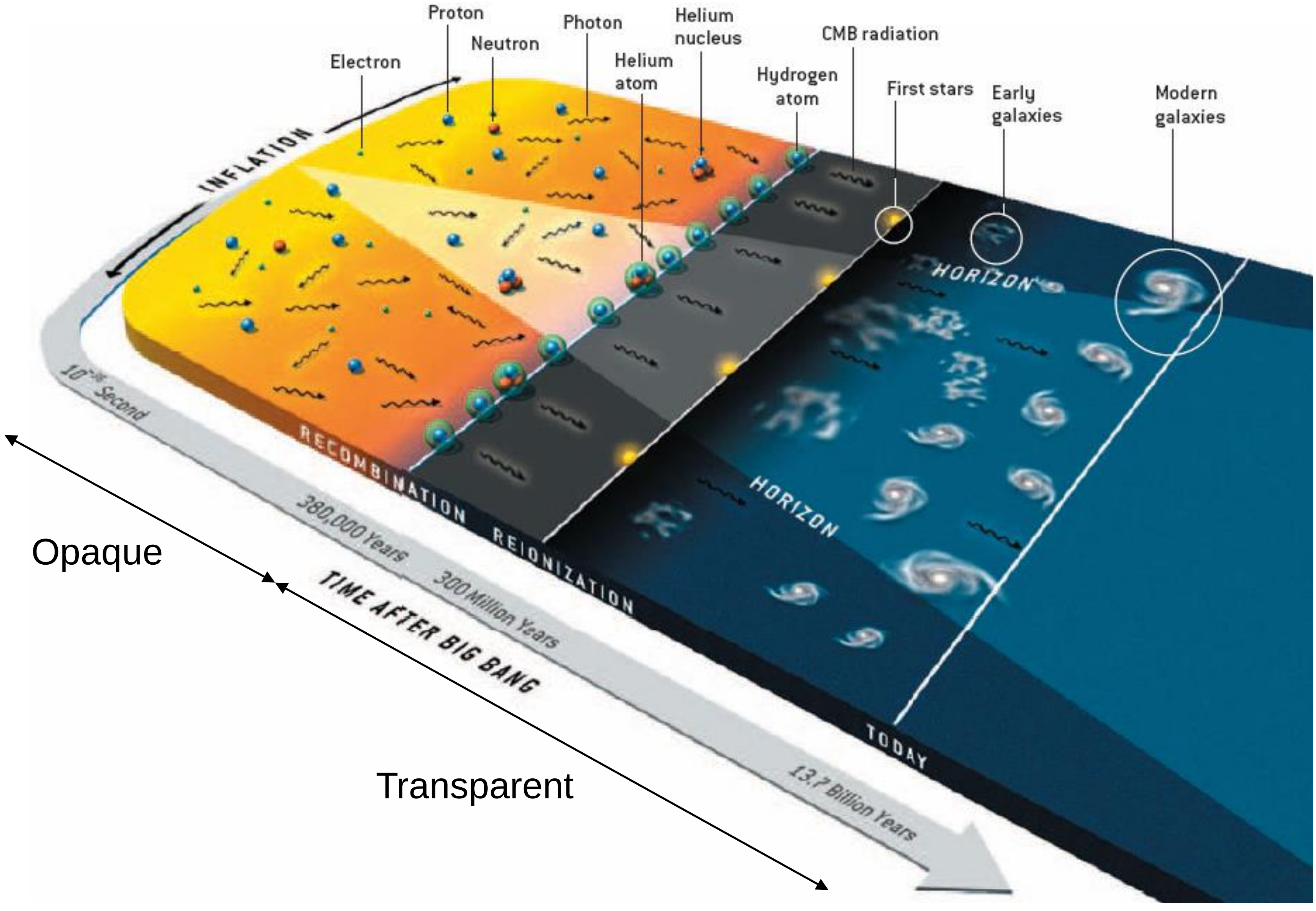
CMB beyond a single power spectrum:
Non-Gaussianity and frequency dependence



University of Sussex

Antony Lewis
http://cosmologist.info/

Evolution of the universe

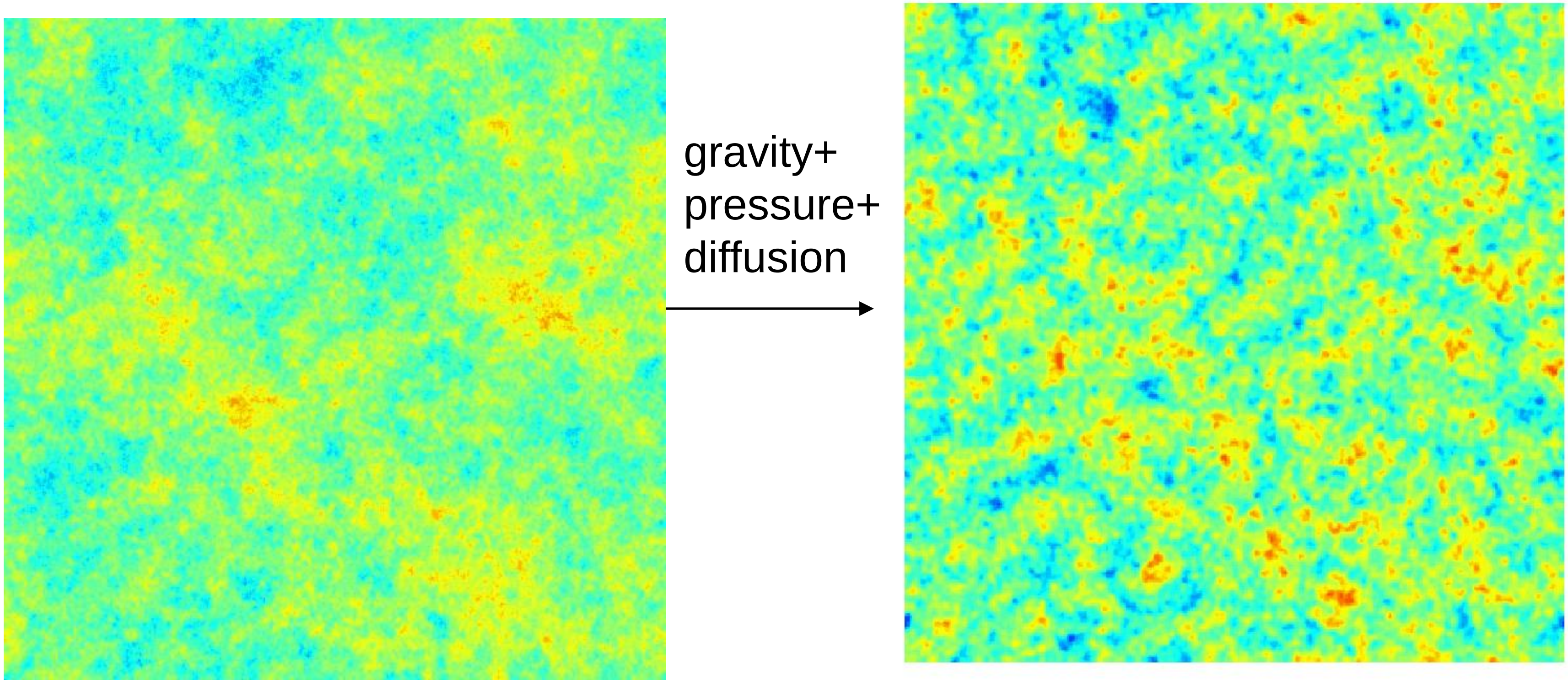


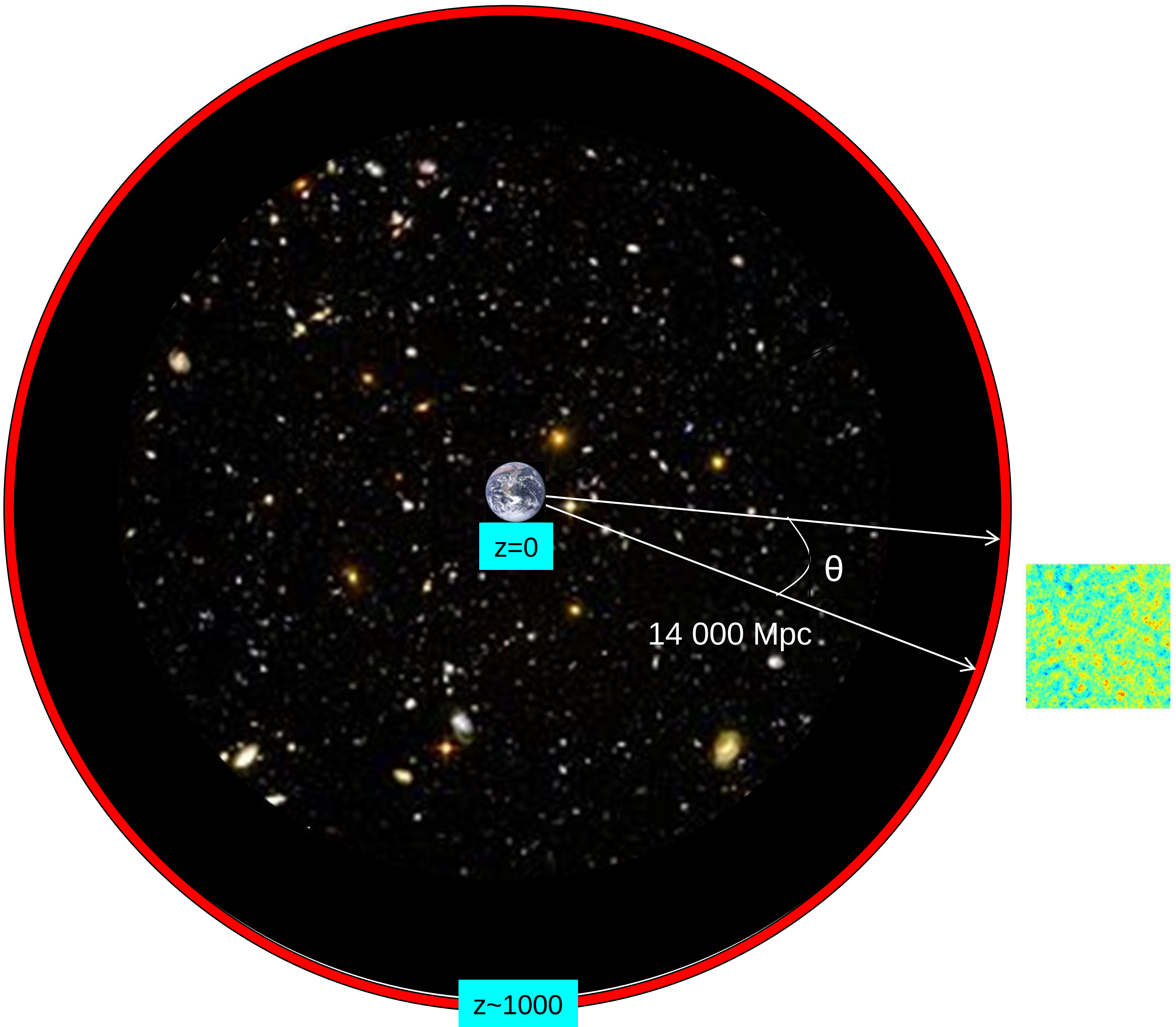
Hu & White, Sci. Am., 290 44 (2004)

CMB temperature

End of inflation

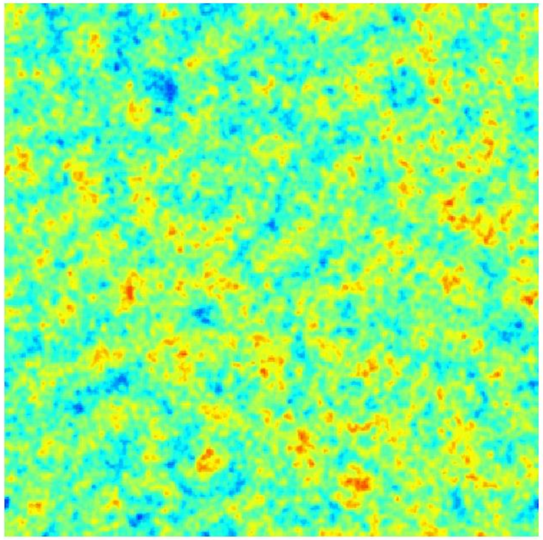
Last scattering surface

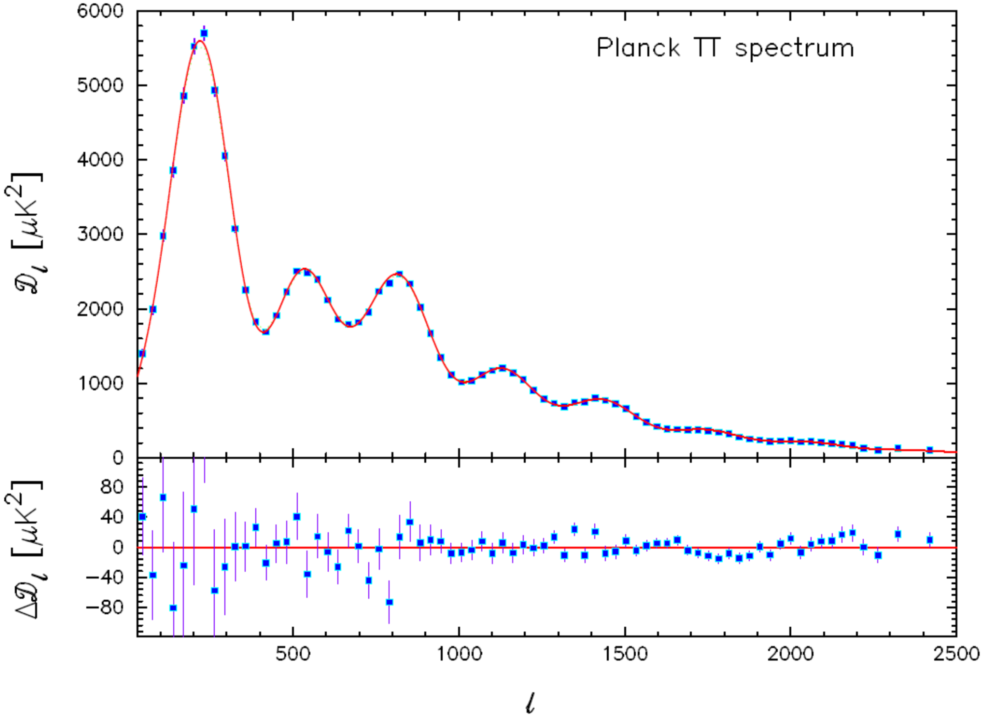




z=0
θ
14 000 Mpc
z~1000

Observed CMB blackbody power spectrum





Observations → **Constrain theory of early universe + evolution parameters and geometry**

Beyond Gaussianity – general possibilities

Flat sky approximation: $\Theta(x) = \frac{1}{2\pi}\int d^2l\,\Theta(l)e^{ix\cdot l}$ $(\Theta = T)$

Gaussian + statistical isotropy

$$\langle\Theta(l_1)\Theta(l_2)\rangle = \delta(l_1 + l_2)C_l$$

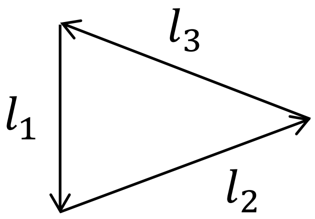
- power spectrum encodes all the information
- modes with different wavenumber are independent

Higher-point correlations

Gaussian: can be written in terms of C_l

Non-Gaussian: non-zero connected n-point functions

Bispectrum



$$\boldsymbol{l_1} + \boldsymbol{l_2} + \boldsymbol{l_3} = \mathbf{0}$$

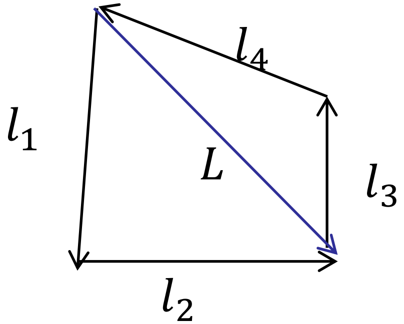
Flat sky approximation: $\langle \Theta(l_1)\Theta(l_2)\Theta(l_3)\rangle = \frac{1}{2\pi}\delta(l_1 + l_2 + l_3) b_{l_1 l_2 l_3}$

If you know $\Theta(l_1), \Theta(l_2)$, sign of $b_{l_1 l_2 l_3}$ tells you which sign of $\Theta(l_3)$ is more likely

Trispectrum

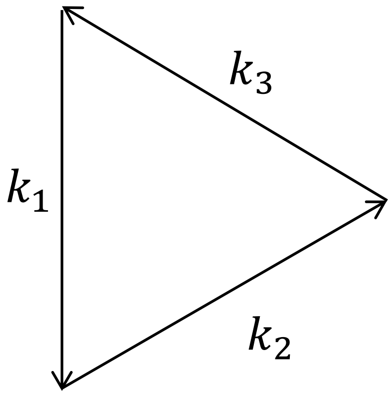
$$\langle \Theta(\mathbf{l}_1)\Theta(\mathbf{l}_2)\Theta(\mathbf{l}_3)\Theta(\mathbf{l}_4)\rangle_C = (2\pi)^{-2}\delta(\mathbf{l}_1 + \mathbf{l}_2 + \mathbf{l}_3 + \mathbf{l}_4) T(\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3, \mathbf{l}_4)$$

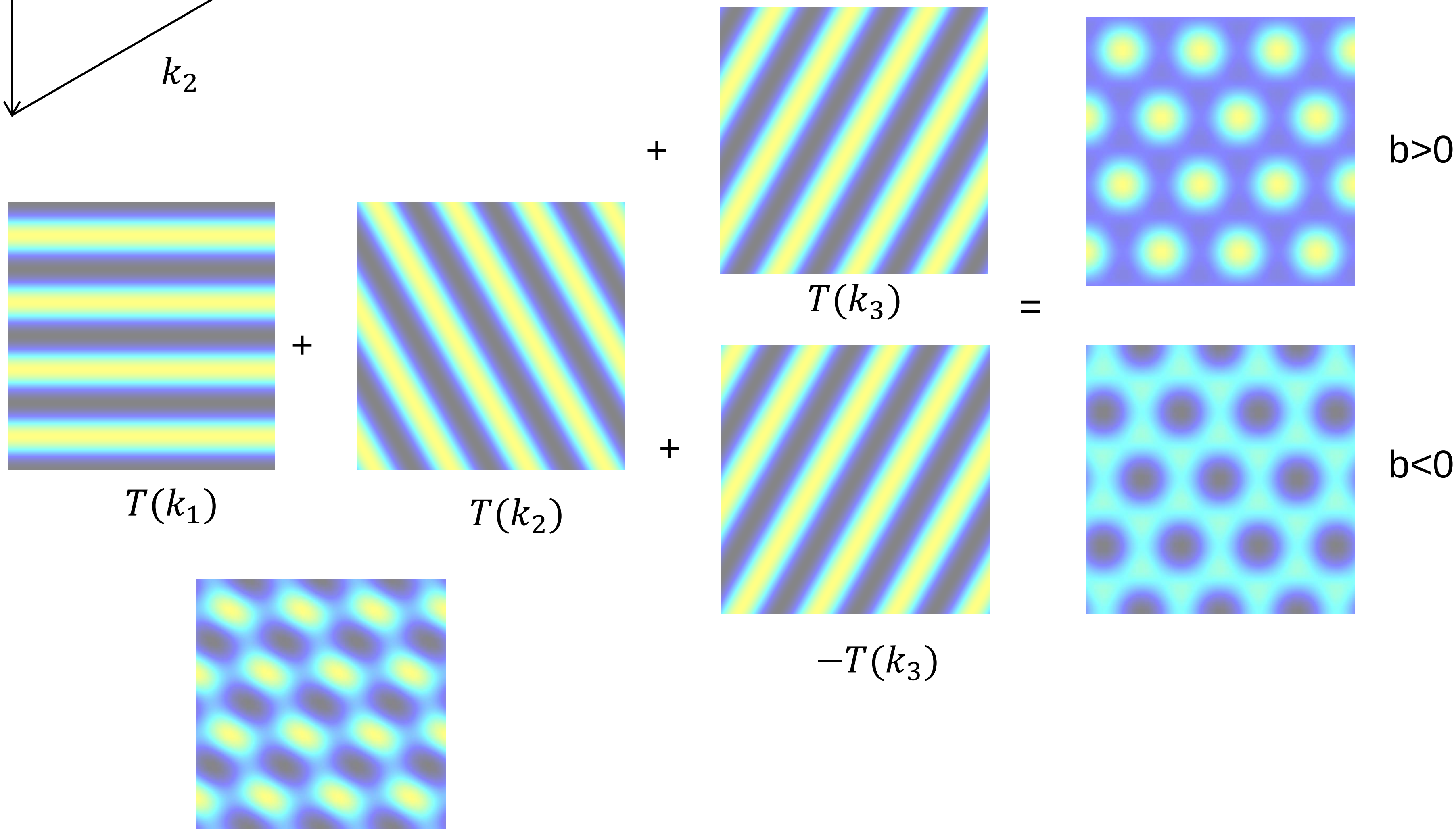
$$\langle \Theta(\mathbf{l}_1)\Theta(\mathbf{l}_2)\Theta(\mathbf{l}_3)\Theta(\mathbf{l}_4)\rangle_C = \frac{1}{2}\int \frac{d^2\mathbf{L}}{(2\pi)^2}\delta(\mathbf{l}_1 + \mathbf{l}_2 + \mathbf{L})\delta(\mathbf{l}_3 + \mathbf{l}_4 - \mathbf{L})\mathbb{T}^{(\ell_1\ell_2)}_{(\ell_3\ell_4)}(L) + \text{perms.}$$



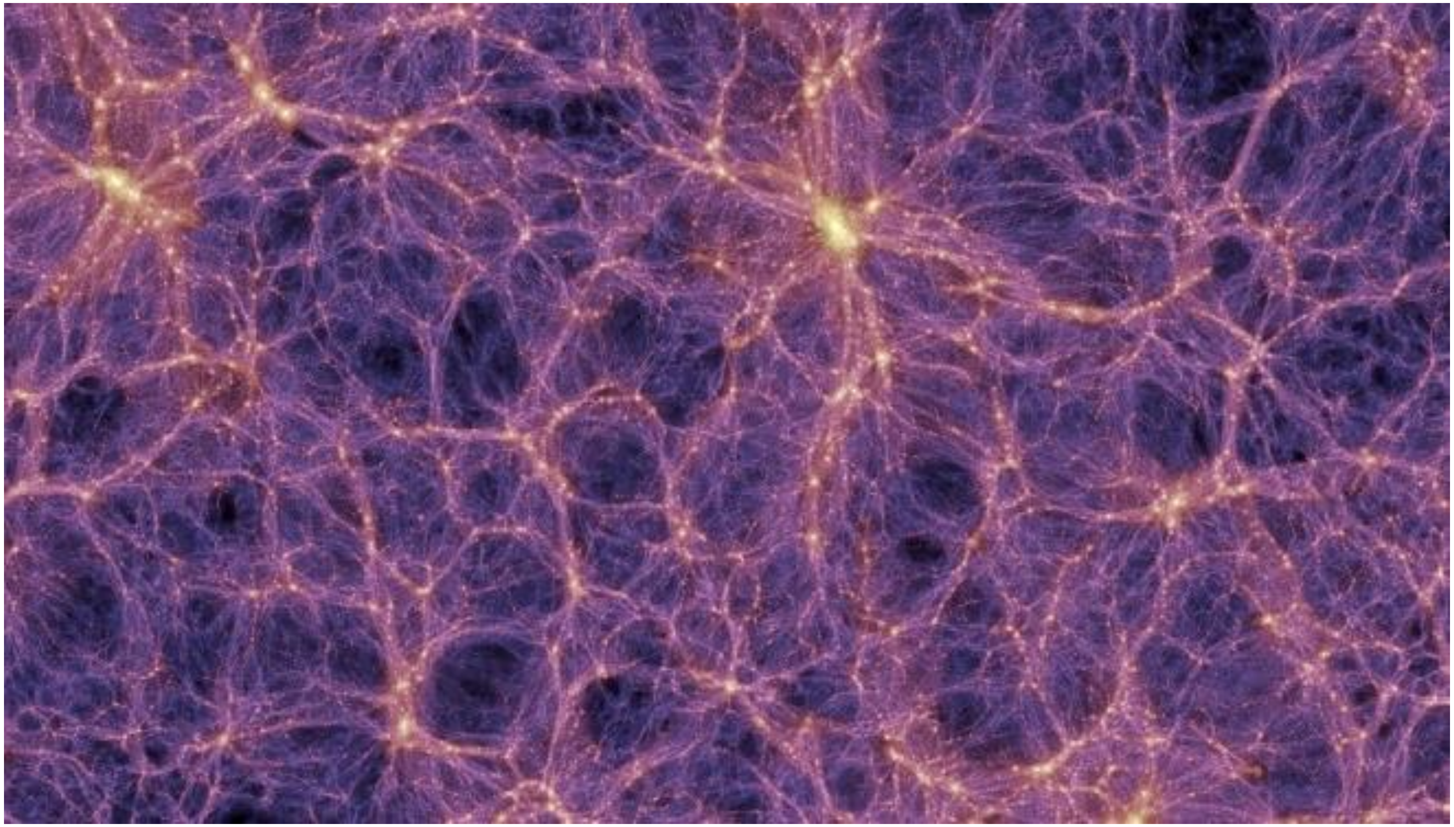
N-spectra…

Equilateral $k_1 + k_2 + k_3 = 0, |k_1| = |k_2| = |k_3|$



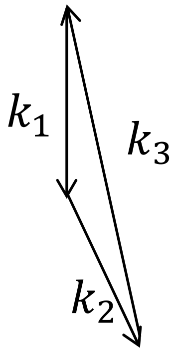


AL: The Real Shape of Non-Gaussianities, arXiv:1107.5431

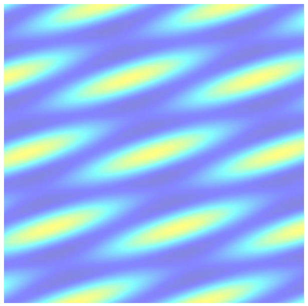


Millennium simulation

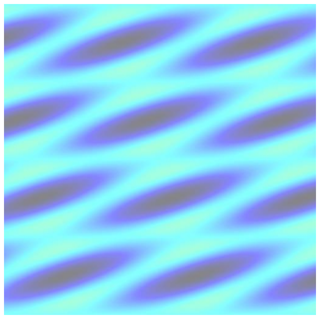
Near-equilateral to flattened:



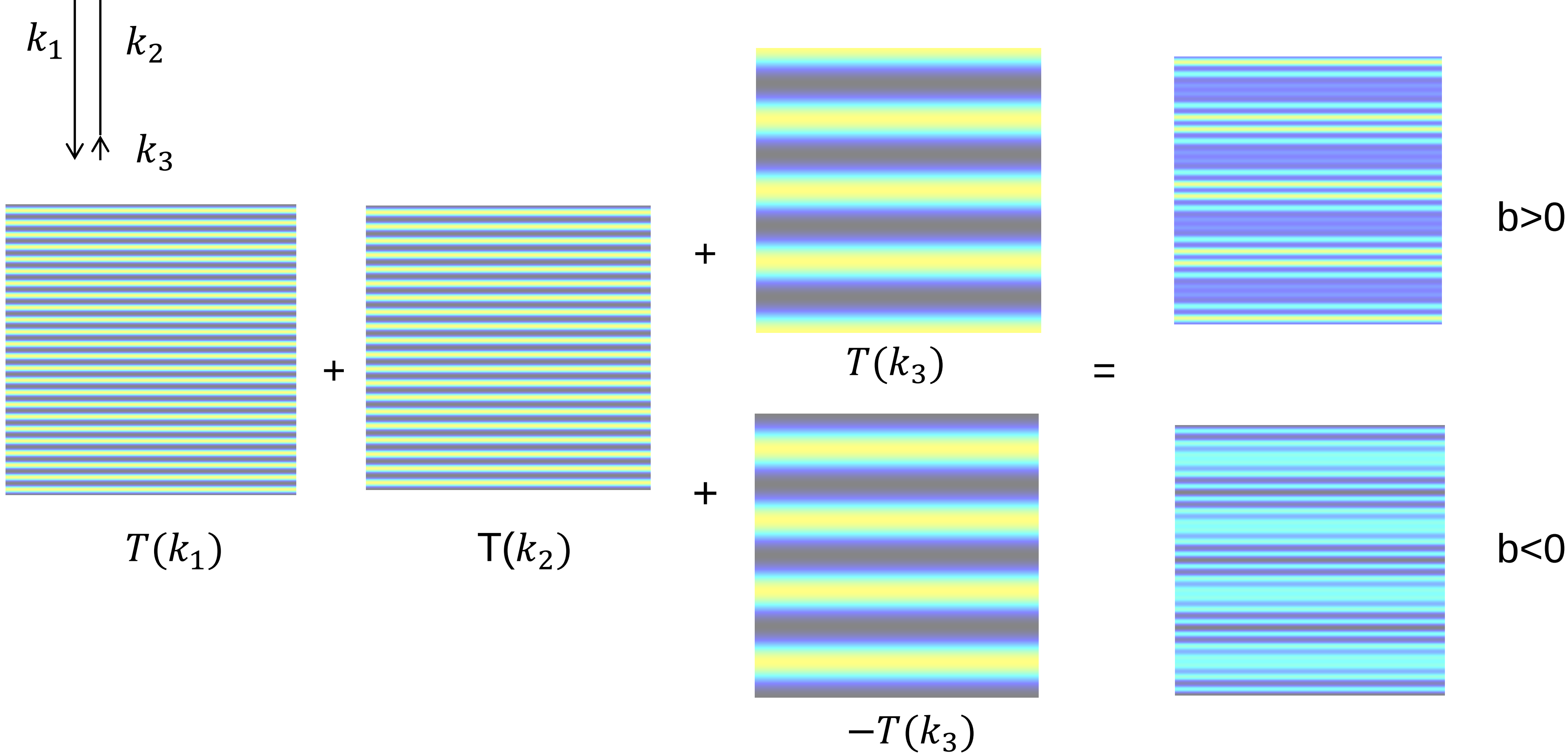
b>0



b<0



Local (squeezed) $k_1 + k_2 + k_3 = 0, \quad k_1 \ll k_2, k_3 \quad k_2 \sim -k_3$

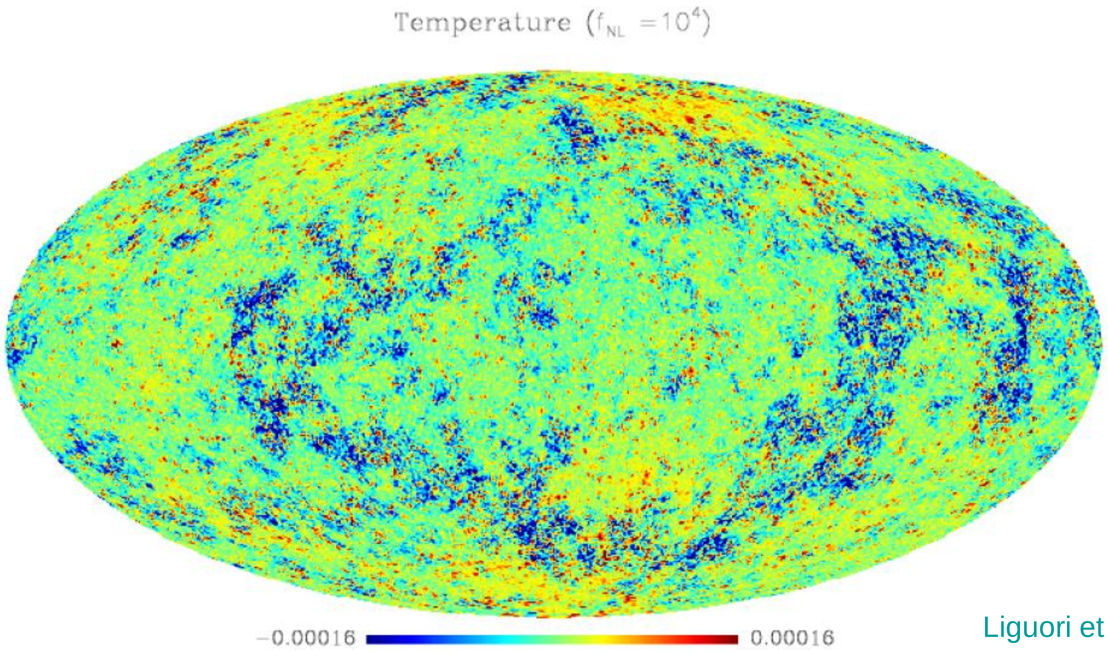


Squeezed bispectrum is a *correlation* of small-scale power with large-scale modes

Primordial local non-Gaussianity

e.g. $\zeta = \zeta_0 \left(1 + \frac{6}{5} f_{NL} \zeta_{0,l}\right)$

$\Rightarrow T \sim T_g (1 + \frac{6}{5} f_{NL} \zeta_{*,l})$



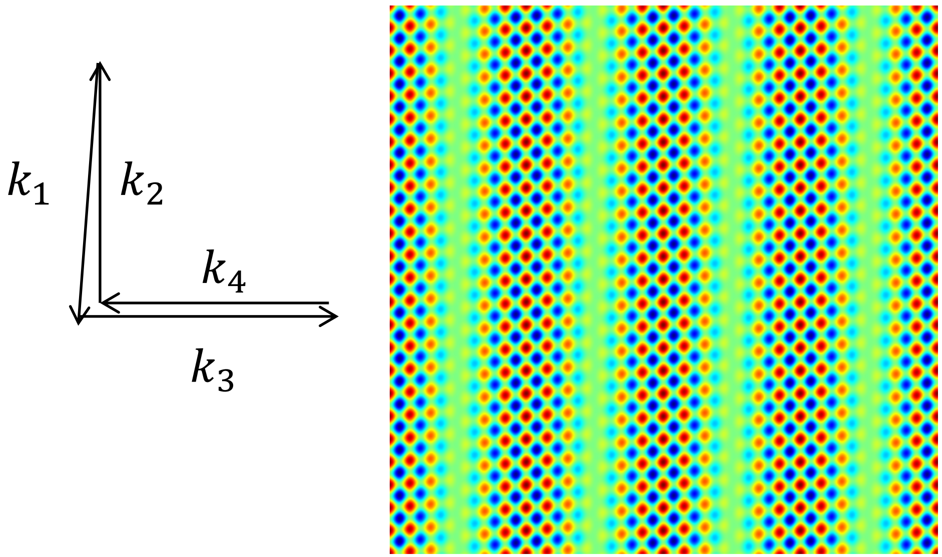
Liguori et al 2007

Single-field slow-roll inflation: $f_{NL} \sim 0$

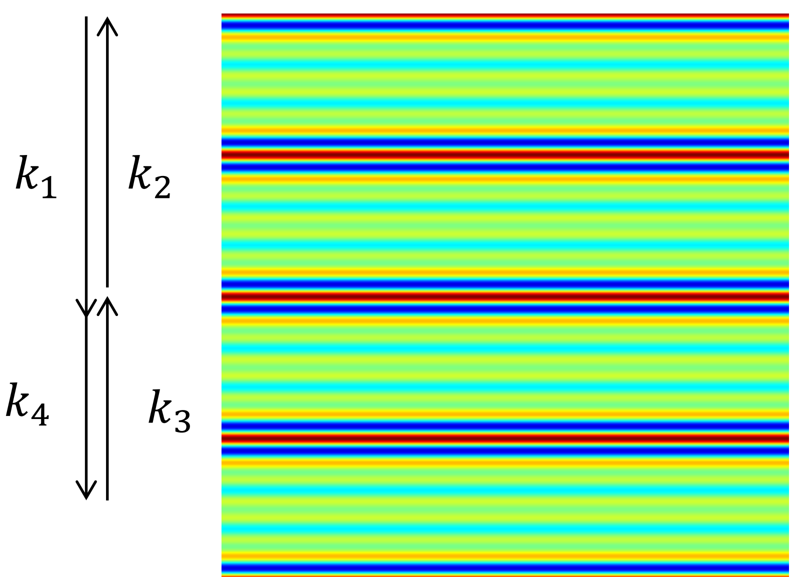
$\Rightarrow$ Any significant detection would rule out large classes of inflation models

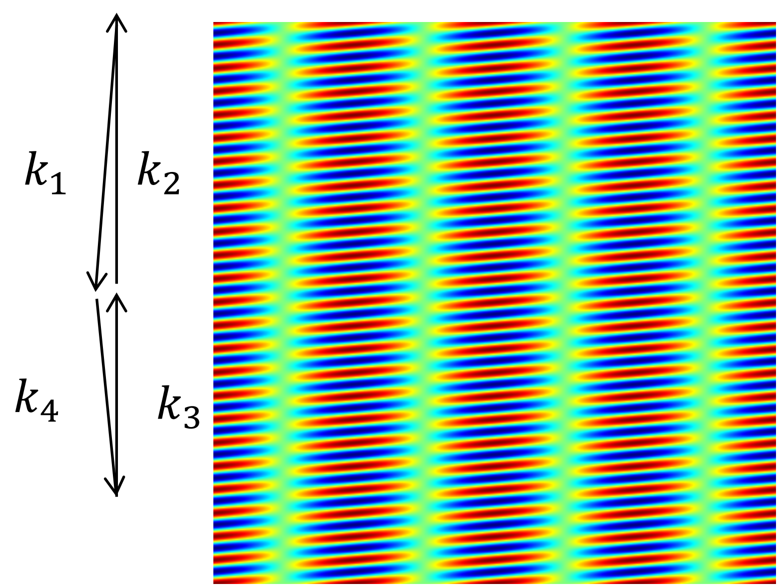
<u>New information that is not present in the power spectrum</u>

Diagonal squeezed trispectra $|k_1| \sim |k_2|, |k_3| \sim |k_4|, |k_1 + k_2| = |k_3 + k_4| \ll |k_2|, |k_3|$



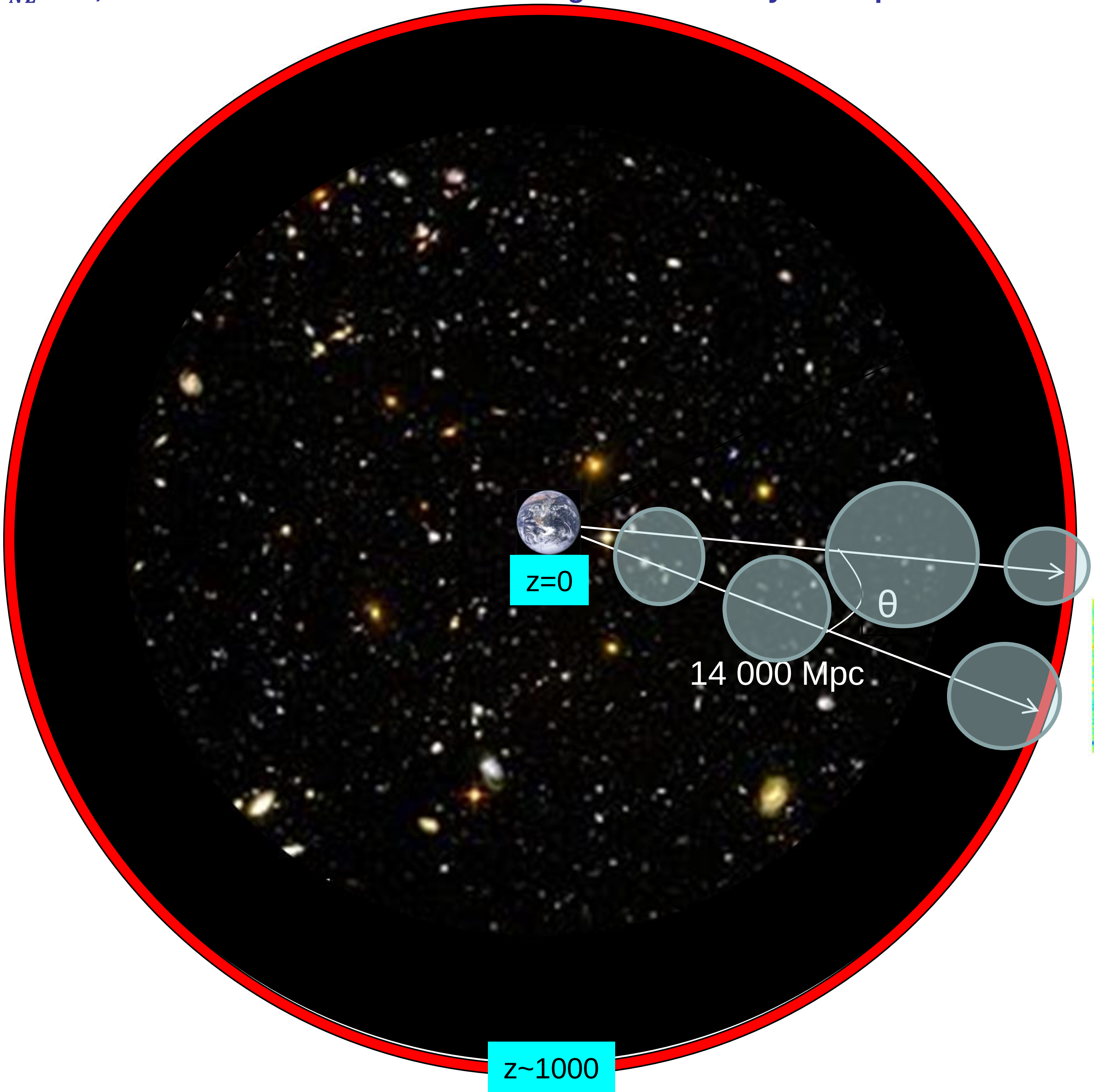
Trispectrum = power spectrum of modulation

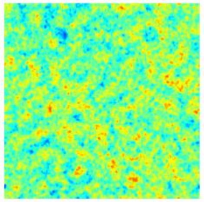




Squeezed trispectrum measures the *spatial modulation* of small-scale power

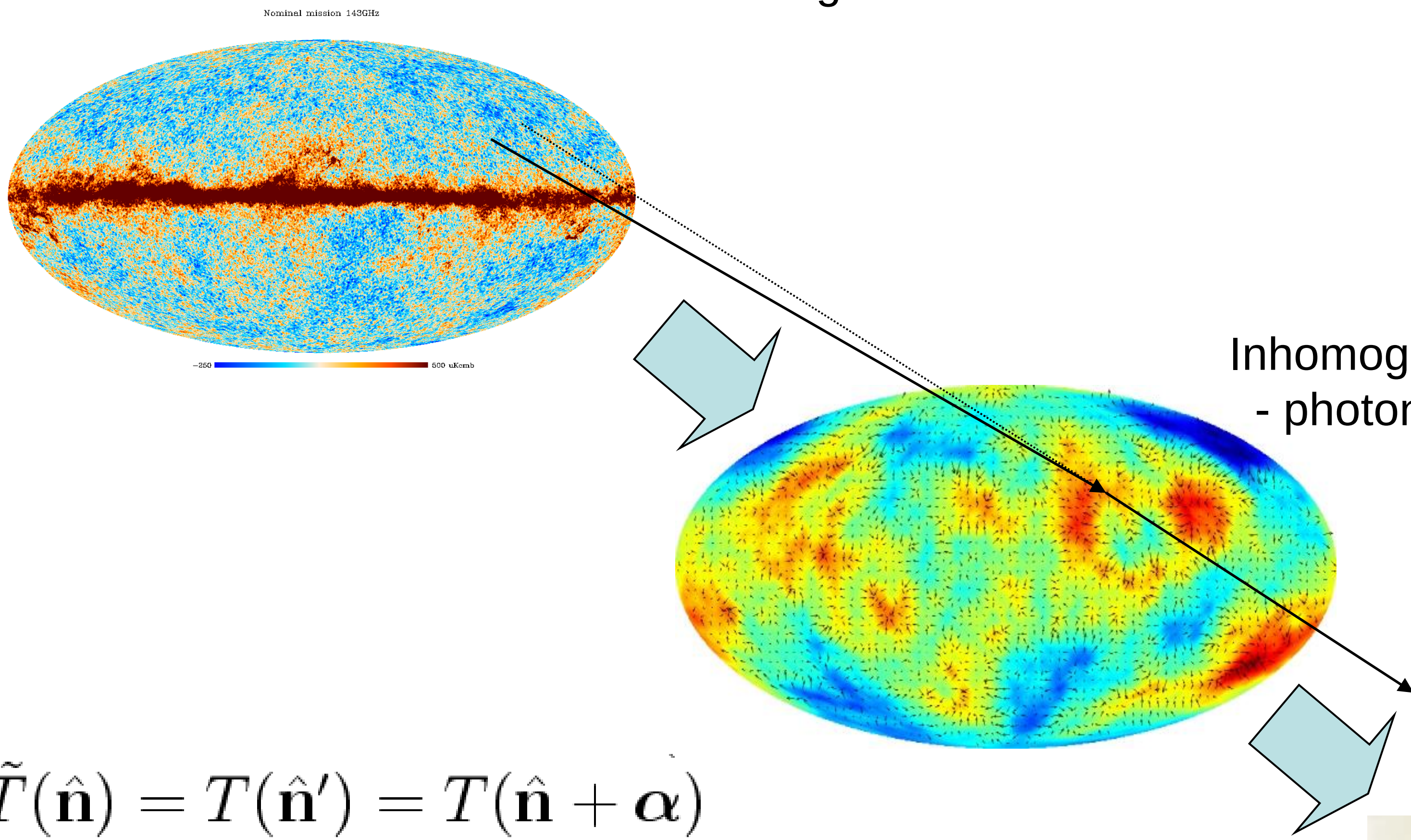
But even with $f_{NL} = 0$, we observe CMB at last scattering modulated by other perturbations





e.g. CMB Lensing: modulation due to large-scale gravitational lenses

Last scattering surface



Inhomogeneous universe
- photons deflected

Observer

$$\tilde{T}(\hat{\mathbf{n}}) = T(\hat{\mathbf{n}}') = T(\hat{\mathbf{n}} + \boldsymbol{\alpha})$$

$$\boldsymbol{\alpha} = \nabla\psi$$

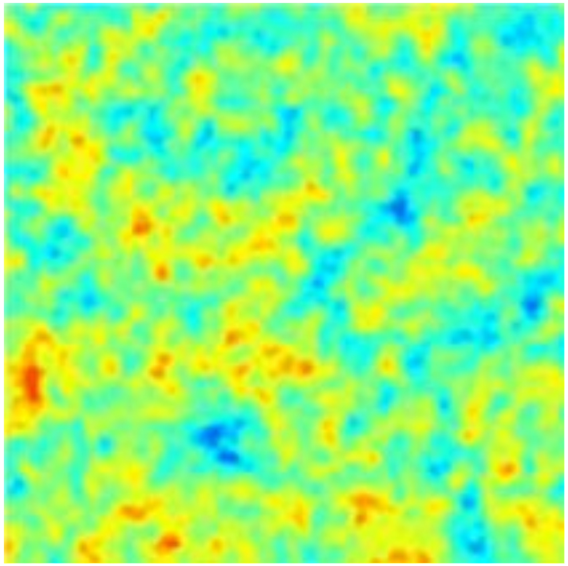
$$\psi(\hat{\mathbf{n}}) = -2\int_0^{\chi_*} \mathrm{d}\chi\, \Psi(\chi\hat{\mathbf{n}};\eta_0-\chi)\,\frac{f_K(\chi^*-\chi)}{f_K(\chi^*)f_K(\chi)}$$

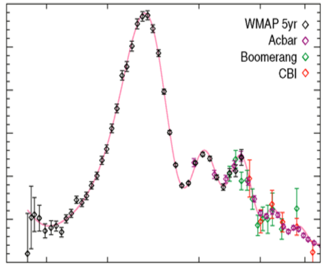
How to measure squeezed non-Gaussianity? Modulation reconstruction

e.g. can reconstruct and hence measure the lensing field

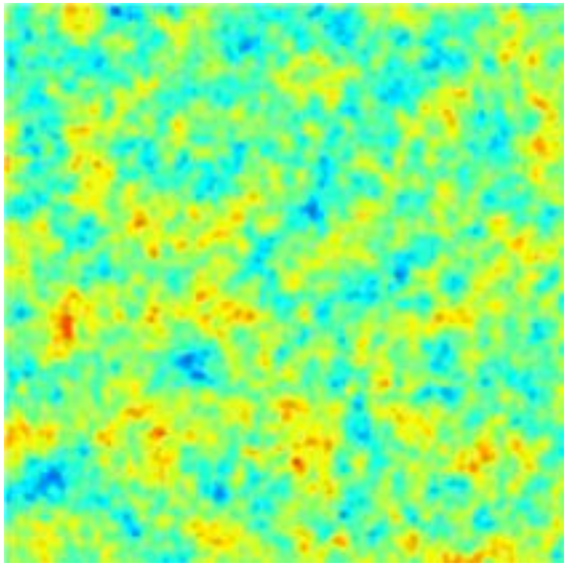
Fractional magnification ~ convergence $\kappa = -\nabla \cdot \boldsymbol{\alpha}/2$

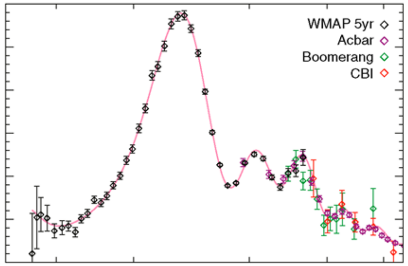
Magnified



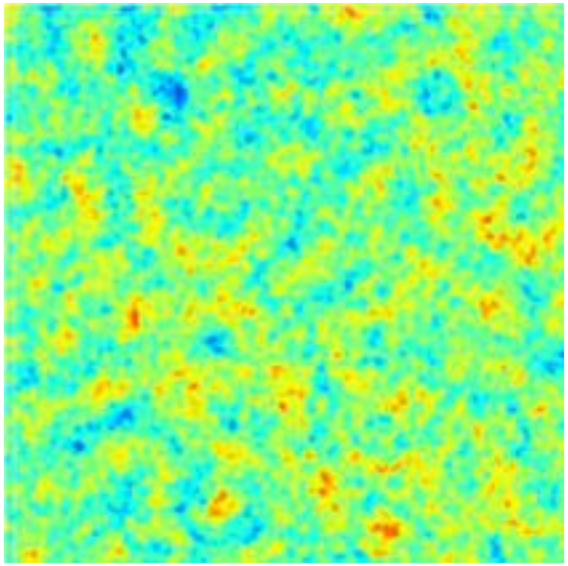


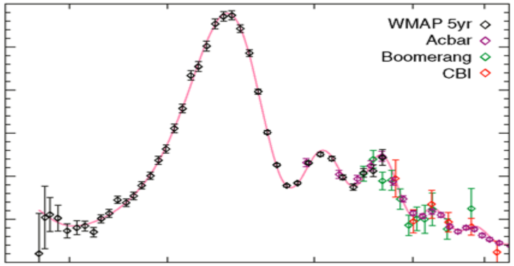
Unlensed



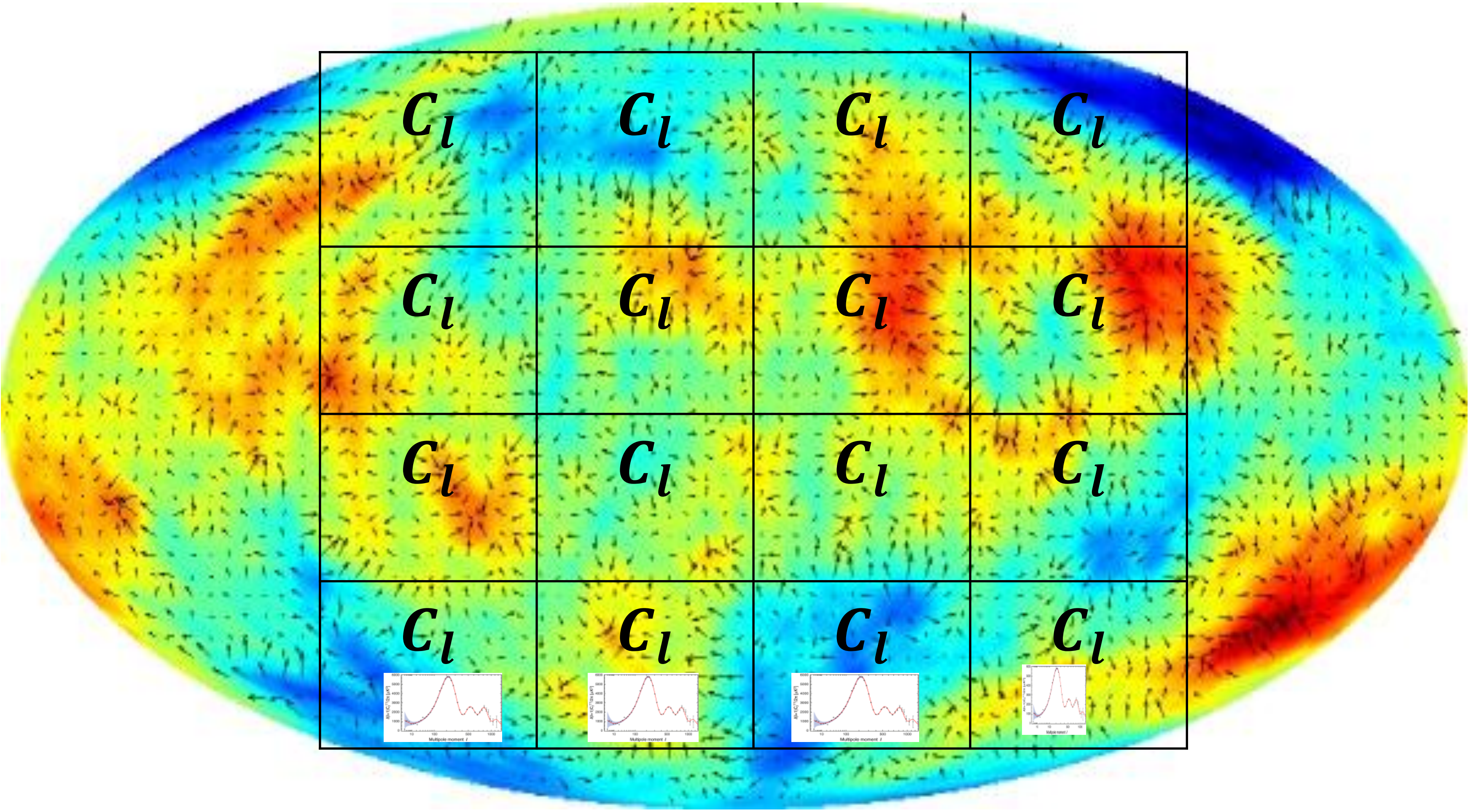


Demagnified





Lensing reconstruction
-concept



Lensing reconstruction

- Maths and algorithm sketch

For a *given* (fixed) lensing field, $T \sim P(T|X)$:

X here is lensing potential, deflection angle, or κ

Flat sky approximation: modes correlated for $\mathbf{k}_2 \neq \mathbf{k}_3 \Rightarrow$ use off-diagonal correlation

First-order series expansion in the lensing field:

$$\langle \tilde{T}(\mathbf{k}_2)\tilde{T}(\mathbf{k}_3)\rangle_{P(\tilde{T}|X)} \approx \int \mathrm{d}\mathbf{K}\, X(\mathbf{K})^* \underbrace{\left\langle \frac{\delta}{\delta X(\mathbf{K})^*}\left(\tilde{T}(\mathbf{k}_2)\tilde{T}(\mathbf{k}_3)\right)\right\rangle}_{\mathcal{A}(K,k_2,k_3)\delta(K+k_2+k_3)} \approx \mathcal{A}(K,k_2,k_3)\; X(\mathbf{K})^*|_{\mathbf{K}=-\mathbf{k}_2-\mathbf{k}_3}$$

function easy to calculate for $X(\mathbf{K}) = 0$

$$A(L,l_1,l_2) \sim \left(\mathbf{l}_1 \cdot \mathbf{L}\tilde{C}_{l_1} + \mathbf{l}_2 \cdot \mathbf{L}\tilde{C}_{l_2}\right)$$

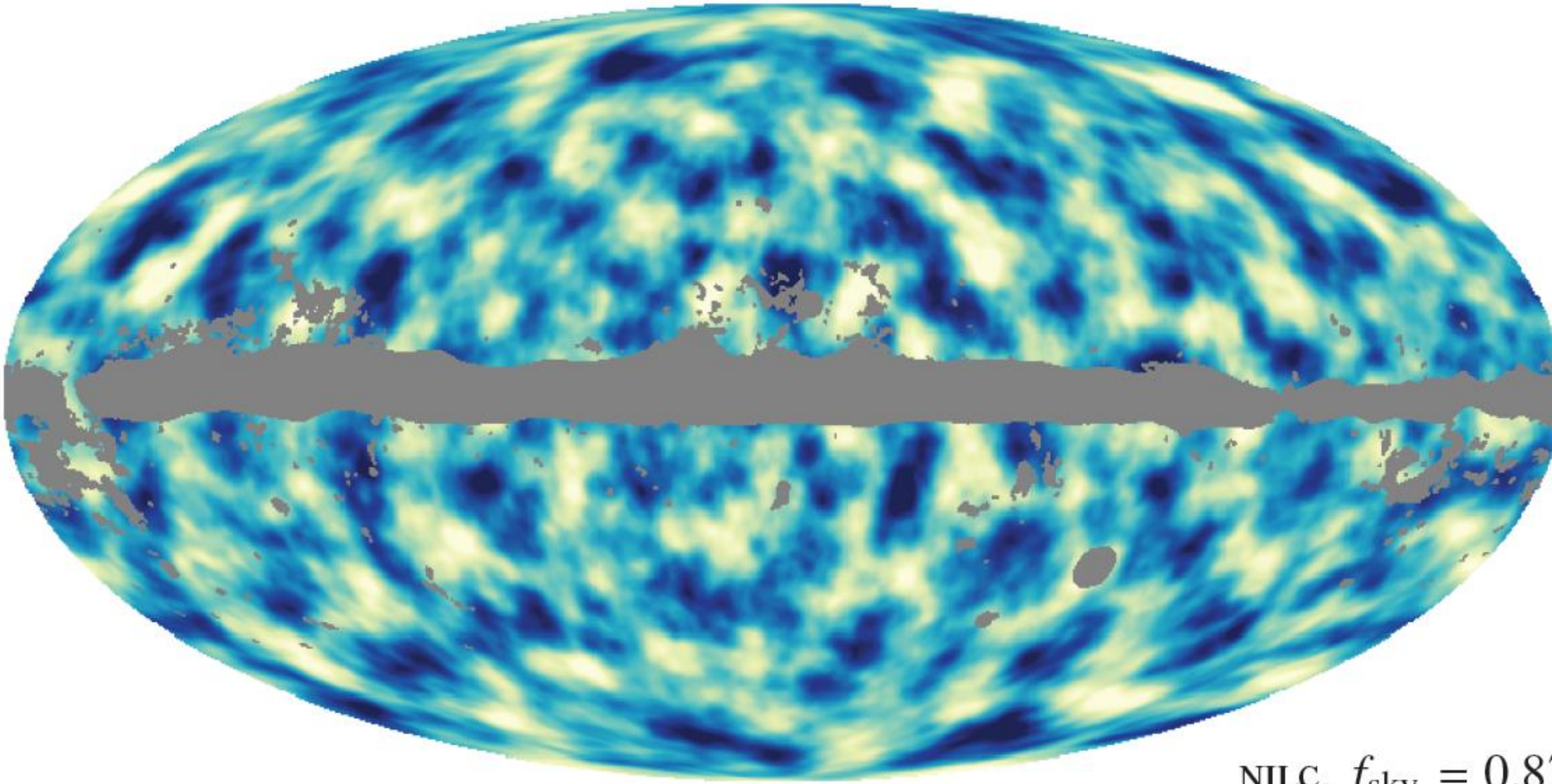
Can reconstruct the modulation field X

Full sky analysis similar, summing modes with optimal weights gives

Quadratic estimator: $\hat{\psi}^*_{l_1 m_1} = N^{(0)}_{l_1} \sum_{l_2 l_3}^{l_1 \le l_2 \le l_3} \Delta^{-1}_{l_1 l_2 l_3} \mathcal{A}^{TT}_{l_1 l_2 l_3} \sum_{m_2 m_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \frac{\tilde{T}_{l_2 m_2}\tilde{T}_{l_3 m_3}}{\tilde{C}^{TT}_{\mathrm{tot}\; l_2}\tilde{C}^{TT}_{\mathrm{tot}\; l_3}}$

Zaldarriaga, Hu, Hanson, and others.

Planck lensing potential reconstruction: estimate of modulation field $\hat{\psi}$



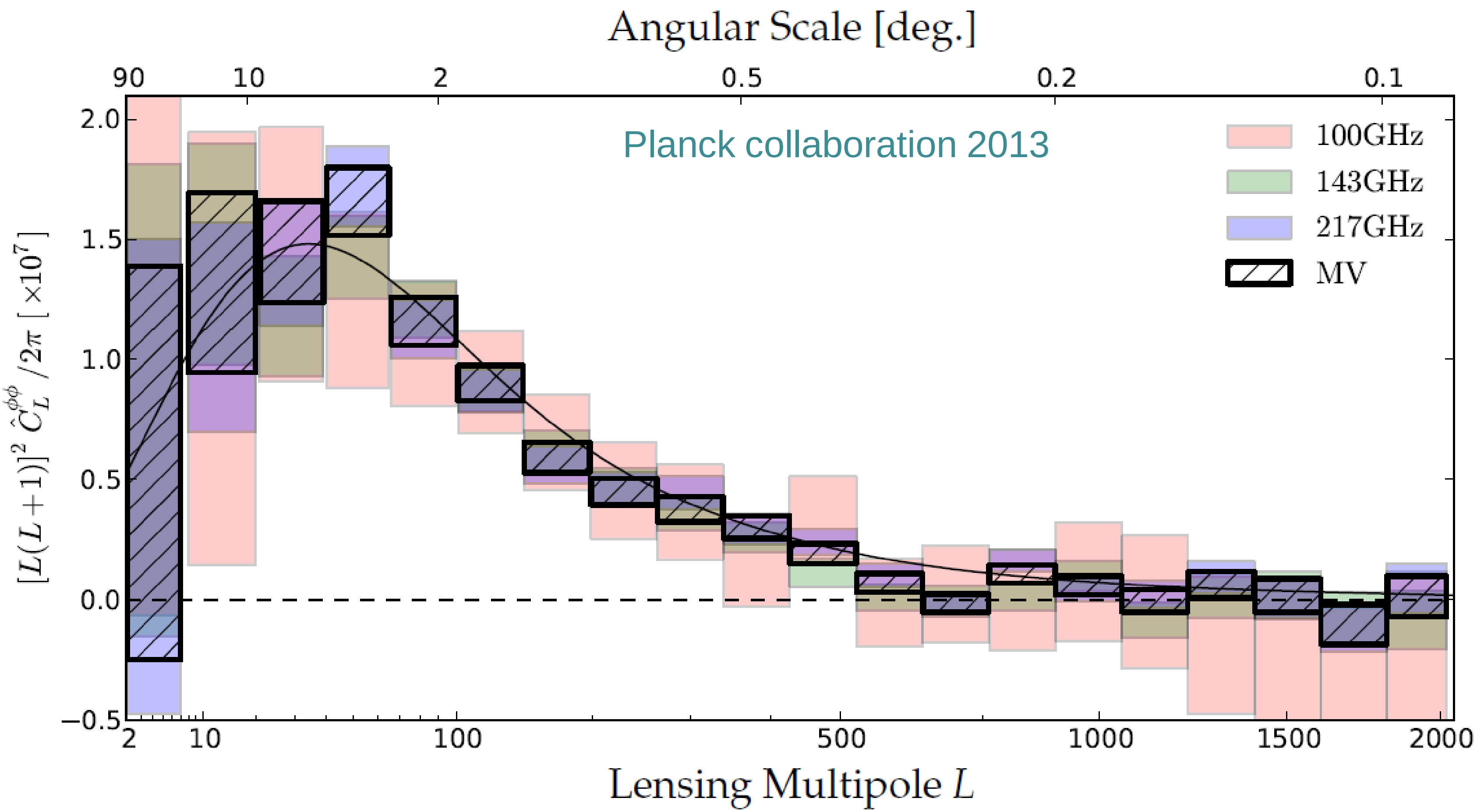
NILC, $f_{\rm sky} = 0.87$

Planck collaboration 2013

Note – about half signal, half reconstruction noise, not all structures are real
map is effectively Wiener filtered

Correlation of $\hat{\psi}$ with T $\Rightarrow$ Bispectrum; $\hat{\psi}$ power spectrum $\Rightarrow$ Trispectrum

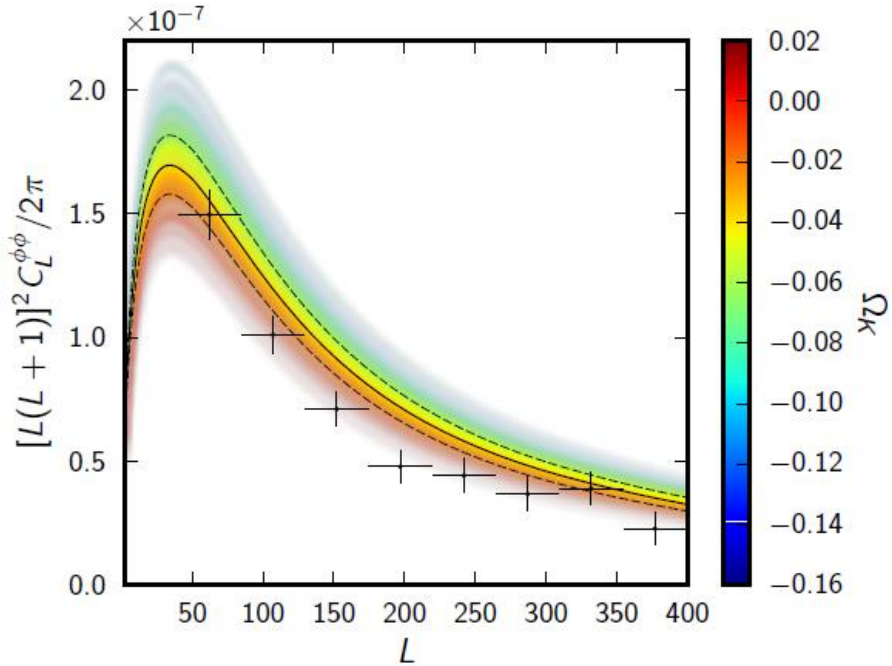
Power spectrum of reconstruction ⇒ $C_l^{\psi\psi}$

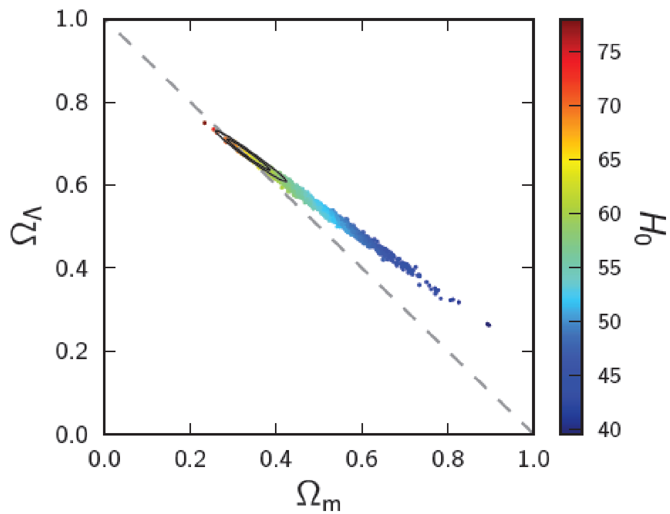


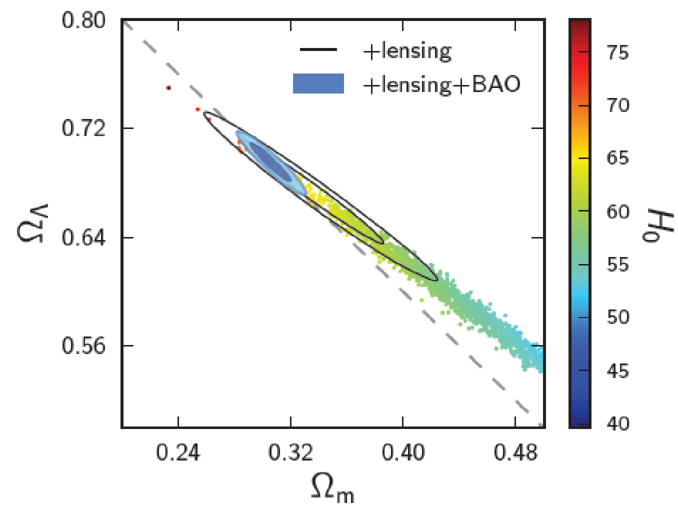
Measured at high significance: probes perturbations along the line of sight to recombination

Extra information in lensing can help break parameter degeneracies

Colour: Planck TT constraint
Crosses: Planck lensing



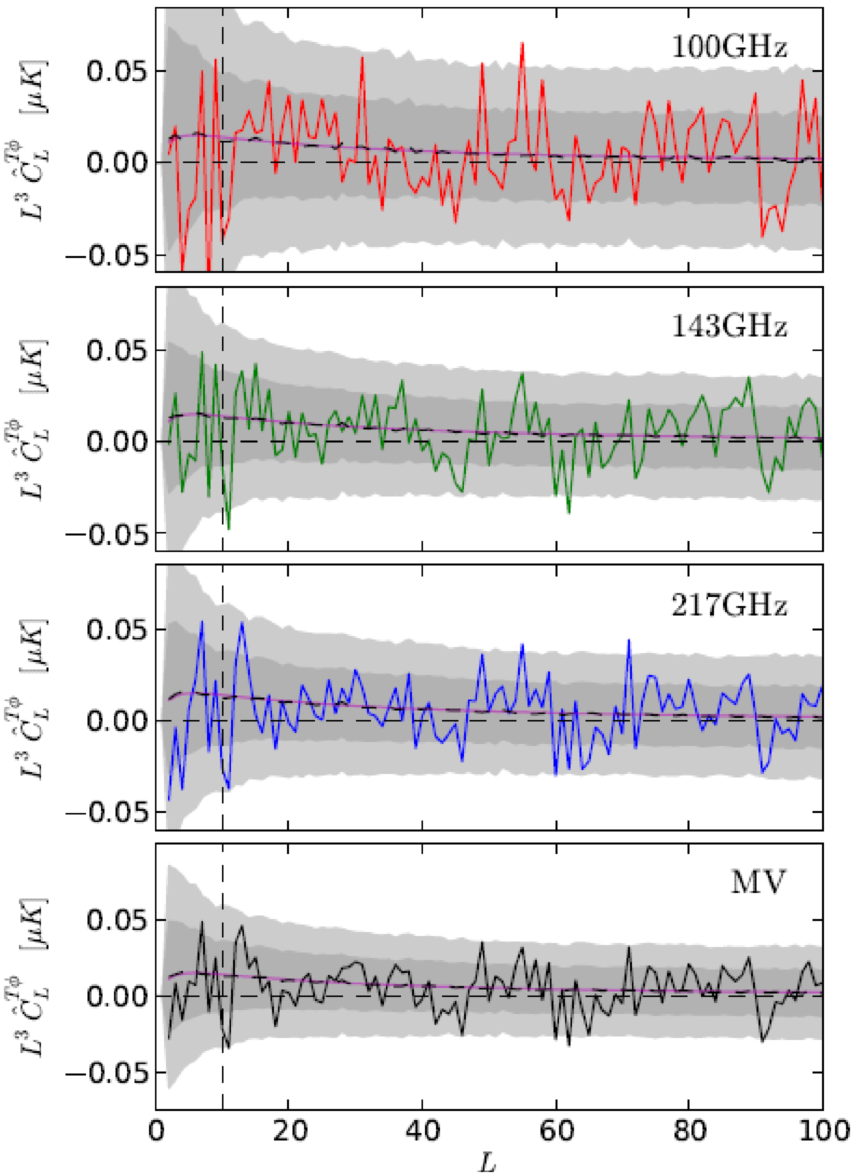




Trispectrum: power of the modulation $\Rightarrow$ $C_l^{\psi\psi}$

Bispectrum: modulation $\times$ temperature $\Rightarrow$ $C_l^{T\psi}$

Planck: first detection of lensing bispectrum!



Large cosmic variance and reconstruction noise, but 'detected' at $\sim 2.5\sigma$

Table 2. Results for the amplitude of the ISW-lensing bispectrum from the SMICA, NILC, SEVEM, and C-R foreground-cleaned maps, for the KSW, binned, and modal (polynomial) estimators; error bars are 68% CL.

	SMICA	NILC	SEVEM	C-R
KSW	0.81 ± 0.31	0.85 ± 0.32	0.68 ± 0.32	0.75 ± 0.32
Binned	0.91 ± 0.37	1.03 ± 0.37	0.83 ± 0.39	0.80 ± 0.40
Modal	0.77 ± 0.37	0.93 ± 0.37	0.60 ± 0.37	0.68 ± 0.39

Planck collaboration 2013

Primordial bispectrum?

Table 8. Results for the $f_{\rm NL}$ parameters of the primordial local, equilateral, and orthogonal shapes, determined by the KSW estimator from the SMICA foreground-cleaned map. Both independent single-shape results and results marginalized over the point source bispectrum and with the ISW-lensing bias subtracted are reported; error bars are 68% CL.

	Independent KSW	ISW-lensing subtracted KSW
SMICA		
Local	9.8 ± 5.8	**2.7 ± 5.8**
Equilateral	-37 ± 75	**-42 ± 75**
Orthogonal	-46 ± 39	**-25 ± 39**

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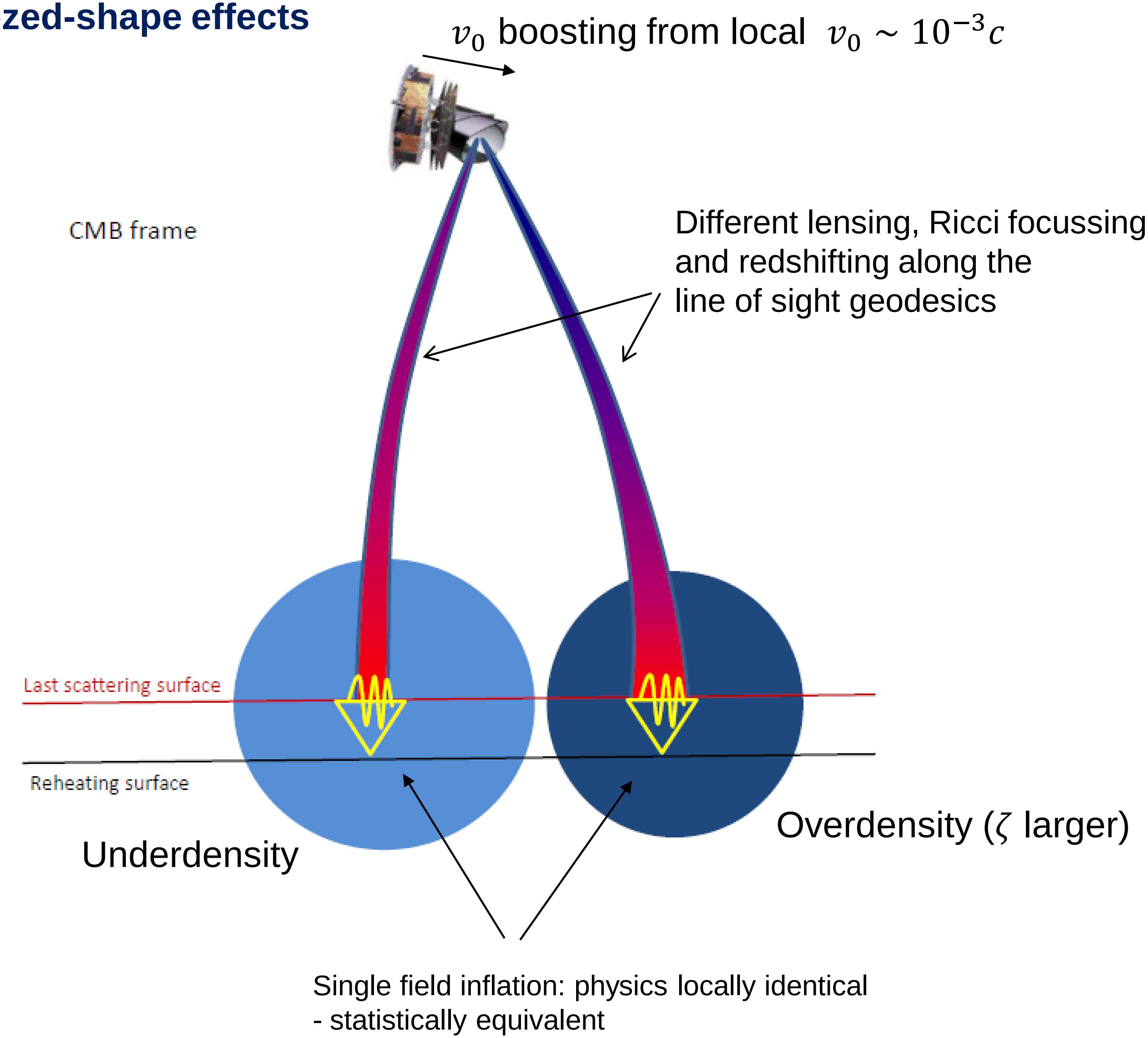
Planck only sees expected lensing-induced modulations
- no evidence for primordial bispectrum

Other squeezed-shape effects



- Redshifting as photons travel through perturbed universe and then Doppler shifted by earth's motion

$$T \rightarrow \left(1 + \frac{\Delta T}{T}\right) T \quad \Rightarrow \quad \Delta T_{\mathrm{small}} \rightarrow (1 + \Delta T_{\mathrm{large}}) \Delta T_{\mathrm{small}}$$

Only Doppler term non-negligible

- Transverse directions also affected:
  perturbations at last scattering are distorted

  Shear + Convergence + Ricci focussing + Aberration

Perturbed angular diameter distance:

$$\mathcal{D}/2 = \chi_* a_* \left[1 + \frac{\delta\chi}{\chi_*} + \underbrace{\frac{\Delta_\gamma}{4} - \Phi} - \kappa + \hat{\mathbf{n}} \cdot \mathbf{v}_A \right]$$

Radial displacement (small, $\delta\chi \ll \chi_*$)

$\zeta_\gamma \equiv \Delta_\gamma/4 - \Phi$

Ricci focussing: δN expansion of ray bundle

Convergence (lensing)

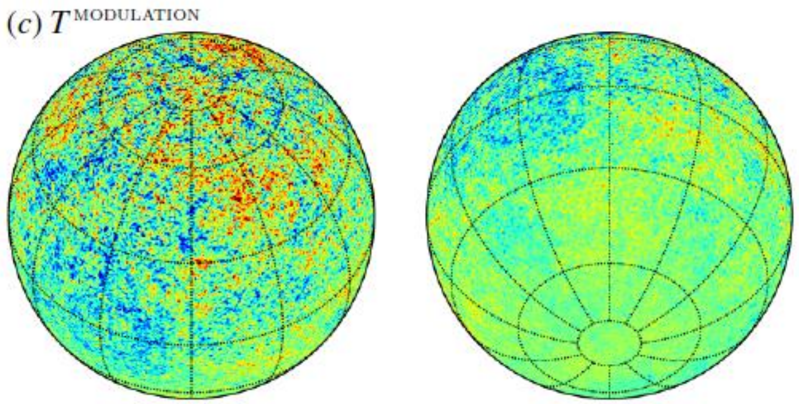
Local aberration

Kinematic dipole
(Doppler aberration ≡ dipole lensing convergence)

Power modulation

$$\Delta\Theta(\hat{\boldsymbol{n}}) \rightarrow \left[1 + \hat{\boldsymbol{n}}\cdot\boldsymbol{v} + T\frac{\mathrm{d}^2 I_\nu/\mathrm{d}T^2}{\mathrm{d}I_\nu/\mathrm{d}T}\hat{\boldsymbol{n}}\cdot\boldsymbol{v}\right]\Delta\Theta(\hat{\boldsymbol{n}})$$
$$= (1 + [x\coth(x/2) - 1]\,\hat{\boldsymbol{n}}\cdot\boldsymbol{v})\,\Delta\Theta(\hat{\boldsymbol{n}}),$$

$$x \equiv h\nu/k_b T$$

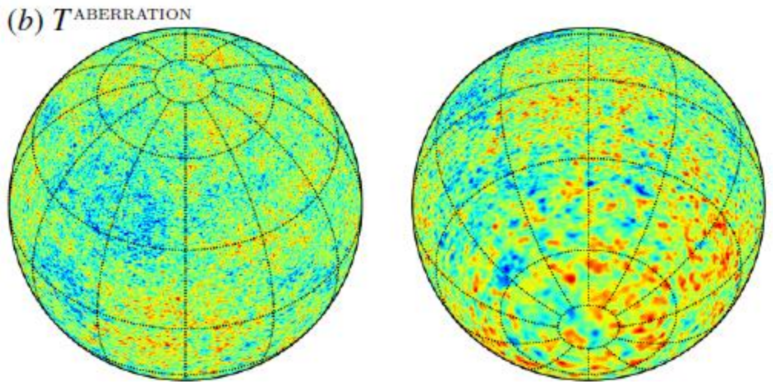


Illustrated for $\frac{v}{c} = 0.85$

Aberration

$$\hat{\boldsymbol{n}} \rightarrow \hat{\boldsymbol{n}} \quad + \nabla(\hat{\boldsymbol{n}}\cdot\boldsymbol{v})$$

- just like a dipole lensing convergence



Challinor & van Leeuwen 2002

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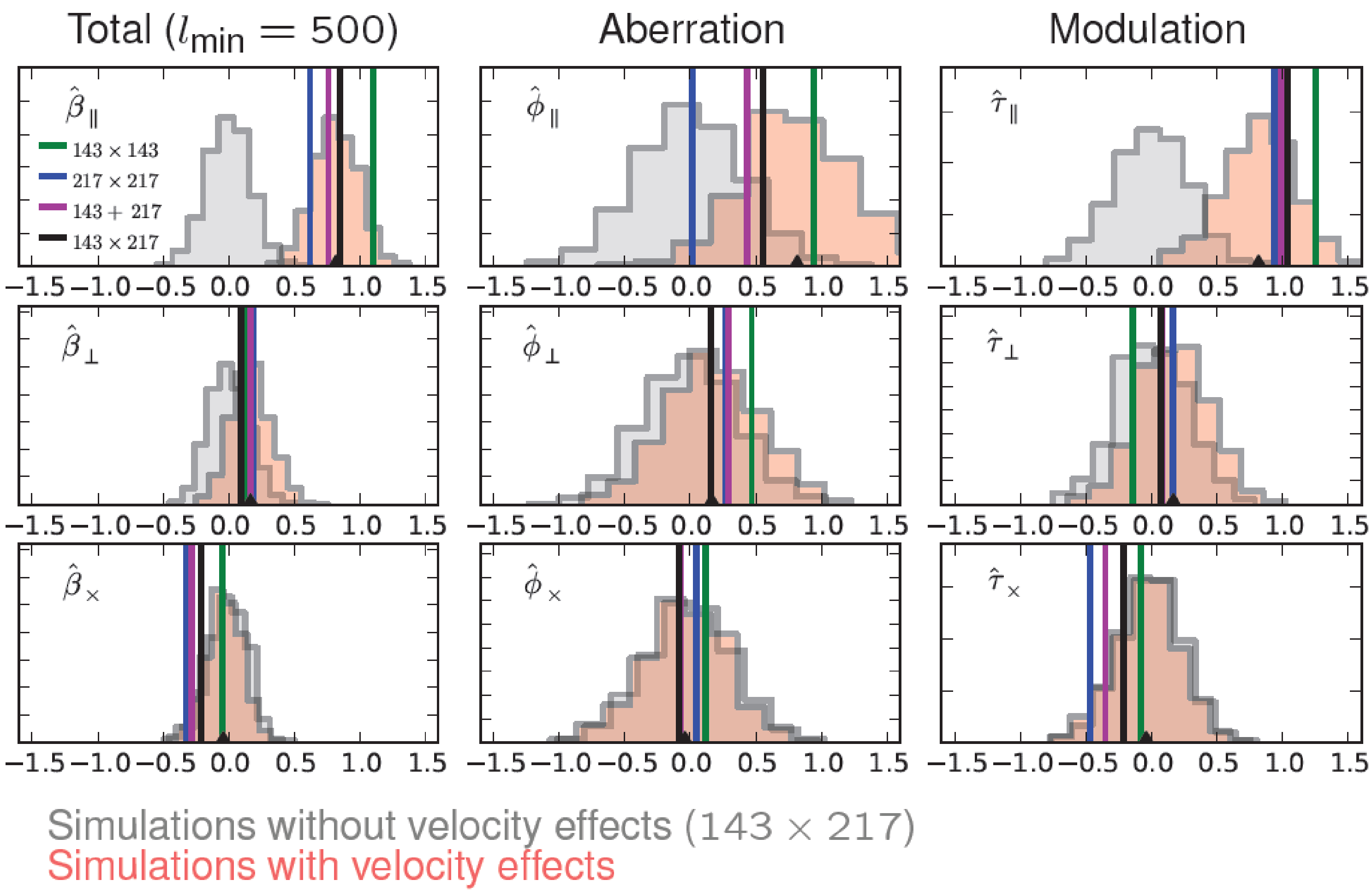


- 5σ detection in 143×217: $v_{\parallel} = (384 \pm 78)\,\mathrm{km\,s^{-1}}$
- Foreground issue at 217×217 in $\widehat{\beta}_\times$ (driven by $\widehat{\tau}_\times$)?

Planck collaboration 2013

The future: Non-blackbody signals in the CMB?

e.g. 1. Spectral distortions in monopole (COBE $\Rightarrow$ small)
2. Frequency-dependent anisotropies

1. Spectral distortions. E.g. μ-distortion

$$f_\gamma(E,T) = \frac{2}{(2\pi)^3}\frac{1}{e^{(E-\mu_\gamma)/T}-1}$$

Full thermal equilibrium: $\mu_\gamma = 0$

Kinetic equilibrium: $\mu_\gamma \neq 0$ if energy deposited but photon number cannot change

Measurement of μ can measure energy injection well before recombination

e.g.

- Dark matter and relic decay/annihilation
- Integral of small-scale primordial power spectrum (via. silk damping energy release)

Rayleigh scattering

blue sky thinking for future CMB observations

arXiv:1307.8148; previous work: Takahara et al. 91, Yu, et al. astro-ph/0103149

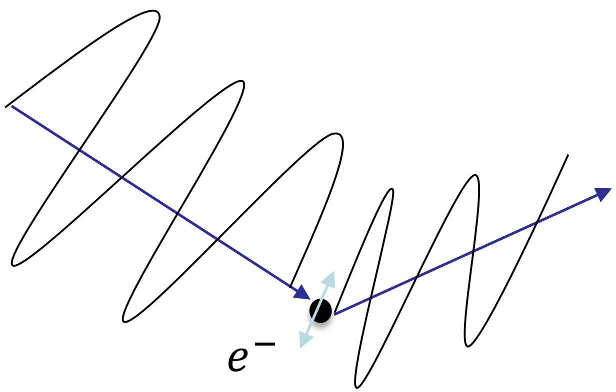
http://en.wikipedia.org/wiki/Rayleigh_scattering

Classical dipole scattering

Oscillating dipole $\boldsymbol{p} = p_0 \sin(\omega t)\,\hat{\boldsymbol{z}} \Rightarrow$ radiated power $\propto \omega^4 p_0^2 \sin^2\theta\, \mathrm{d}\Omega$

Thomson Scattering

$$m_e \ddot{z} = -eE_z \sin\omega t$$
$$\Rightarrow \boldsymbol{p} = \frac{-e^2 E_z}{m_e \omega^2} \sin\omega t\ \hat{\boldsymbol{z}}$$



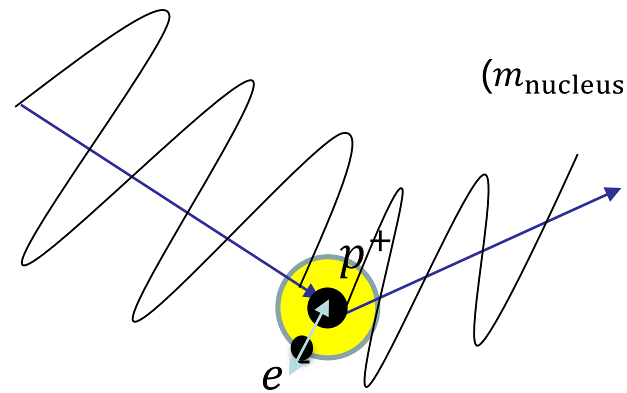
Frequency independent
Given by fundamental constants:

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\epsilon_0 m_e c^2}\right)^2$$

Rayleigh Scattering

$$m_e \ddot{z} = -eE_z \sin\omega t - m_e \omega_0 z$$
$$\Rightarrow \boldsymbol{p} = \frac{-e^2 E_z}{m_e(\omega^2 - \omega_0^2)} \sin\omega t\ \hat{\boldsymbol{z}}$$

$(m_{\text{nucleus}} \gg m_e)$



Frequency dependent
Depends on natural frequency ω_0 of target

$$\sigma_R \approx \frac{\omega^4}{\omega_0^4}\sigma_T \qquad (\omega \ll \omega_0)$$

http://farside.ph.utexas.edu/teaching/em/lectures/node95.html

Photon scattering rate

Total cross section ≈ $\Gamma(\nu) = n_e\sigma_T + \sigma_R(\nu)\left[n_H + R_{He}n_{He}\right]$

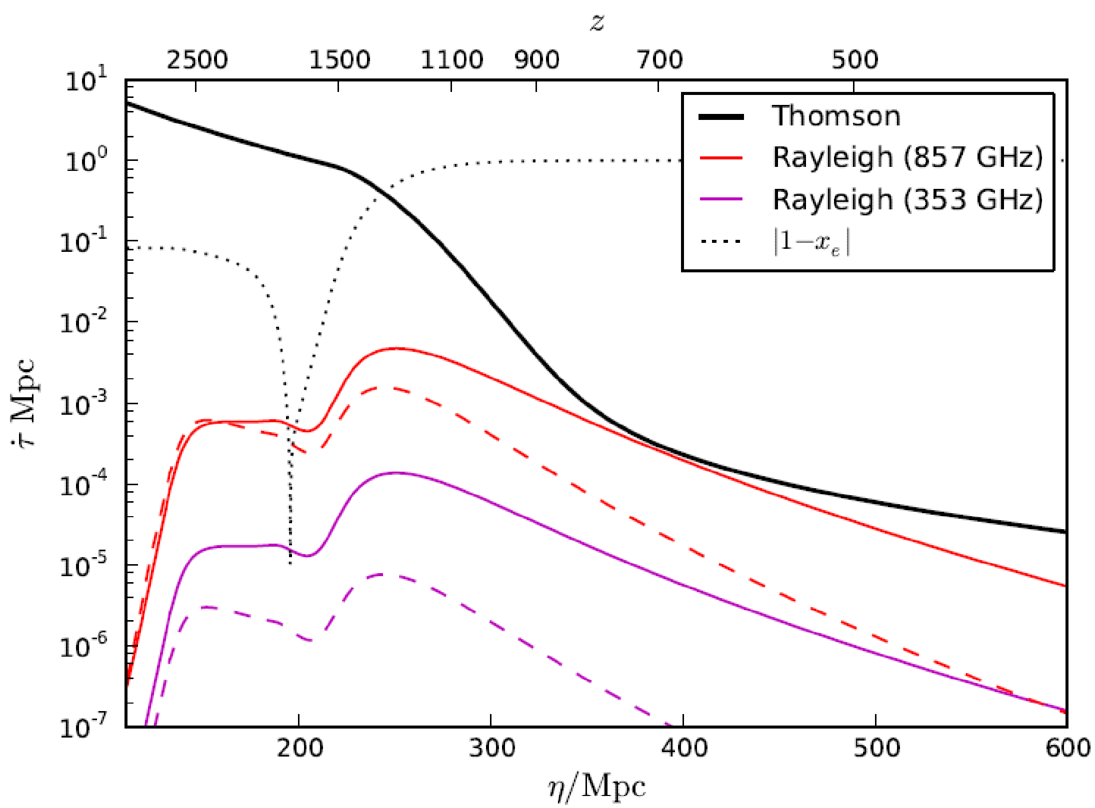
$$\sigma_R(\nu) = \left[\left(\frac{\nu}{\nu_{\rm eff}}\right)^4 + \frac{638}{243}\left(\frac{\nu}{\nu_{\rm eff}}\right)^6 + \frac{1626820991}{136048896}\left(\frac{\nu}{\nu_{\rm eff}}\right)^8 + \dots\right]\sigma_T \qquad \nu_{\rm eff} \equiv \sqrt{\frac{8}{9}}\, c\, R_A \approx 3.1\times 10^6 \mathrm{GHz} \quad , R_{He} \approx 0.1$$

(Lee 2005: Non-relativistic quantum calculation, for energies well below Lyman-alpha)

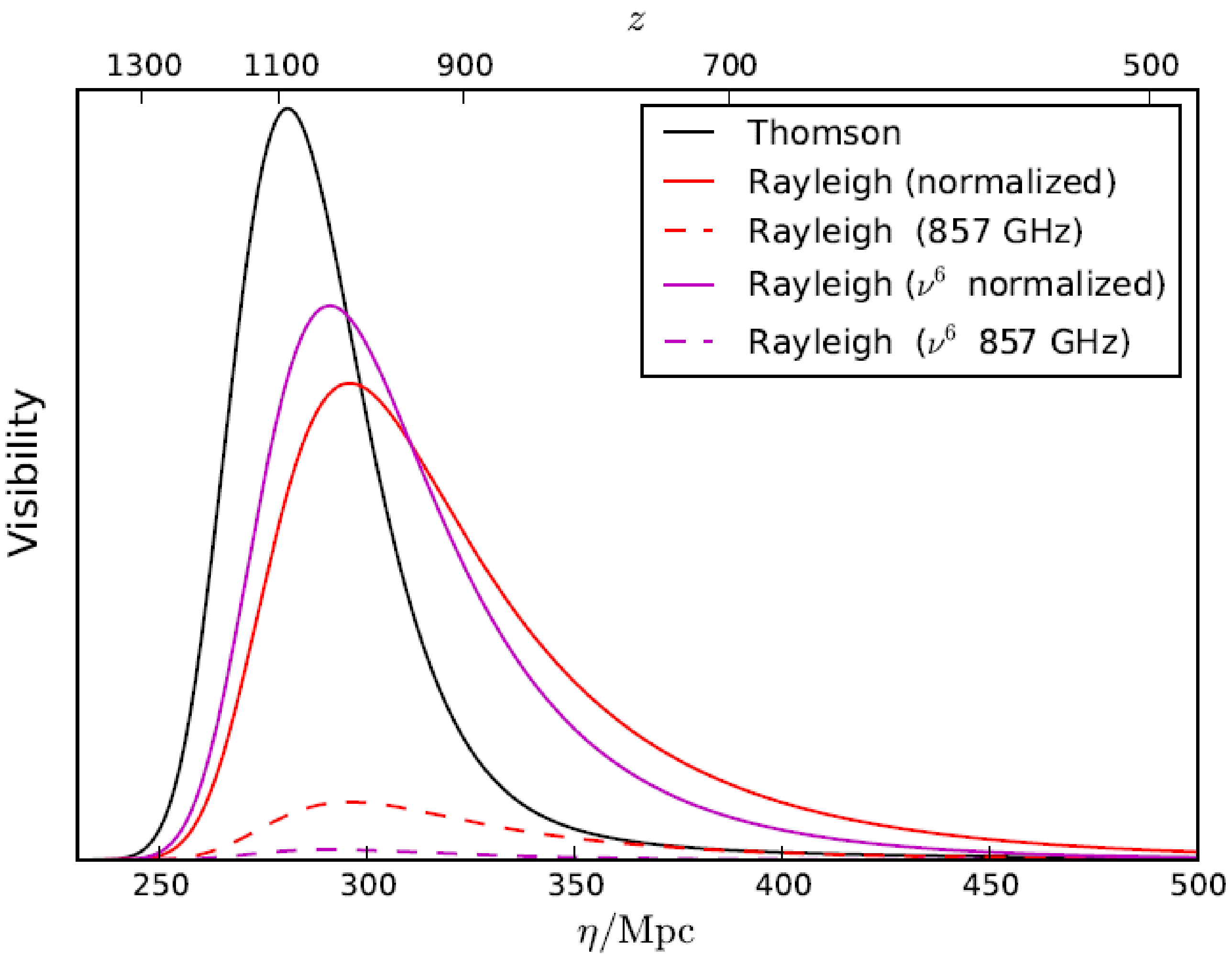
$$\dot\tau = \Gamma/(1+z)$$

Rayleigh only negligible compared to Thomson for

$$n_H\left(\frac{(1+z)\nu_{\rm obs}}{3\times10^6\mathrm{GHz}}\right)^4 \ll n_e$$



Visibility

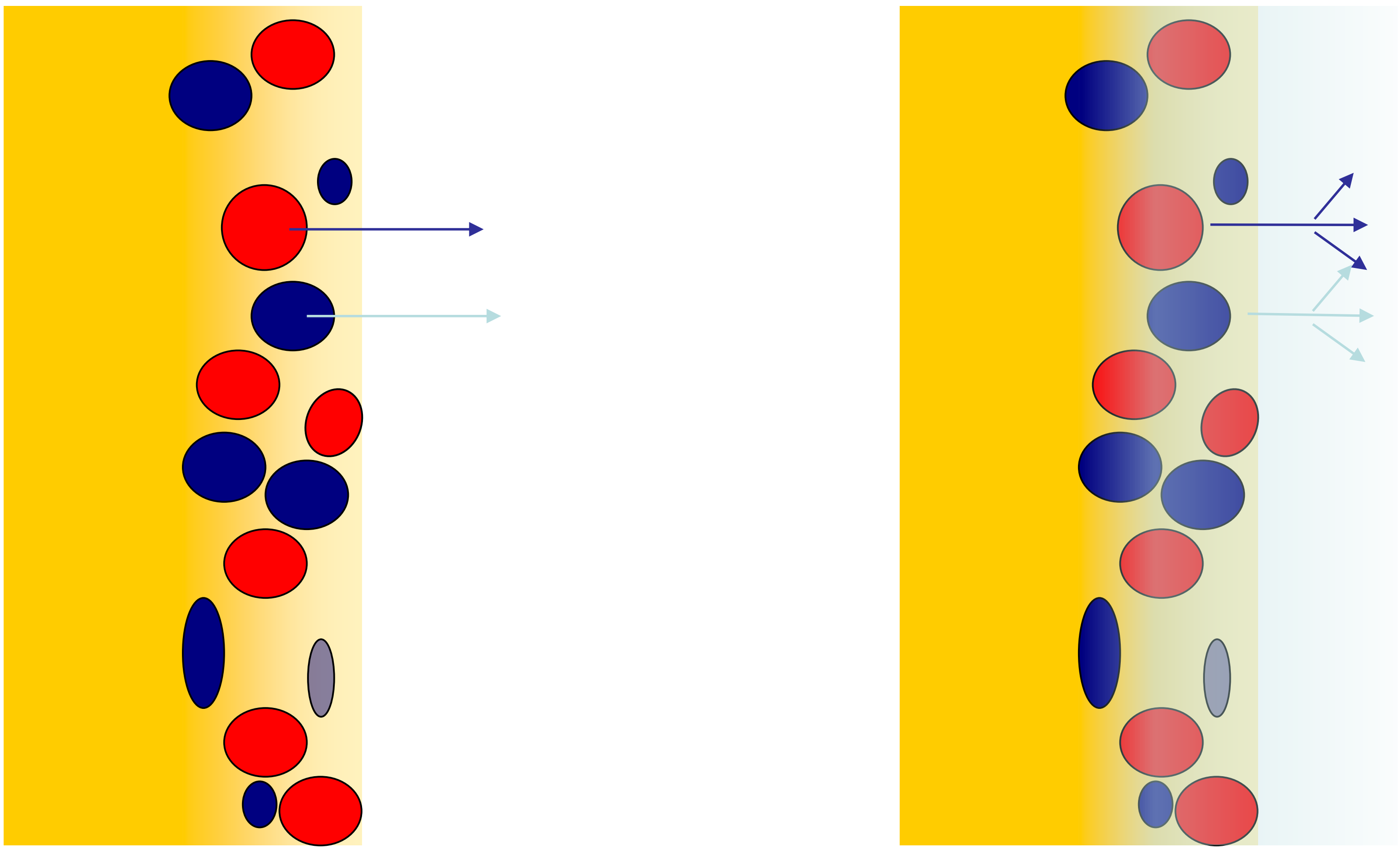


z
1300
1100
900
700
500
Thomson
Rayleigh (normalized)
Rayleigh (857 GHz)
Rayleigh (ν^6 normalized)
Rayleigh (ν^6 857 GHz)
Visibility
250
300
350
400
450
500
η/Mpc

Small-scale CMB

Primary signal

Primary + Rayleigh signal



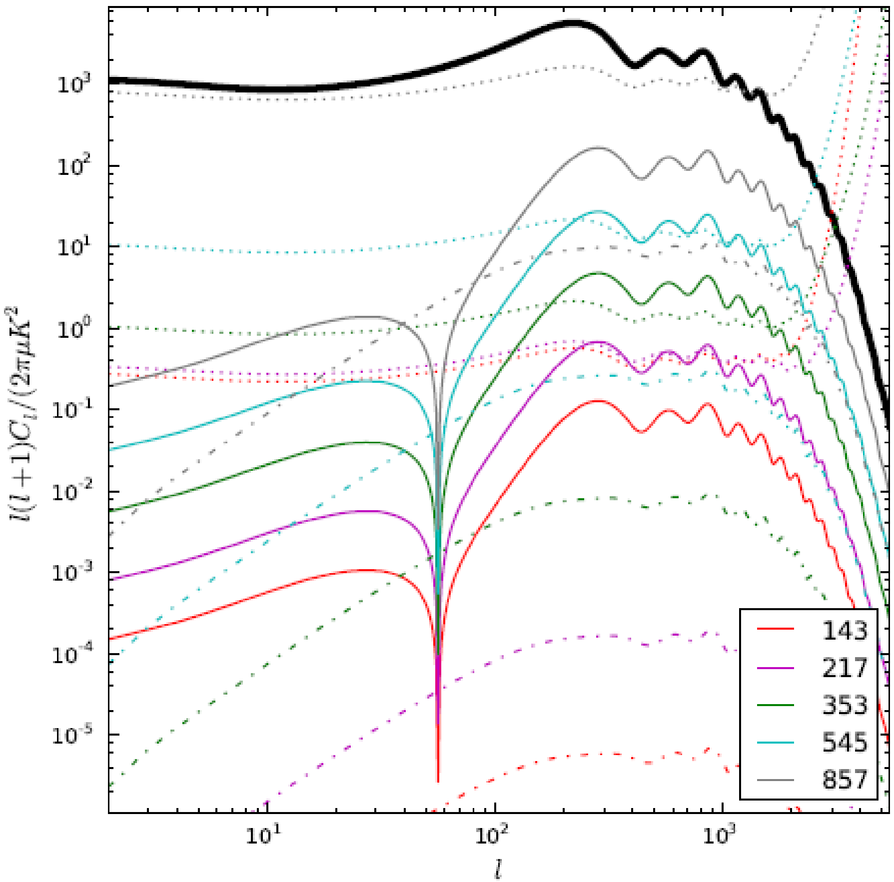
Small-scale CMB cont.

Hot spots are red, cold spots are blue
Polarization is scattered and is red too

Rayleigh difference signal: (photons scattered in to line of sight) – (scattered out)
$\sim \tau_R \, \Delta T$ ⇨ very correlated to primary ΔT

Rayleigh temperature power spectrum

$$(\text{Primary+Rayleigh})^2 = \text{Primary}^2 + 2\ \text{Primary} \times \text{Rayleigh} + \text{Rayleigh}^2$$



Solid: Rayleigh × Primary
Dot-dashed: Rayleigh × Rayleigh

Dots: naïve Planck sensitivity to the cross per $\Delta l/l$ = 10 bin
(possibly 5σ with Planck full mission)

Small-scale signal is highly correlated to primary

Can hope to isolate using
Low frequency × High frequency

Note: not limited by cosmic variance of primary anisotropy
– multi-tracer probe of same underlying perturbation realization

Test of expansion and ionization history at recombination

Large-scale CMB temperature

Rayleigh signal only generated by sub-horizon scattering
(no Rayleigh monopole background to distort by anisotropic photon redshifting)

$$\frac{\Delta T_0}{T}(\hat{\mathbf{n}}) = \frac{\Delta_\gamma(\eta_*)}{4} + \underbrace{\Psi(\eta_*) - \Psi_0}_{\text{Sachs-Wolfe}} + \underbrace{\hat{\mathbf{n}} \cdot (\mathbf{v}_o - \mathbf{v})}_{\text{Doppler}} + \underbrace{\int_{\eta_*}^{\eta_0} d\eta (\Psi' + \Phi')}_{\text{ISW}}$$

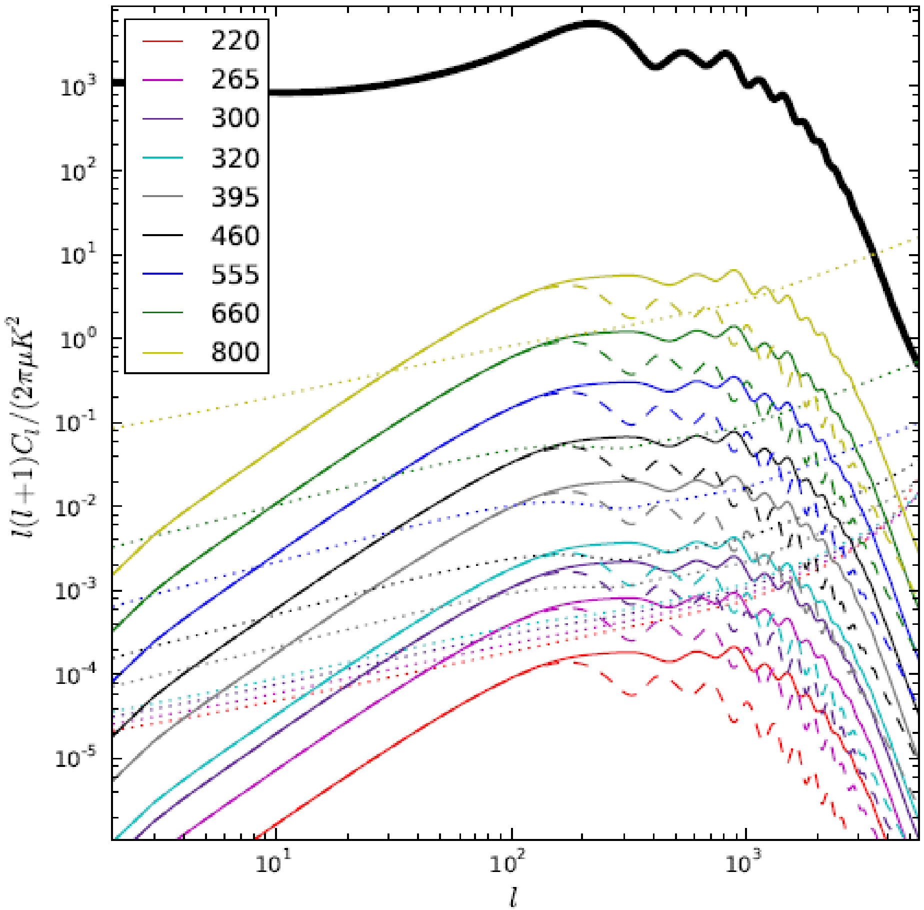
Temperature perturbation at recombination
(Newtonian Gauge)

Rayleigh scattering probes Doppler terms independently of SW/ISW

Measure new primordial modes with Rayleigh×Rayleigh spectrum?

In principle could double number of modes compared to T+E!

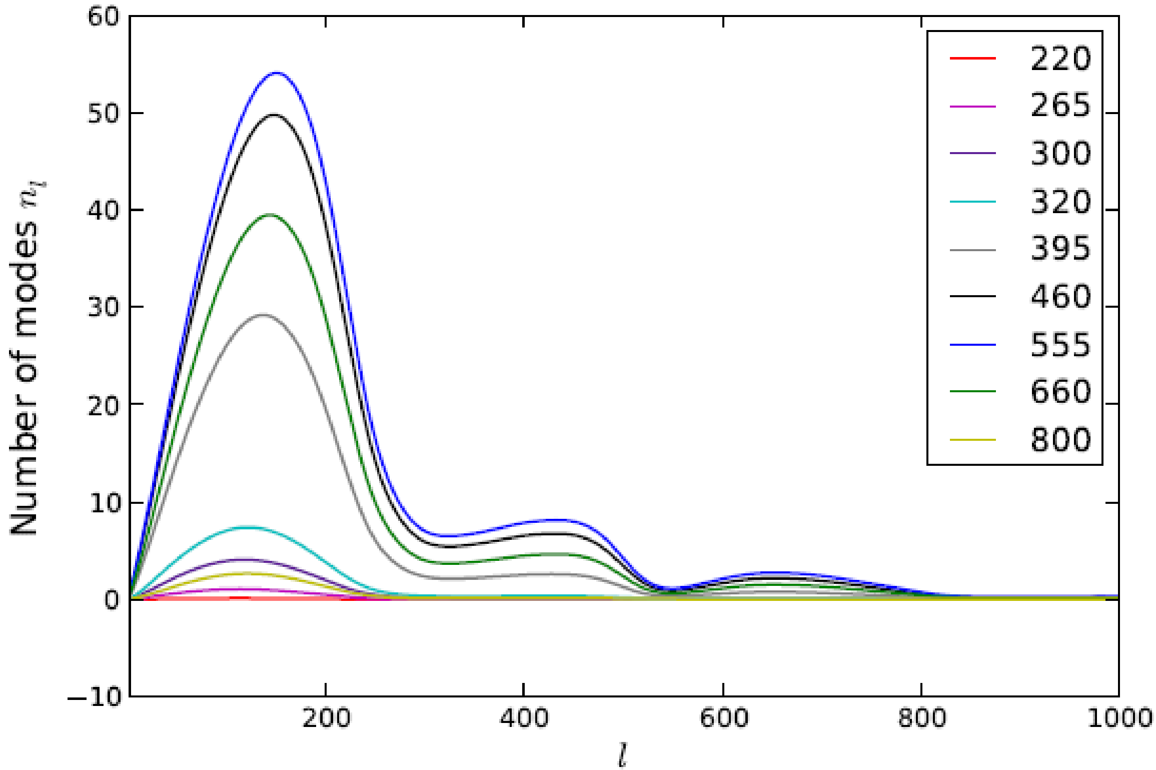
BUT: signal highly correlated to primary on small scales; need the uncorrelated part



Solid: Rayleigh× Rayleigh total; Dashed: uncorrelated part; Dots: error per $\frac{\Delta l}{l} = 10$ bin a from PRISM

Number of new modes with future CMB: e.g. PRISM

Define $n_l \equiv (2l+1) f_{\rm sky} \mathrm{Tr}\left[\left([\boldsymbol{C}_l + \boldsymbol{N}_l]^{-1} \boldsymbol{C}_l \right)^2 \right]$



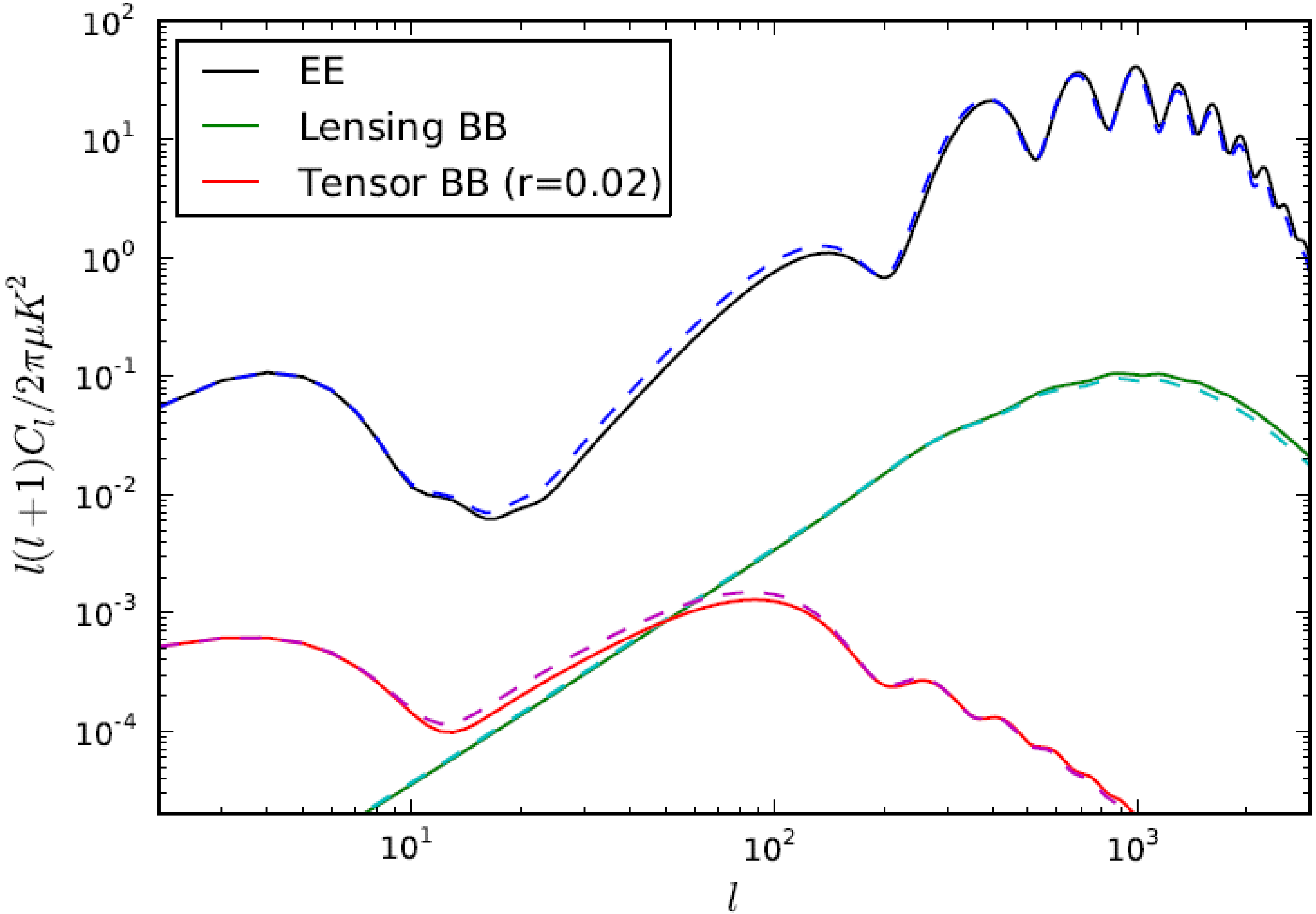
New modes almost all in the $l \leq 500$ temperature signal: total $\approx 10\,000$ extra modes

More horizon-scale information (disentangle Doppler and Sachs-Wolfe terms)

Would need much higher sensitivity to get more modes from polarization/high l

Rayleigh polarization power spectra

Solid: primary　　　Dashed: primary + Rayleigh (857GHz)

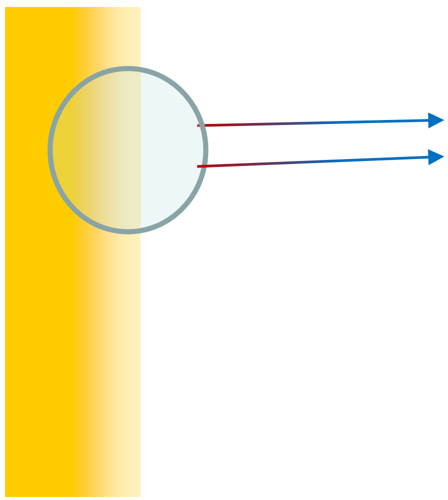


Large-scale polarization from scattering into the line of sight ⇒ polarized CMB sky is blue

but same quadrupole, so highly correlated to primary

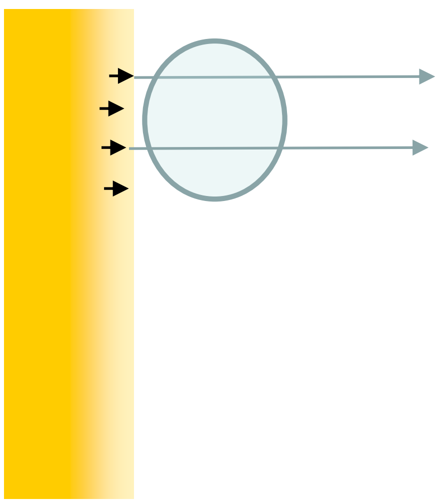
On horizon scales three nearly-independent perturbation modes being probed

$\frac{\Delta T}{T} + \Phi + ISW$
(anisotropic redshifting to constant temperature recombination surface)



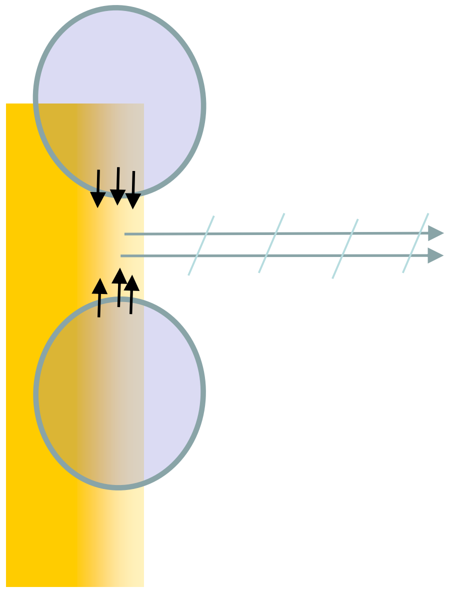
Primary

$\hat{\mathrm{n}} \cdot \mathrm{v_b}$: Doppler



Rayleigh, Primary

Polarization from quadrupole scattering

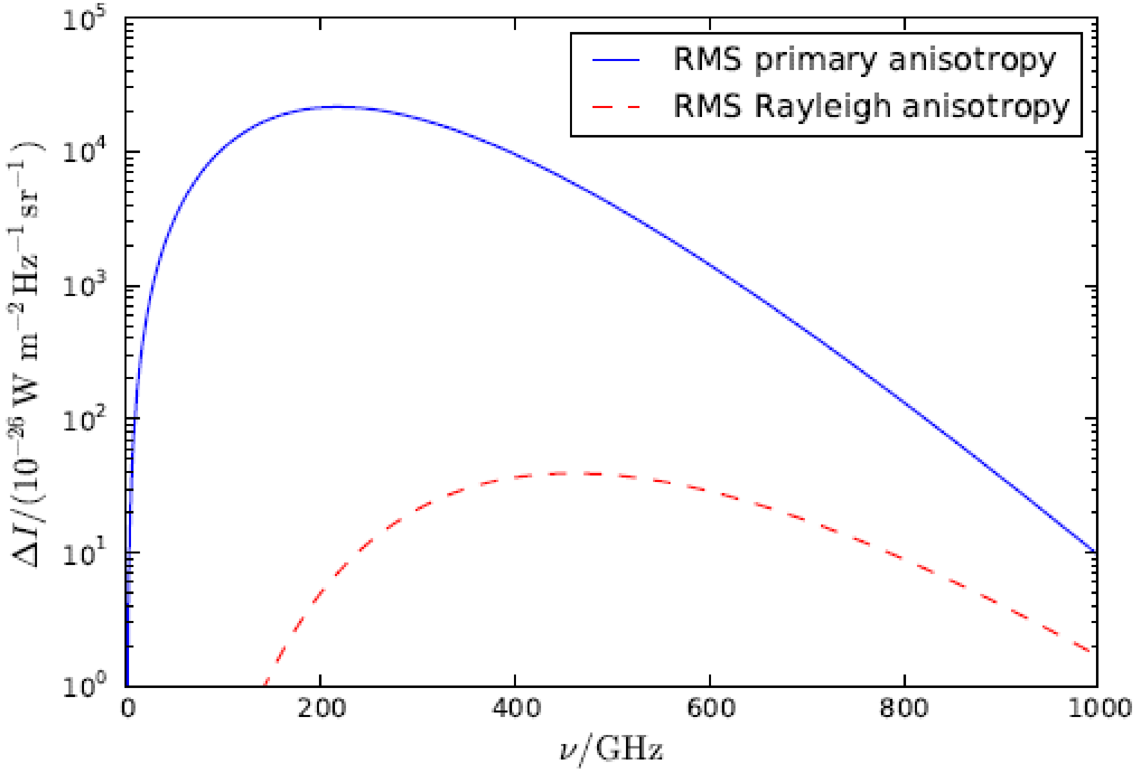


Rayleigh, Primary

Expected signal as function of frequency

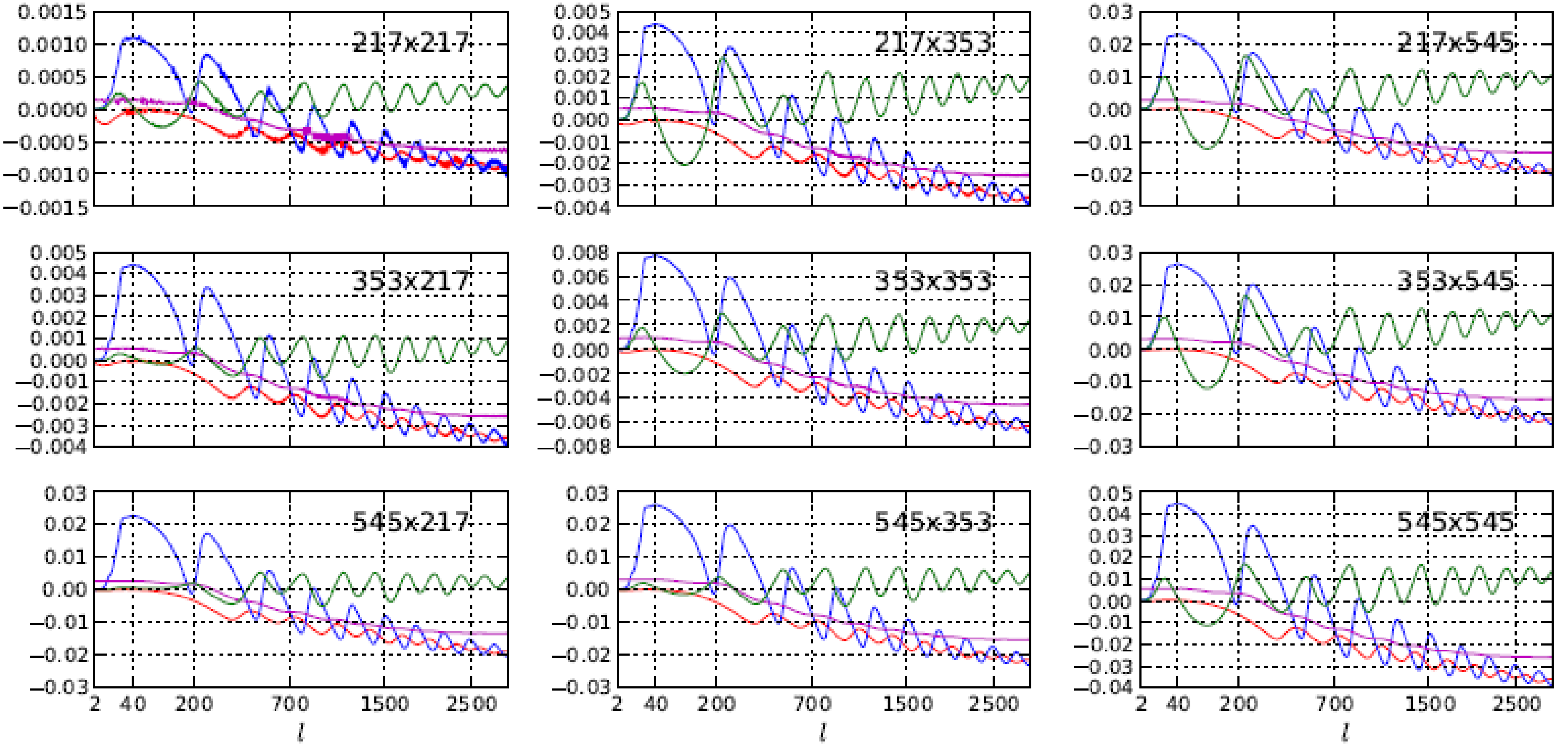
Zero order: uniform blackbody not affected by Rayleigh scattering (elastic scattering, photons conserved)

1st order: anisotropies modified, no longer frequency independent



Need sensitivity at $200\mathrm{GHz} \leq \nu \leq 800\mathrm{GHz}$
(+probably higher for foreground separation efficiency; very hard above 350GHz from ground)

Fractional total C_l differences at realistic frequencies



TT, EE, BB: $\frac{\Delta C_l}{C_l}$ TE: $\frac{\Delta C_l^{TE}}{\sqrt{C_l^{EE} C_l^{TT}}}$

Rayleigh summary

- Significant Rayleigh signal at $\nu \geq 200\ \mathrm{GHz}$; several percent on T, E at $\nu \geq 500\mathrm{GHz}$
- Non-blackbody signal in the anisotropies (but no spectral distortion in monopole)
- Strongly correlated to primary signal on small scales (mostly damping)
  – robust detection via cross-correlation?
- Powerful test of recombination physics/expansion
- Boosts large-scale polarization (except B modes from lensing)
- Multi-tracer probe of last-scattering
  - limited by noise/foregrounds, not cosmic variance
- May be able to provide additional primordial information (10,000+ modes)
  - mostly horizon-scale T modes at recombination from Doppler signal

Conclusions

- Planck TT marks the beginning of the end for C_l^{TT}
  - cosmic variance limited to $L \sim 1600$

- Planck lensing marks end of the beginning for $C_l^{\phi\phi}$

  - first nearly fully-sky maps of the lensing potential, limited by reconstruction noise
  - detected bispectrum; trispectrum measured at high significance
  - must be modelled (+Doppler) to measure primordial non-Gaussianity
  - can break parameter degeneracies (but not yet competitive with e.g. BAO)

  Future: SPT TT (~months), Planck full mission (1 yr), ACTpol, SPTpol

- Non-blackbody future: PRISM, Pixie, … ??

  - Spectral distortions: probe inflation at scales $k \gg 1\mathrm{Mpc}^{-1}$
  - Rayleigh scattering: test expansion, measure more large-scale modes

  (+ 21cm - see talk from 2009!)

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Stephen Wilkins

Observation and modelling of galaxies and clusters