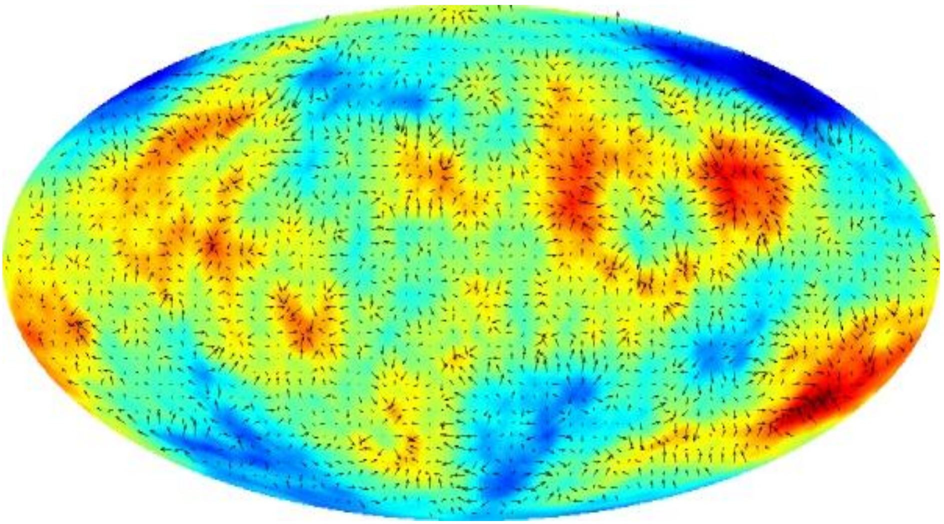
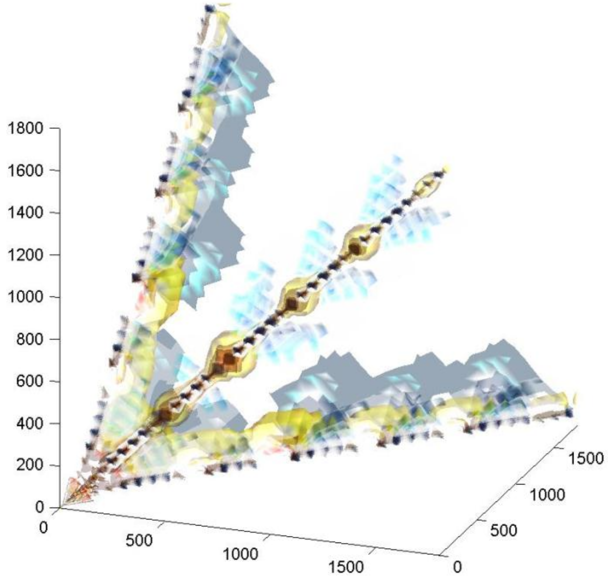
CMB Lensing and other non-Gaussianities



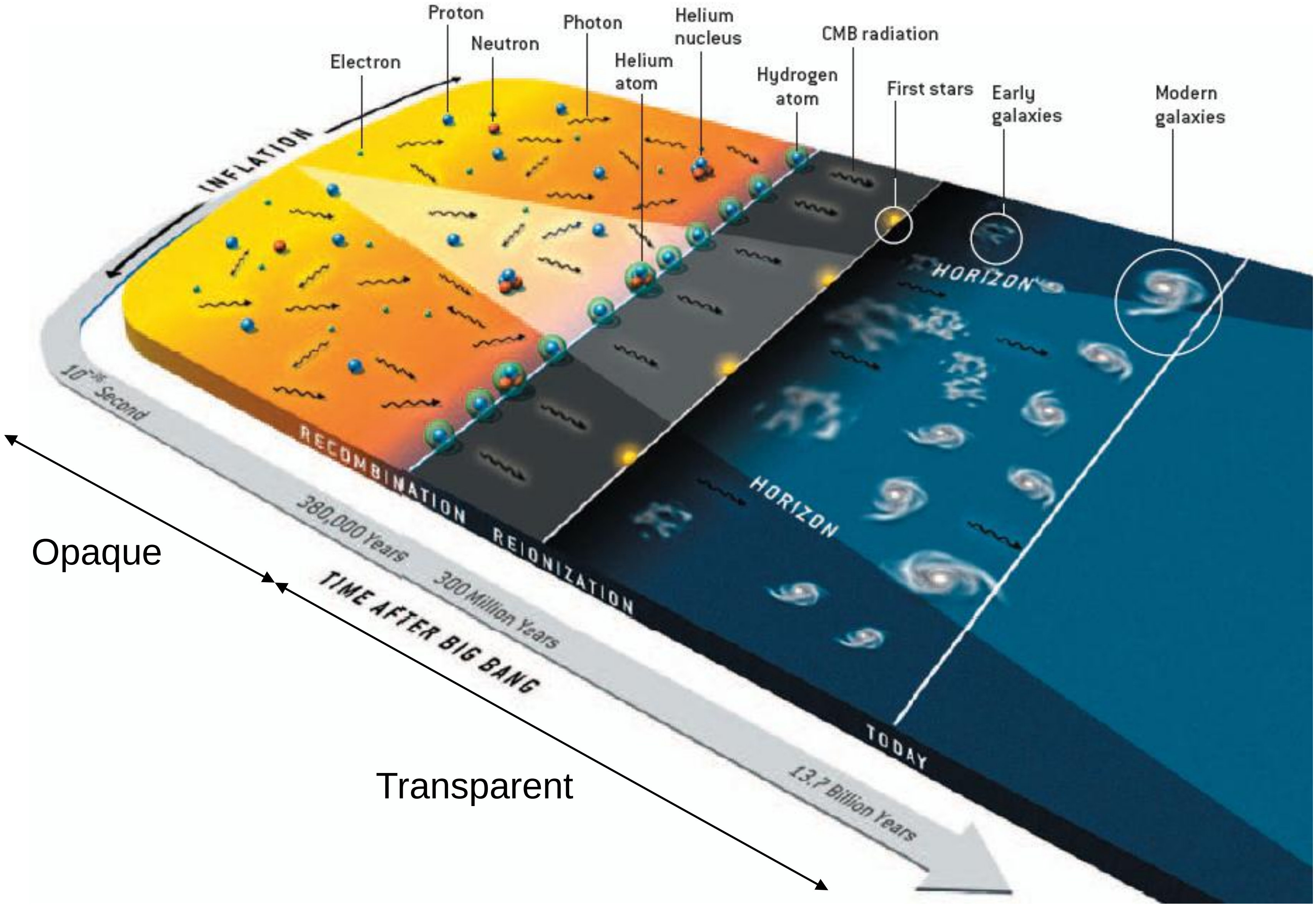


University of Sussex

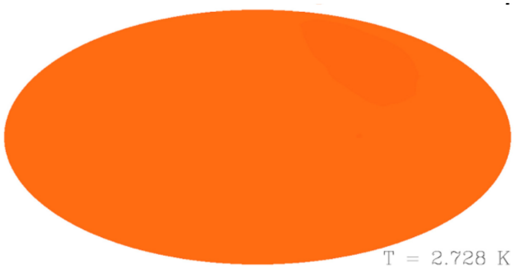
Antony Lewis
http://cosmologist.info/

Lewis, Challinor & Hanson *arXiv:1101.2234*

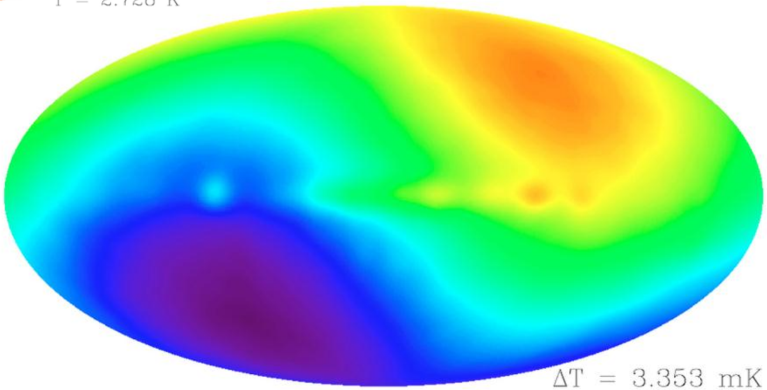
Evolution of the universe



Hu & White, Sci. Am., 290 44 (2004)

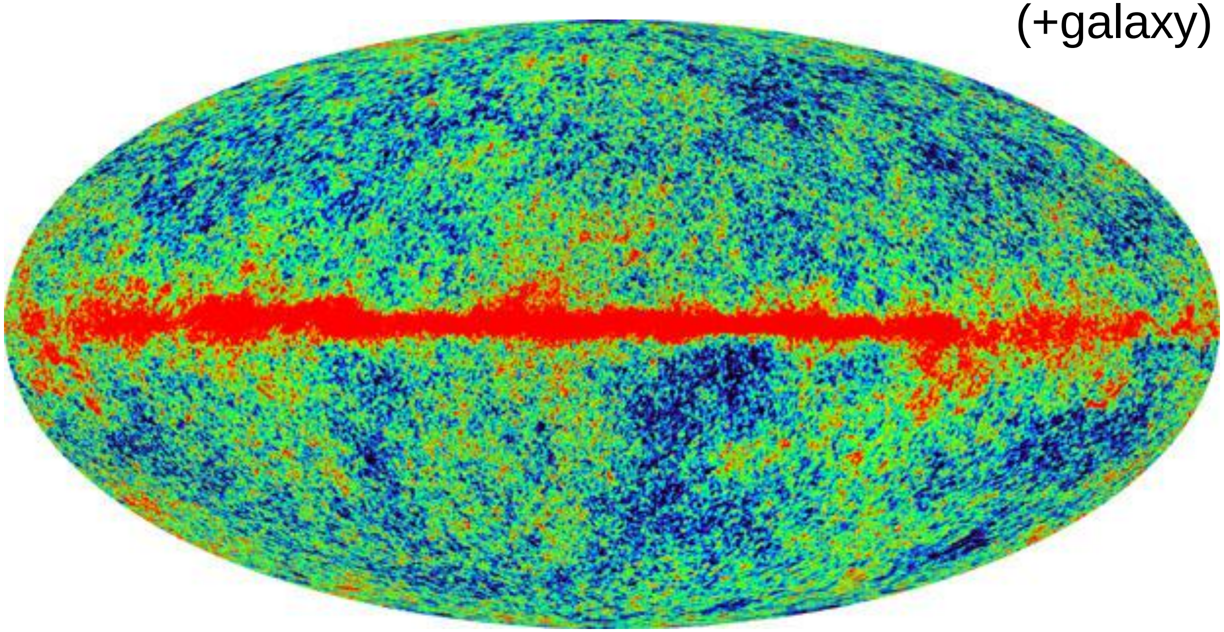


(almost) uniform 2.726K blackbody



Dipole (local motion)

O(10^{-5}) perturbations (+galaxy)

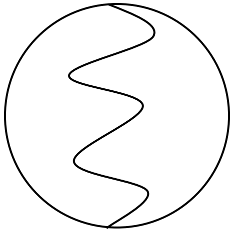


Observations: the microwave sky today

Source: NASA/WMAP Science Team

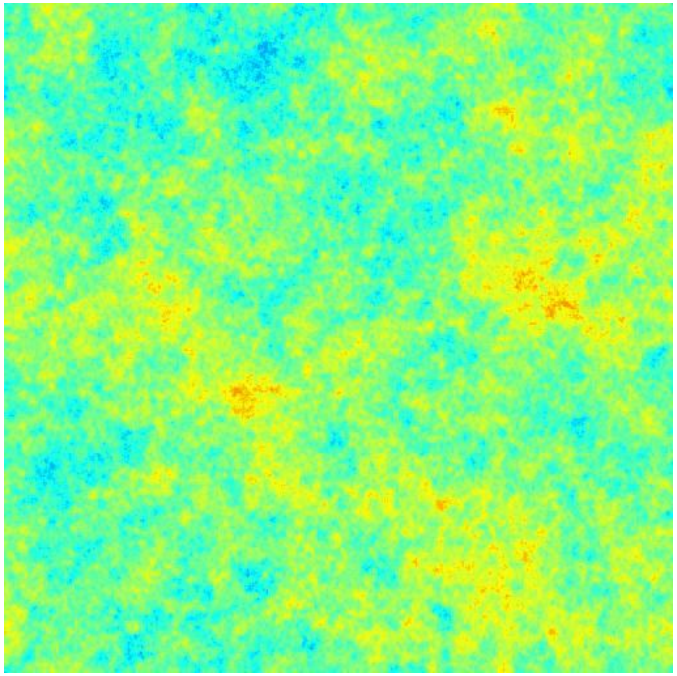
Can we predict the primordial perturbations?

- Maybe..



Quantum Mechanics
"waves in a box"

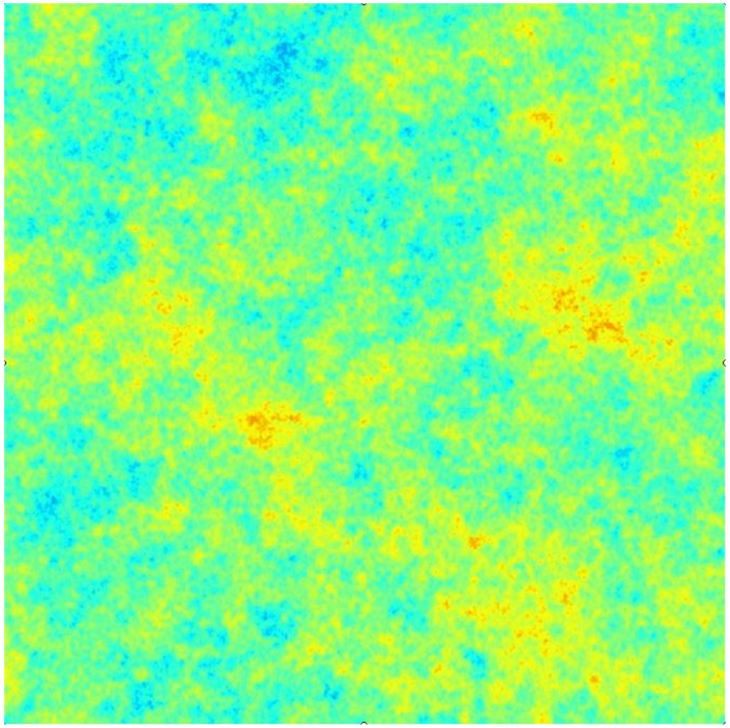
Inflation
make $>10^{30}$ times bigger



After inflation
Huge size, amplitude ~ 10^{-5}

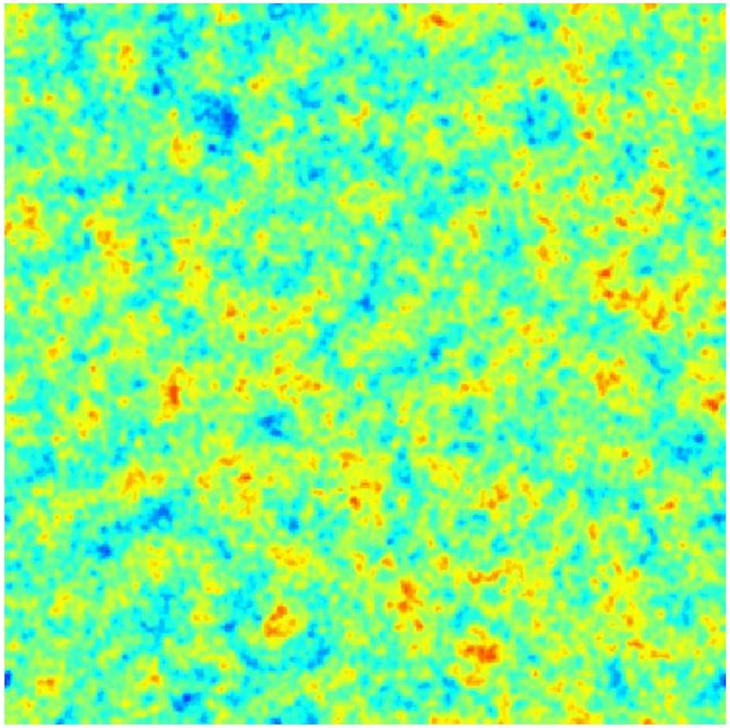
CMB temperature

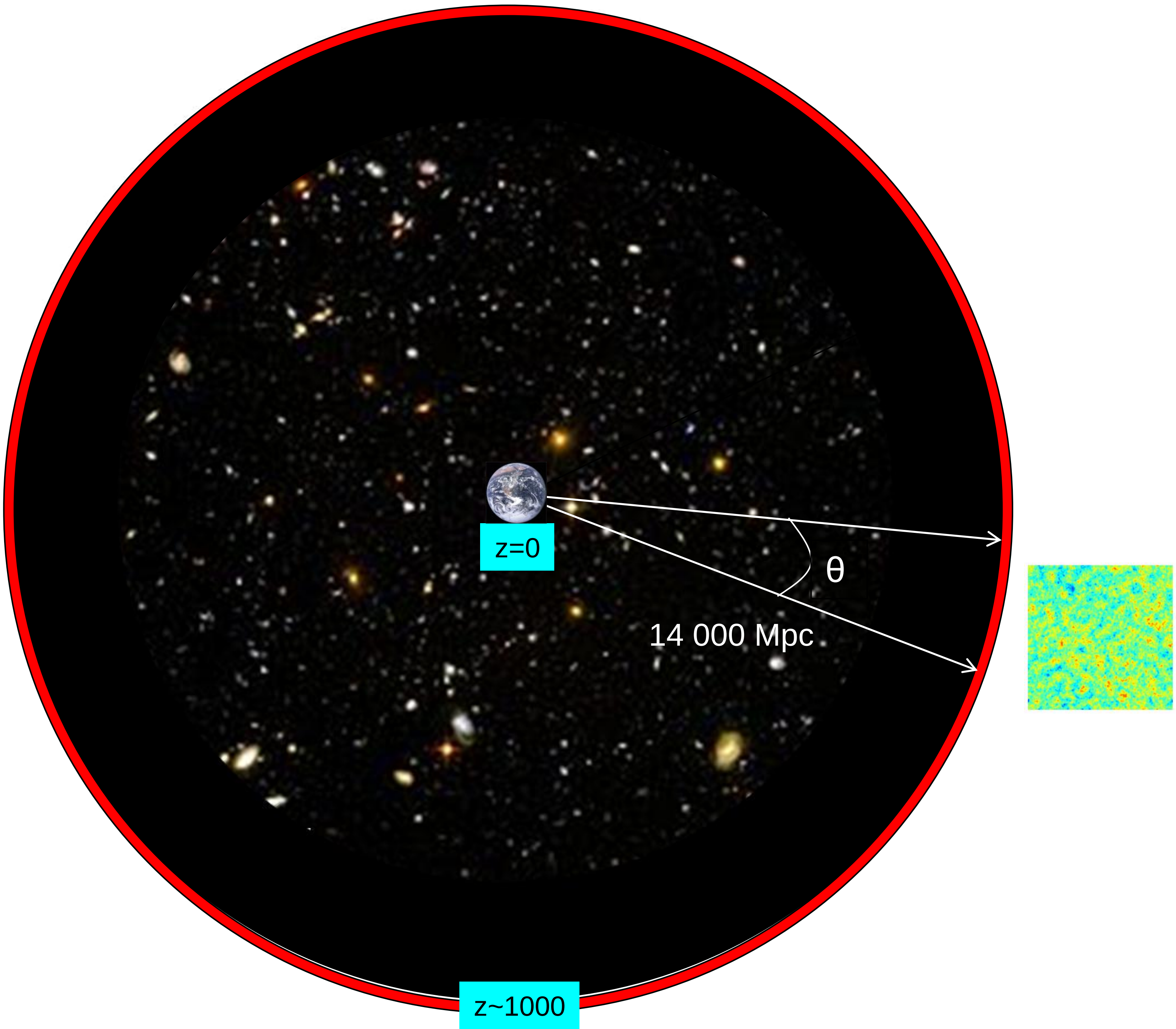
End of inflation



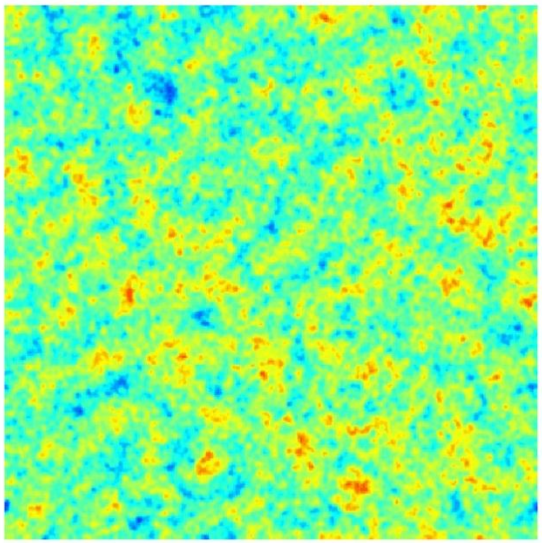
gravity+
pressure+
diffusion

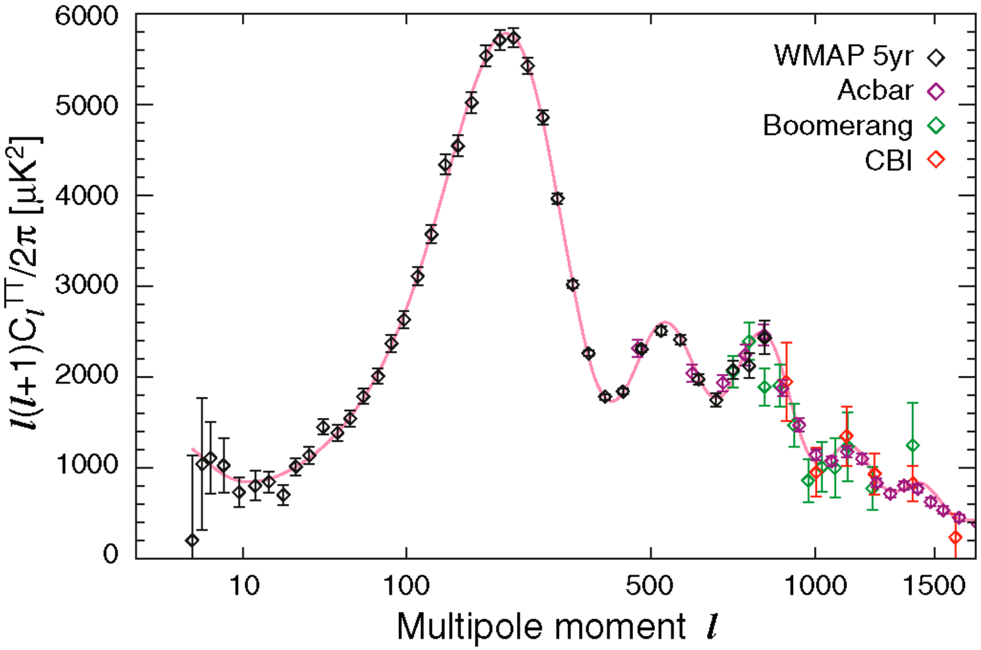
Last scattering surface




z=0
θ
14 000 Mpc
z~1000

Observed CMB temperature power spectrum





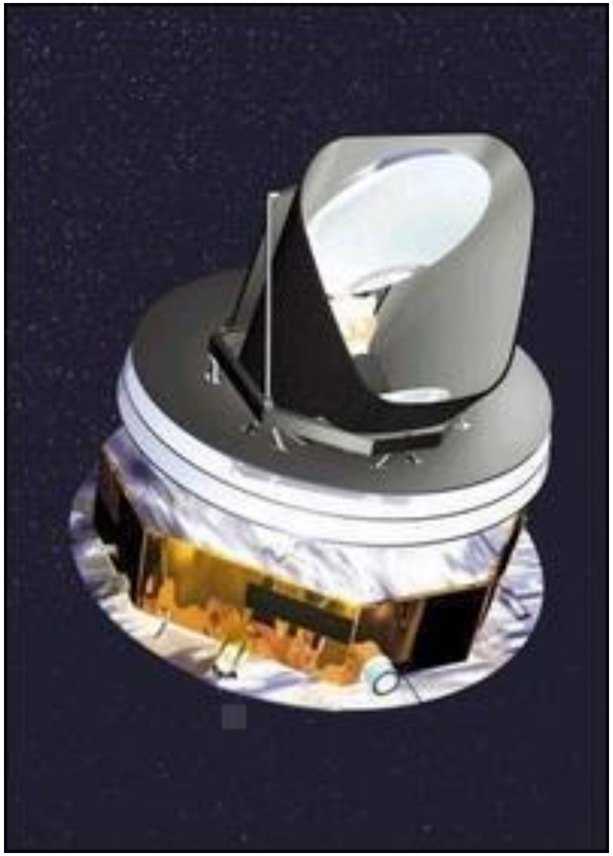
WMAP team

Observations → **Constrain theory of early universe + evolution parameters and geometry**

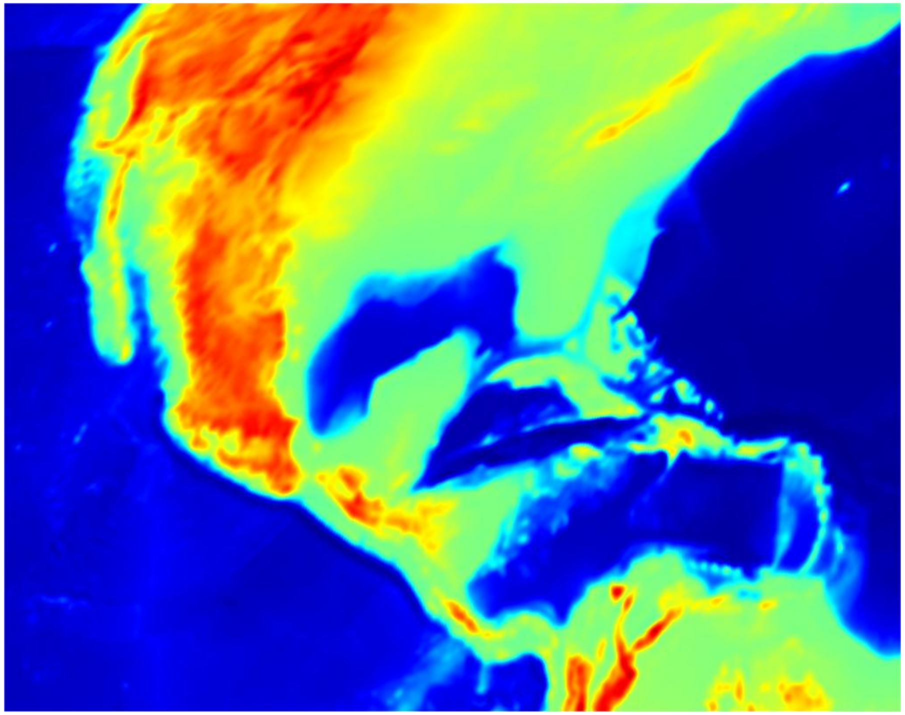
Planck and the future

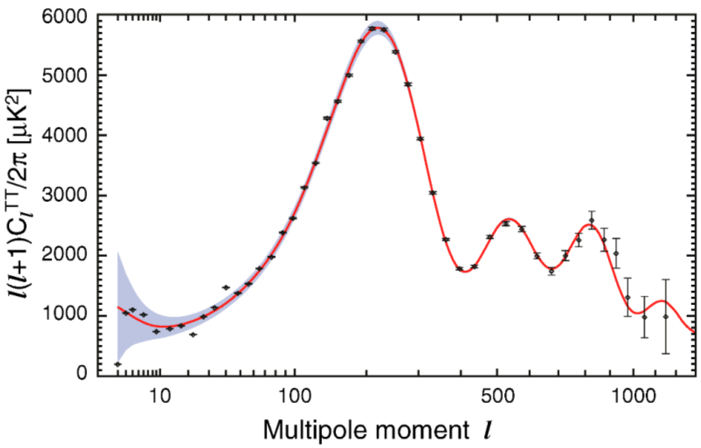
14 May 2009

High sensitivity and resolution
CMB temperature and polarization



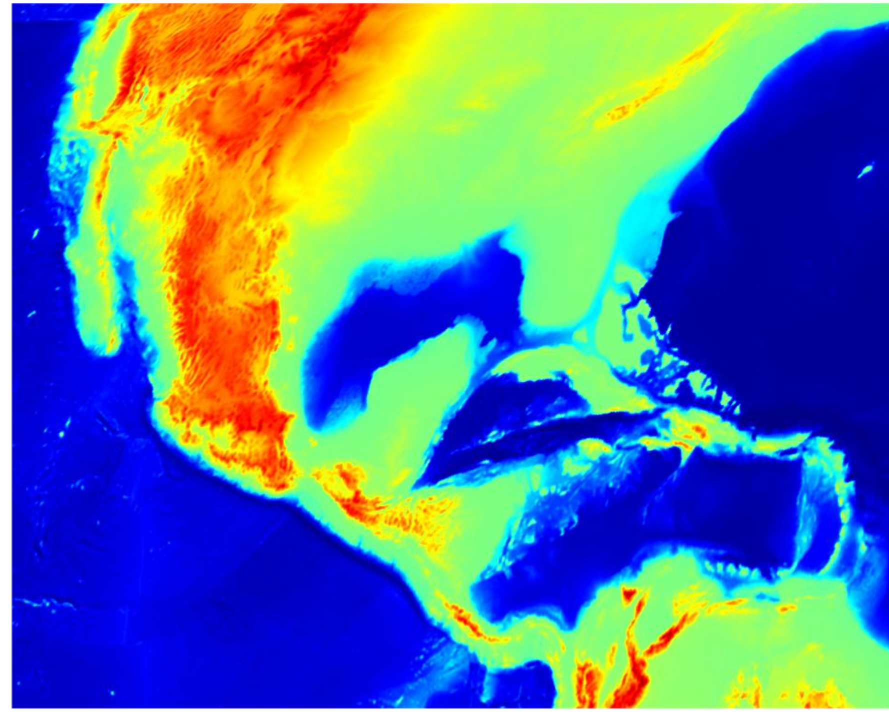
WMAP

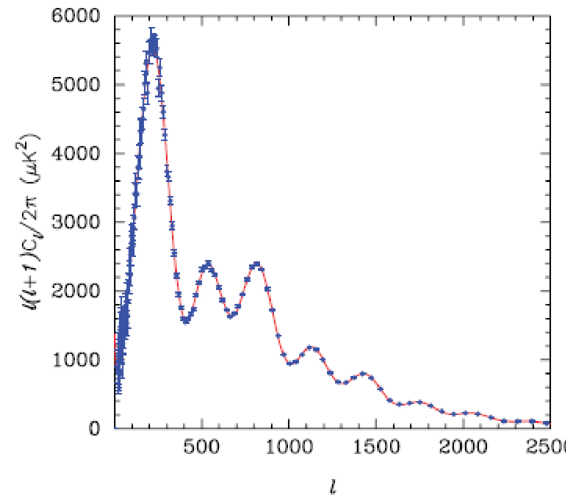




6000
5000
4000
3000
2000
1000
0
10
100
500
1000
Multipole moment l
$l(l+1)C_l^{TT}/2\pi$ [μK^2]

Planck





6000
5000
4000
3000
2000
1000
0
500
1000
1500
2000
2500
l
$l(l+1)C_l/2\pi$ (μK^2)

Parameters. WMAP4 vs Planck

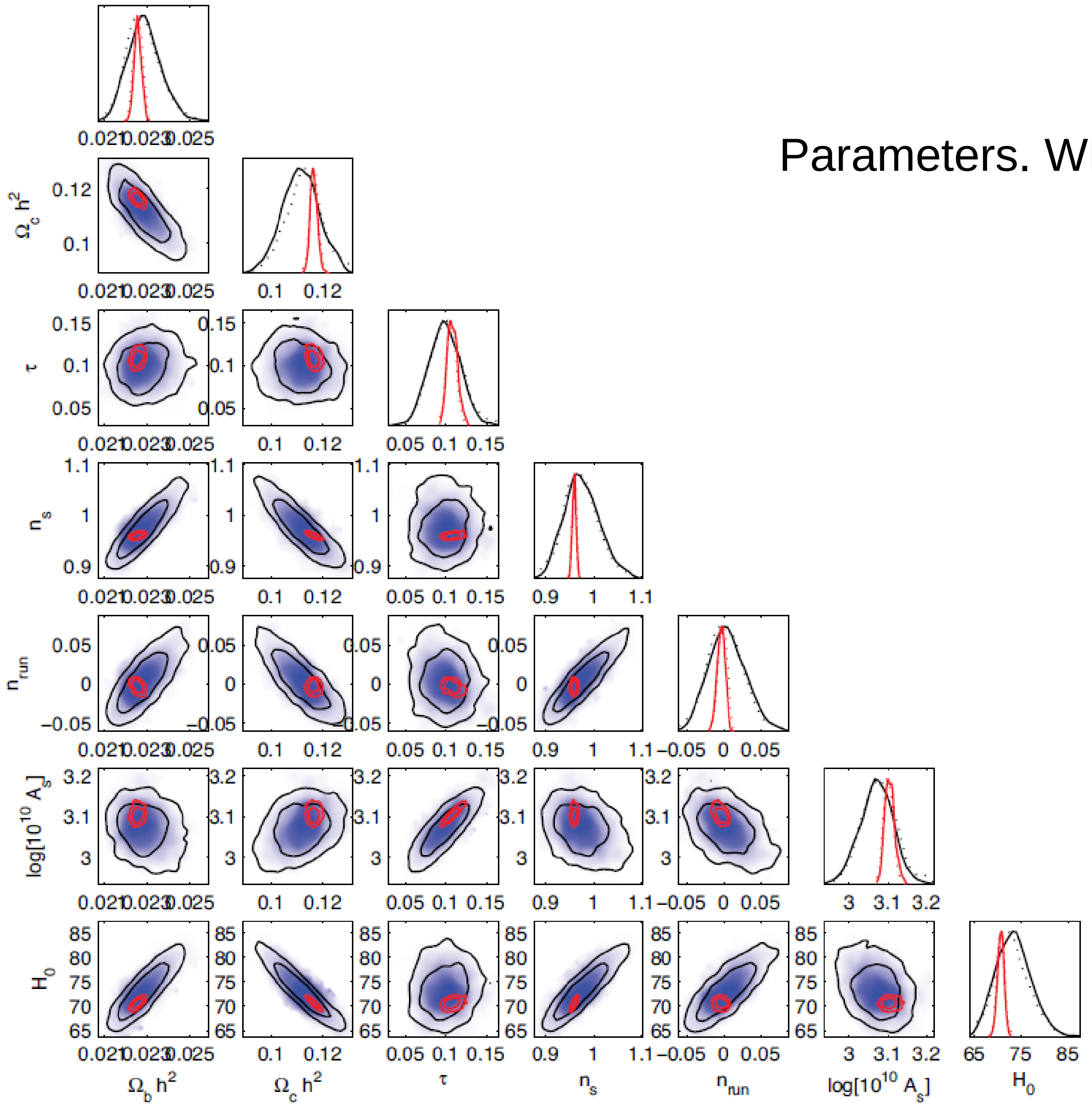
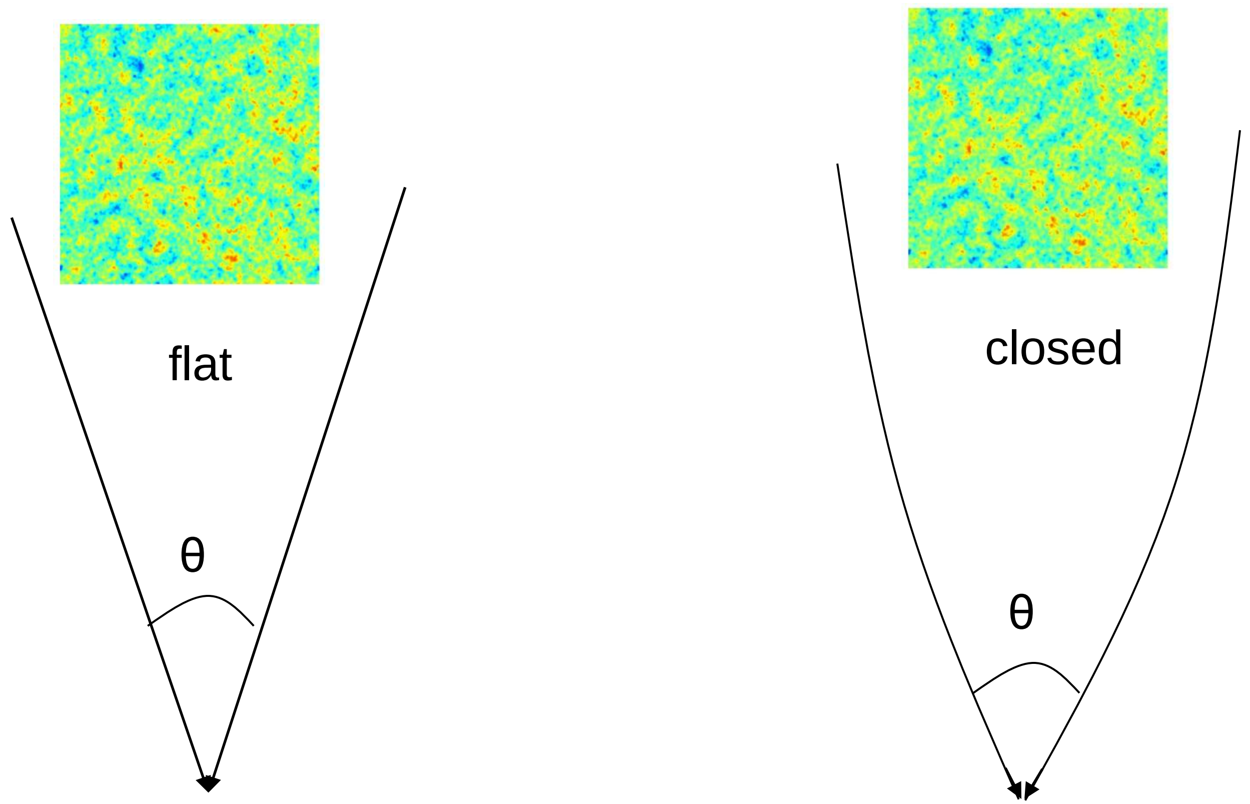
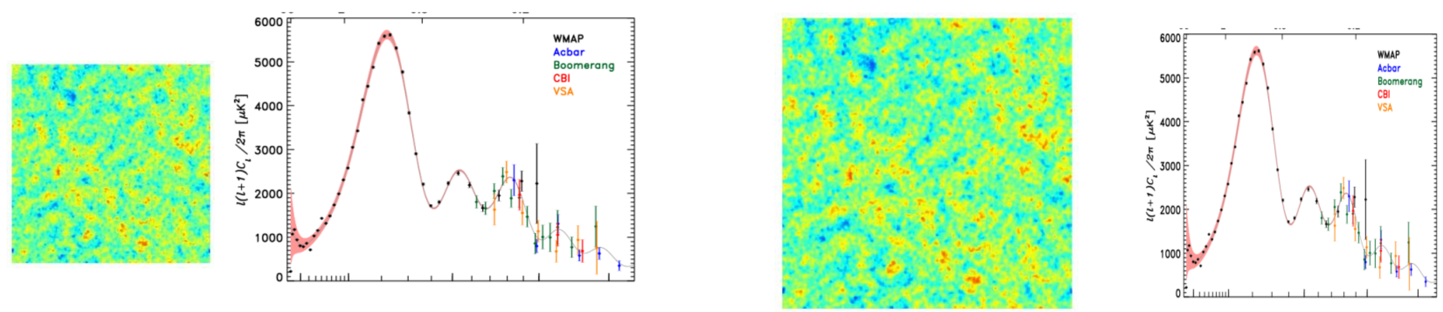


FIG 2.18.—Forecasts of 1 and 2σ contour regions for various cosmological parameters when the spectral index is allowed to run. Blue contours show forecasts for *WMAP* after 4 years of observation and red contours show results for *Planck* after 1 year of observations. The curves show marginalized posterior distributions for each parameter.

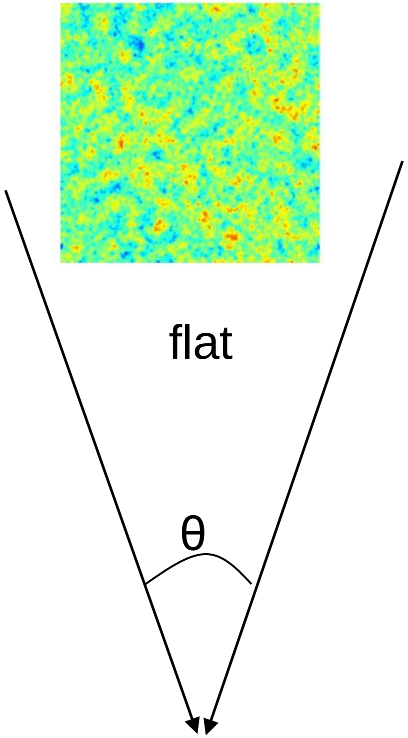
e.g. Geometry: curvature

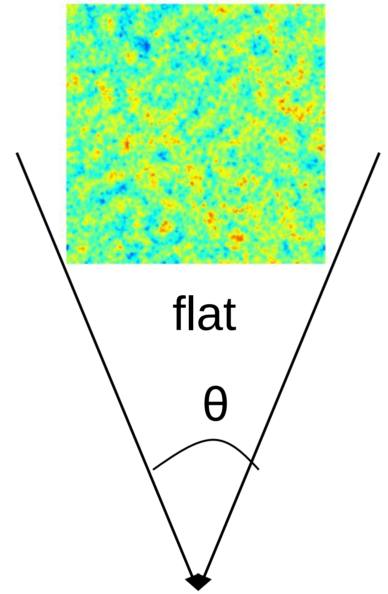


We see:

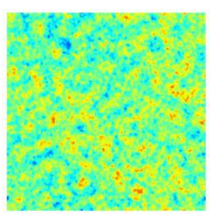


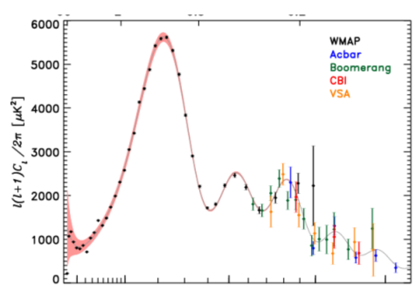
or is it just closer??

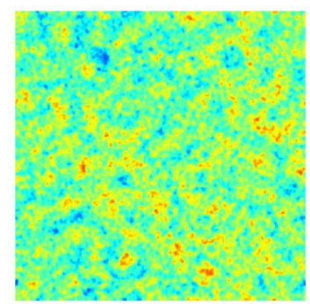


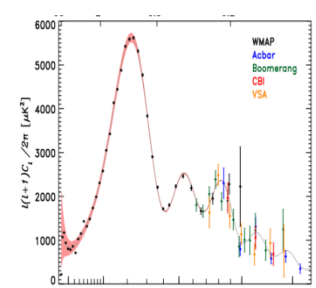


We see:

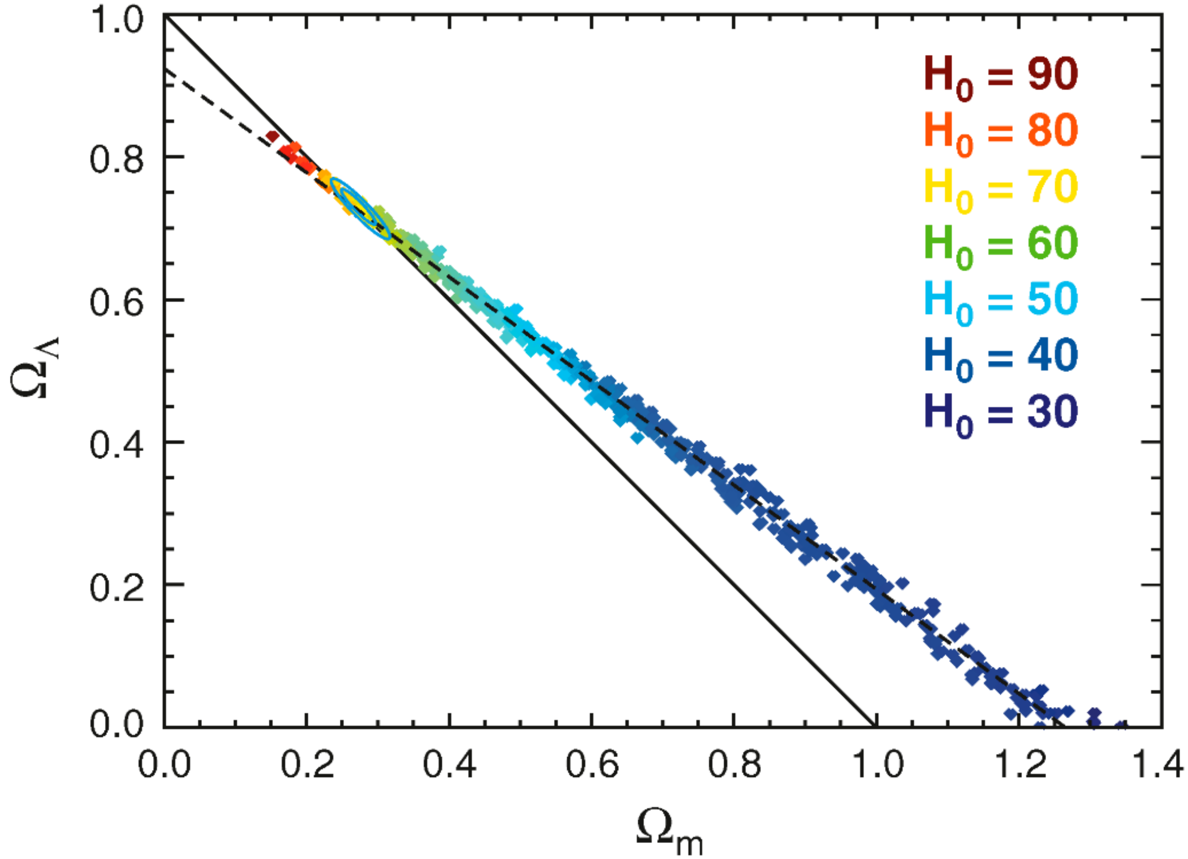








Degeneracies between parameters

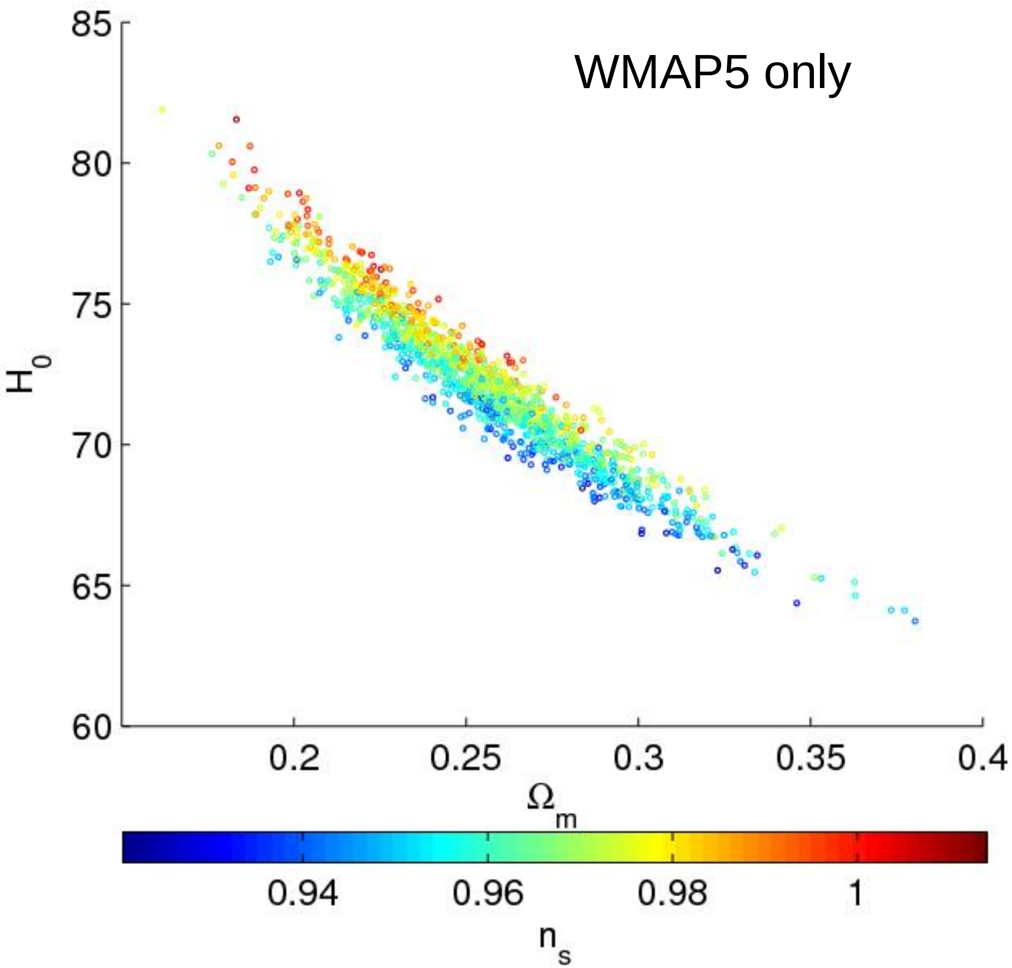


WMAP 7

Need other information to break remaining degeneracies

Constrain *combinations* of parameters accurately

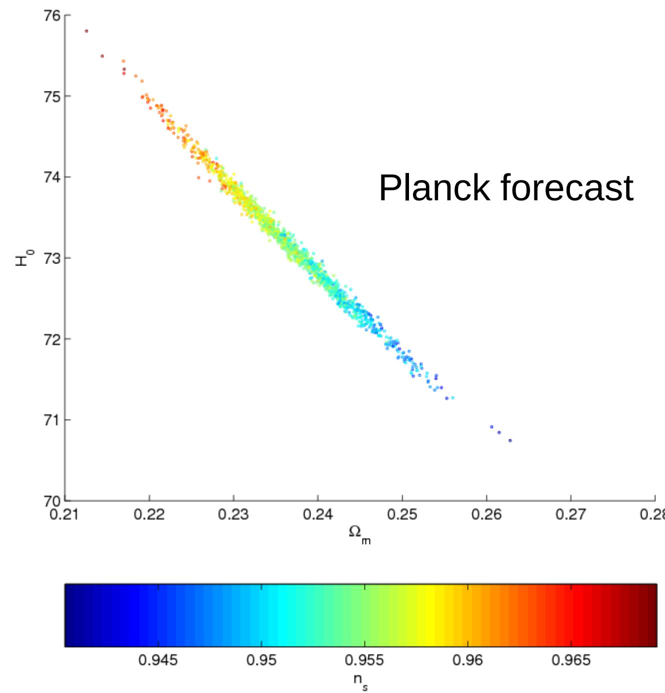
Assume Flat, w=-1



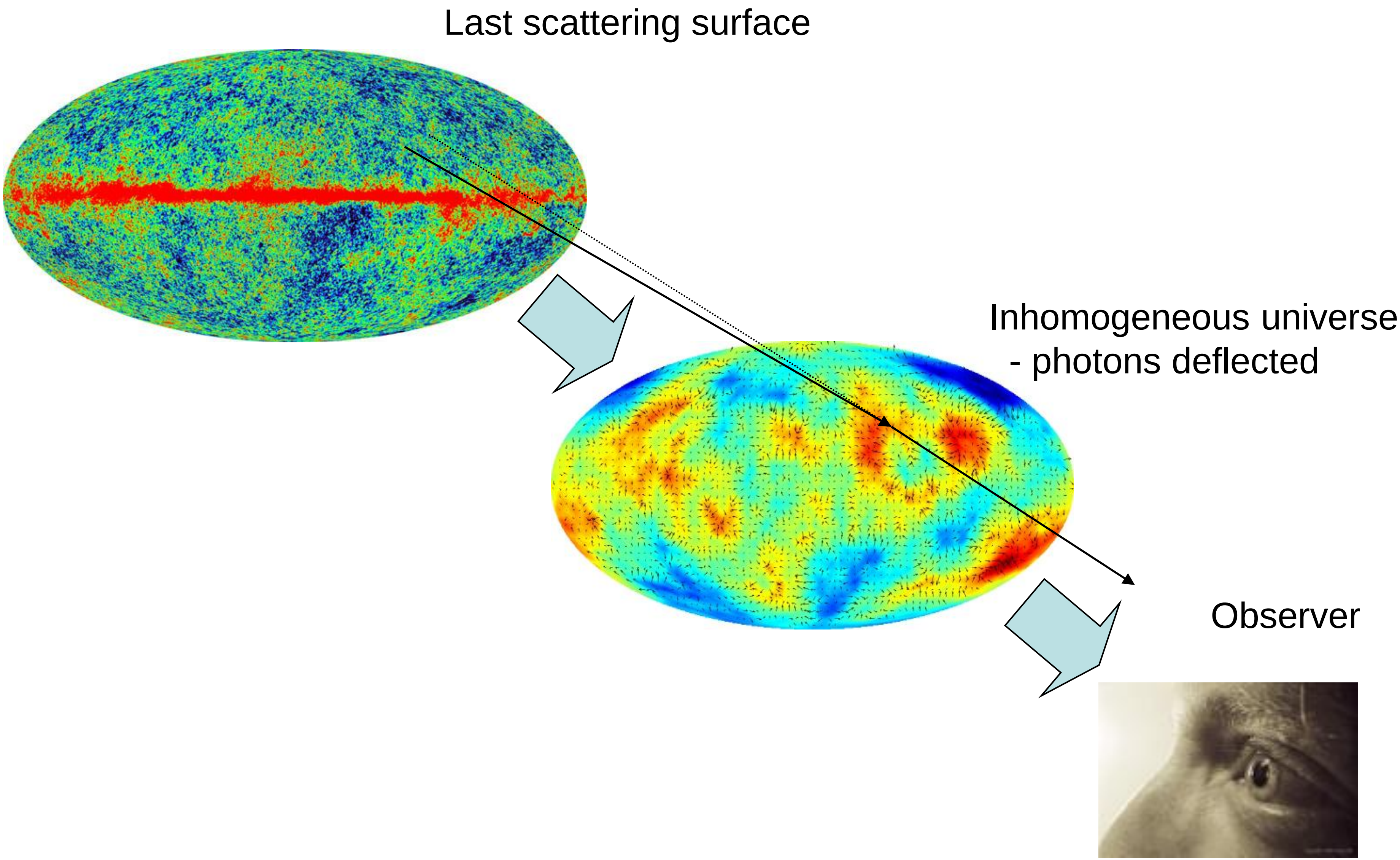
$$\left(\frac{\Omega_m}{0.254}\right)\left(\frac{h}{0.72}\right)^{3.15} = 1.00 \pm 0.03$$

$$\left(\frac{\Omega_m}{0.254}\right)\left(\frac{h}{0.72}\right)^{-3.15} = 1.03 \pm 0.23$$

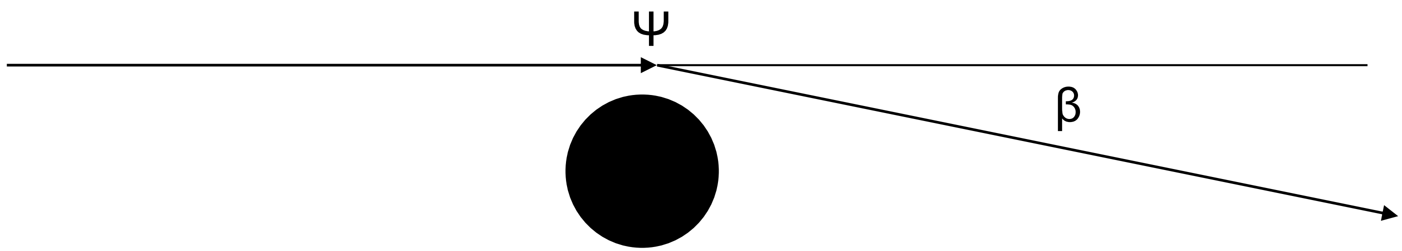
Use other data to break remaining degeneracies



CMB Lensing


Last scattering surface
Inhomogeneous universe
- photons deflected
Observer

Lensing order of magnitudes



Newtonian argument: $\beta = 2\,\Psi$
General Relativity: $\beta = 4\,\Psi$ ($\beta \ll 1$)

Potentials linear and approx Gaussian: $\Psi \sim 2 \times 10^{-5}$

$\beta \sim 10^{-4}$

Characteristic size from peak of matter power spectrum ~ 300Mpc

Comoving distance to last scattering surface ~ 14000 MPc

$\Rightarrow$ pass through ~50 lumps $\Rightarrow$ total deflection ~ $50^{1/2} \times 10^{-4}$

assume uncorrelated

~ 2 arcminutes

(neglects angular factors, correlation, etc.)

So why does it matter?

- 2arcmin: ell ~ 3000

  - On small scales CMB is very smooth so lensing dominates the linear signal

- Deflection angles coherent over 300/(14000/2) ~ 2°

  - comparable to CMB scales

  - expect 2arcmin/60arcmin ~ 3% effect on main CMB acoustic peaks

In detail, lensed temperature depends on deflection angle:

$$\tilde{T}(\hat{\mathbf{n}}) = T(\hat{\mathbf{n}}') = T(\hat{\mathbf{n}} + \boldsymbol{\alpha})$$

$$\boldsymbol{\alpha} = \delta\theta = -2\int_0^{\chi^*} \mathrm{d}\chi \frac{f_K(\chi^* - \chi)}{f_K(\chi^*)} \nabla_\perp \Psi(\chi\hat{\mathbf{n}}; \eta_0 - \chi)$$

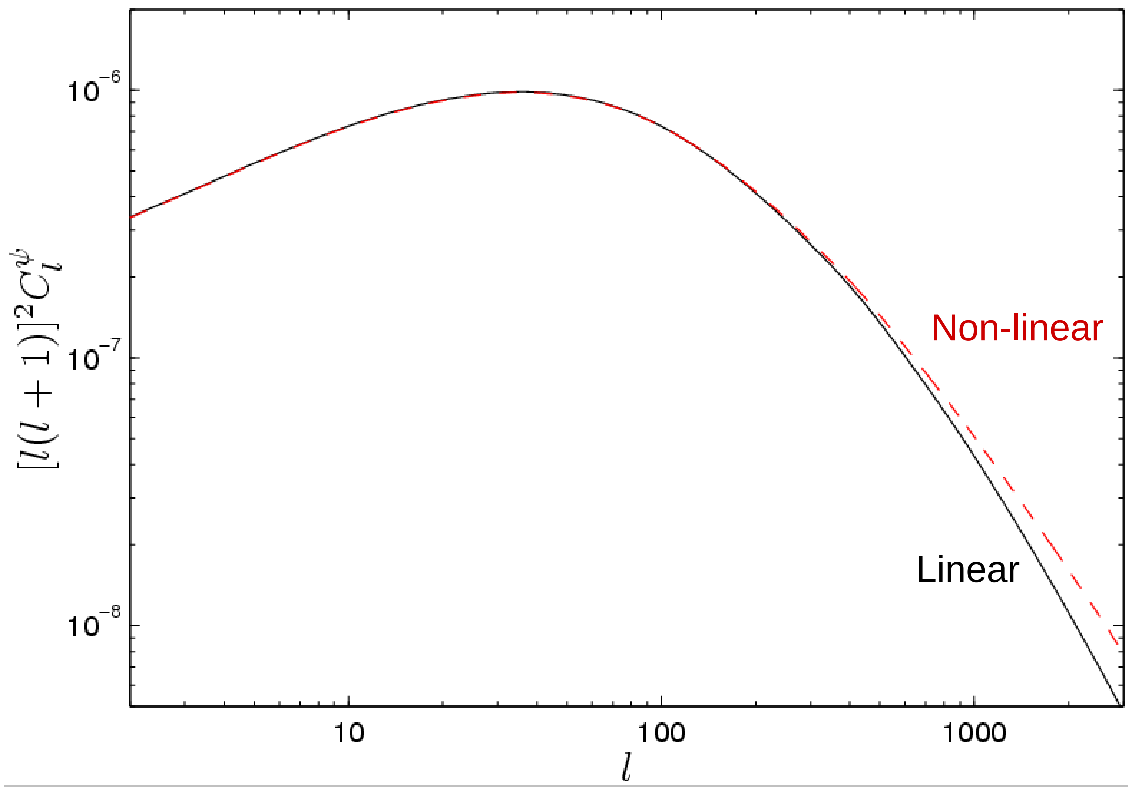
Lensing Potential

Deflection angle on sky given in terms of lensing potential $\boldsymbol{\alpha} = \nabla\psi$

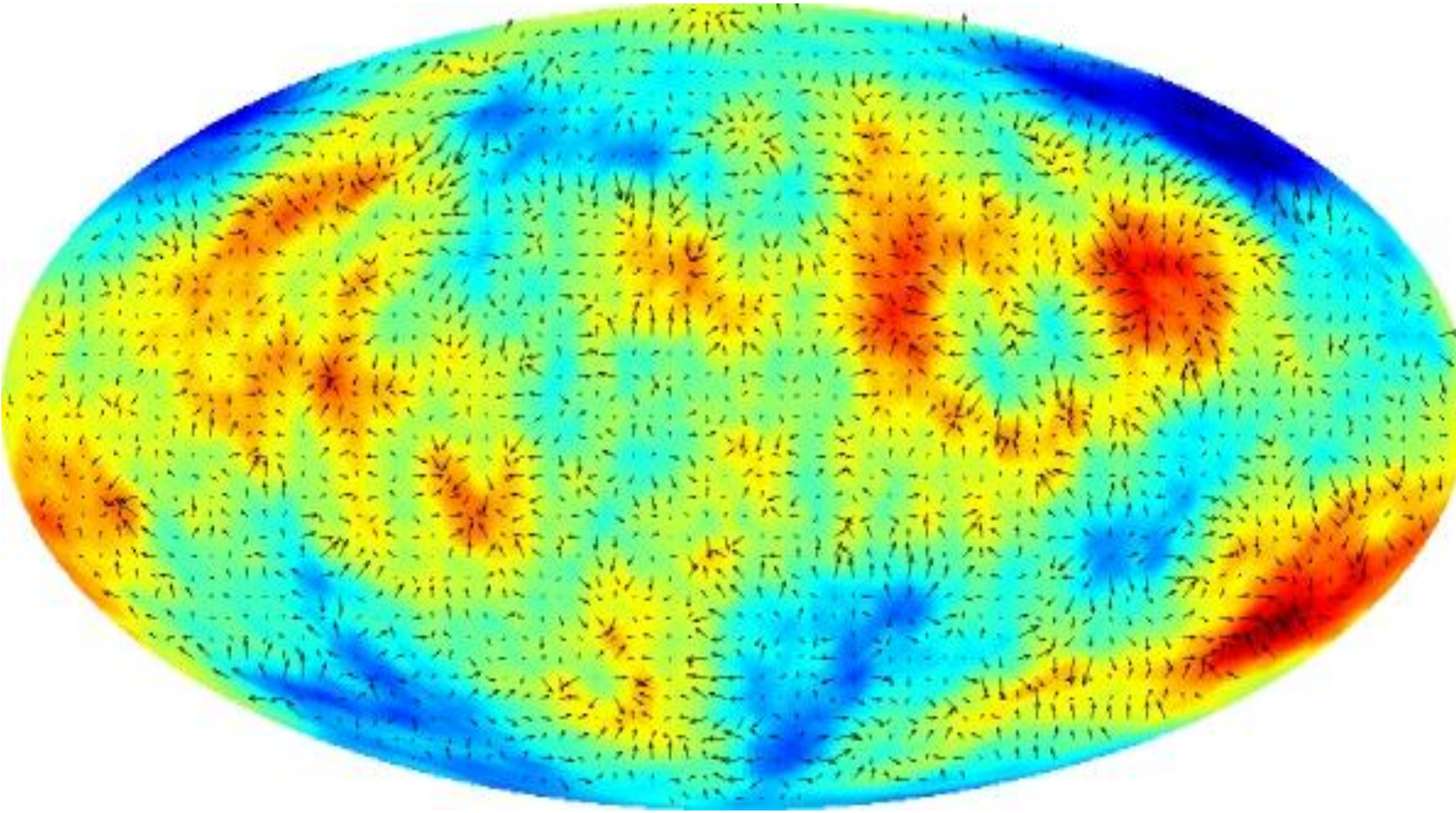
$$\psi(\hat{\mathbf{n}}) = -2\int_0^{\chi_*} \mathrm{d}\chi\, \Psi(\chi\hat{\mathbf{n}}; \eta_0 - \chi) \frac{f_K(\chi^* - \chi)}{f_K(\chi^*) f_K(\chi)}$$

$$\bar{X}(\mathbf{n}) = X(\mathbf{n}') = X(\mathbf{n} + \nabla\psi(\mathbf{n}))$$

Deflection angle power spectrum

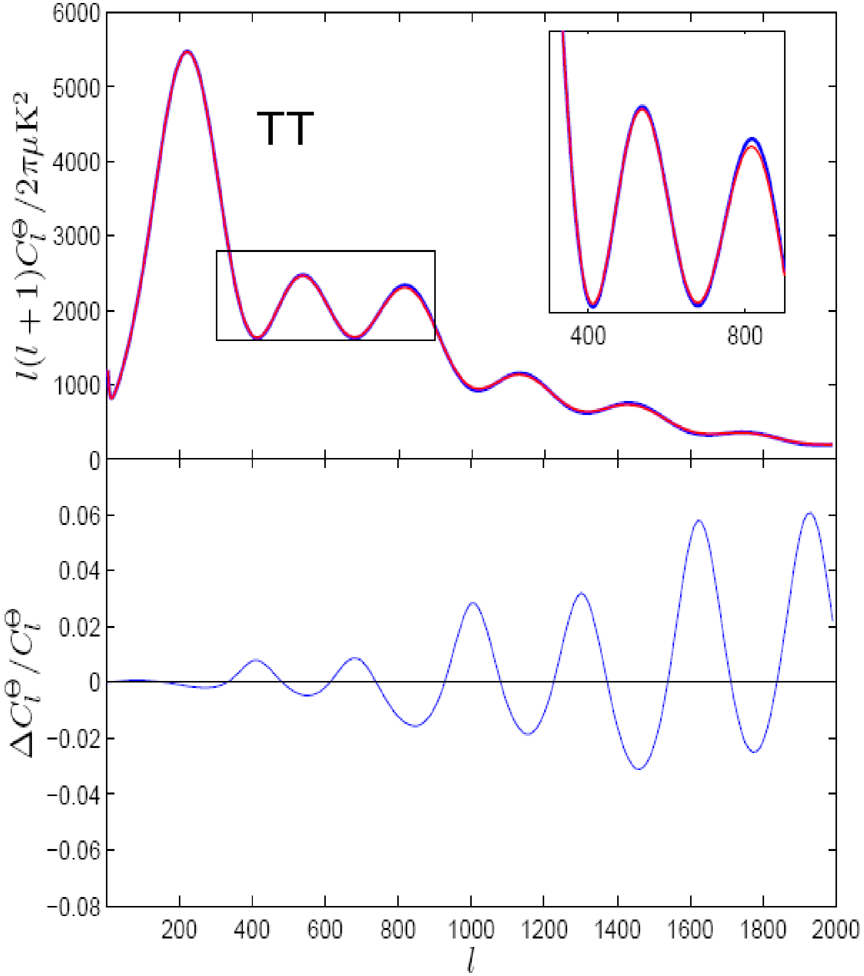


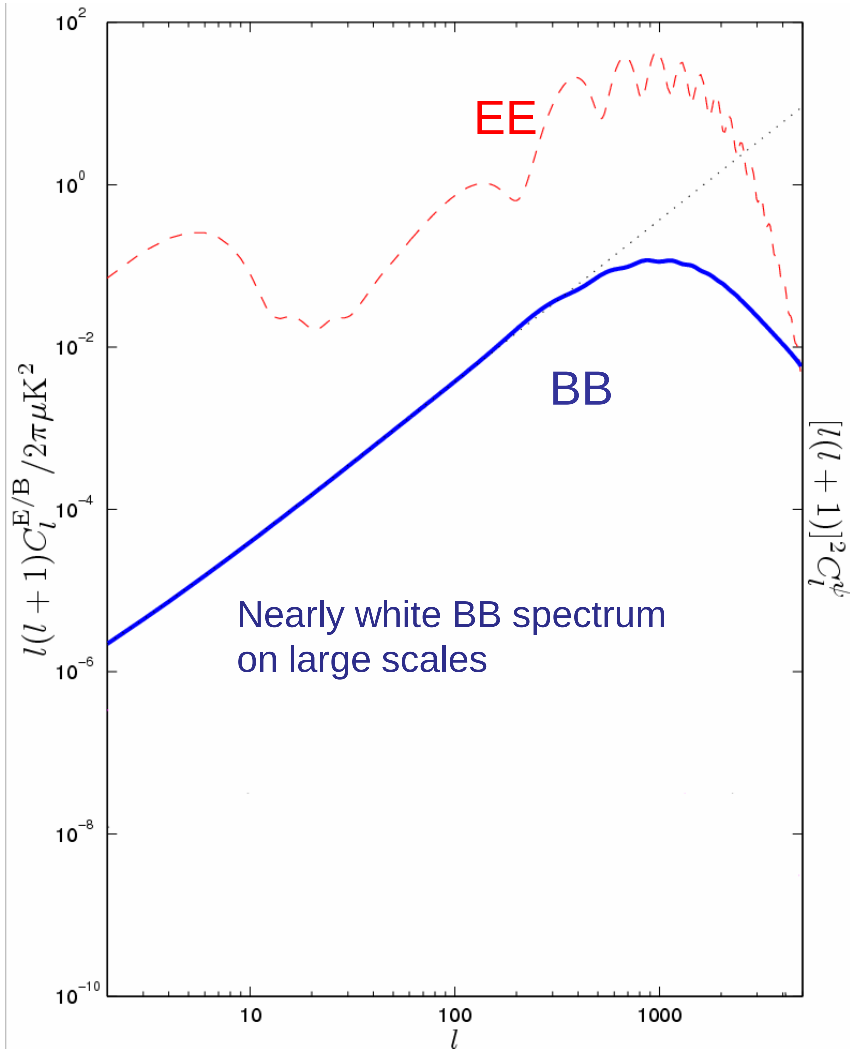
Deflections O(10^{-3}), but coherent on degree scales → important!



Easily simulated assuming Gaussian fields
- just re-map points using Gaussian realisations of CMB and potential

Lensing effect on CMB temperature power spectra





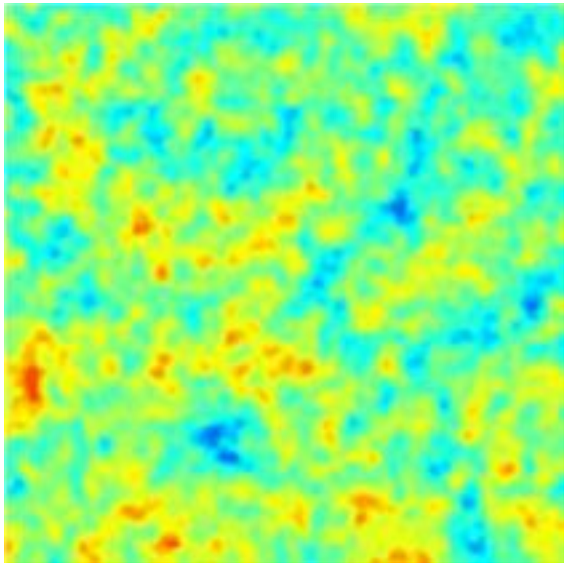
CAMB's 0.1% calculation; http://camb.info : Challinor & Lewis 2005, astro-ph/0502425

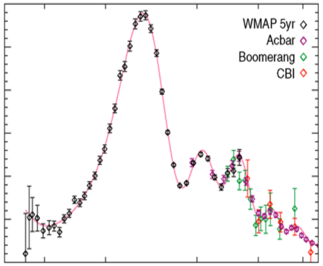
Why lensing is important

- Known effect, small but significant amplitude ($\sim 10^{-3}$)
- Modifies the power spectra on small-scales ($\sim 10^{-2}$)
- Lensing of E gives B-mode polarization (confusion for tensors/strings)
- Anisotropy/non-Gaussianities….

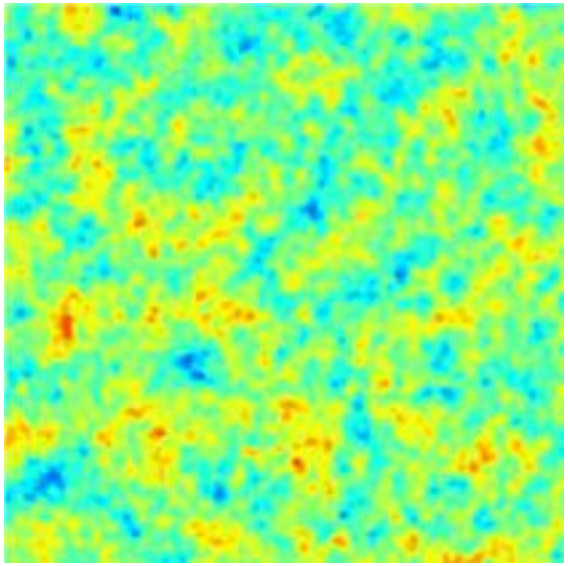
Beyond the power spectrum

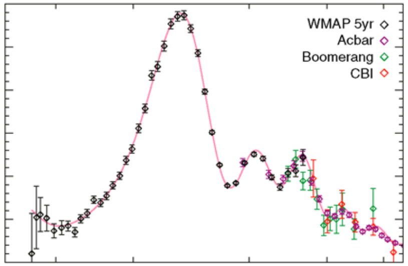
Magnified



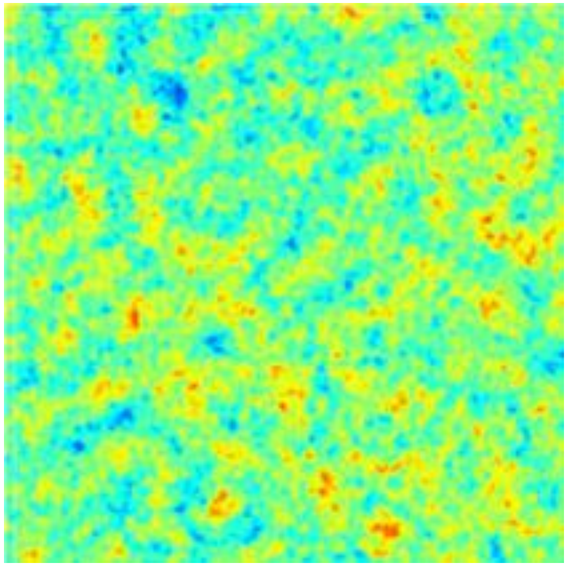


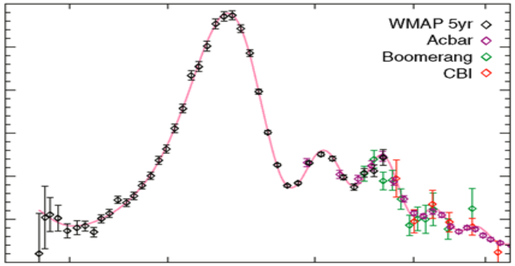
Unlensed





Demagnified





Beyond Gaussianity – general possibilities

Flat sky approximation: $\Theta(x) = \frac{1}{2\pi}\int d^2l\,\Theta(l)e^{ix\cdot l}$ $(\Theta = T)$

Gaussian + statistical isotropy

$$\langle\Theta(l_1)\Theta(l_2)\rangle = \delta(l_1 + l_2)C_l$$

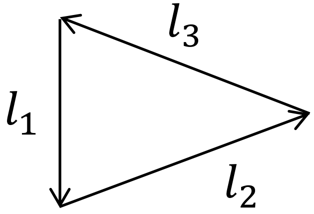
- power spectrum encodes all the information
- modes with different wavenumber are independent

Higher-point correlations

Gaussian: can be written in terms of C_l

Non-Gaussian: non-zero connected n-point functions

Bispectrum



$$\boldsymbol{l_1} + \boldsymbol{l_2} + \boldsymbol{l_3} = \mathbf{0}$$

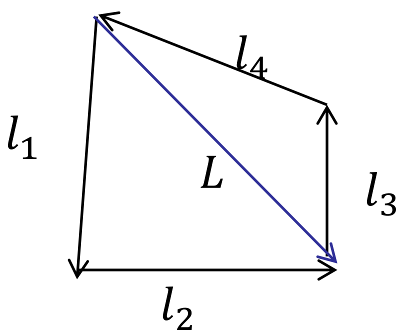
Flat sky approximation: $\langle \Theta(l_1)\Theta(l_2)\Theta(l_3)\rangle = \frac{1}{2\pi}\delta(l_1 + l_2 + l_3) b_{l_1 l_2 l_3}$

If you know $\Theta(l_1), \Theta(l_2)$, sign of $b_{l_1 l_2 l_3}$ tells you which sign of $\Theta(l_3)$ is more likely

Trispectrum

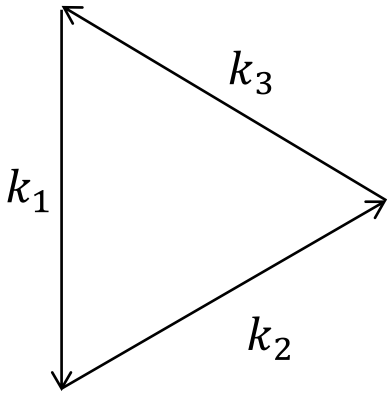
$$\langle \Theta(\mathbf{l}_1)\Theta(\mathbf{l}_2)\Theta(\mathbf{l}_3)\Theta(\mathbf{l}_4)\rangle_C = (2\pi)^{-2}\delta(\mathbf{l}_1 + \mathbf{l}_2 + \mathbf{l}_3 + \mathbf{l}_4) T(\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3, \mathbf{l}_4)$$

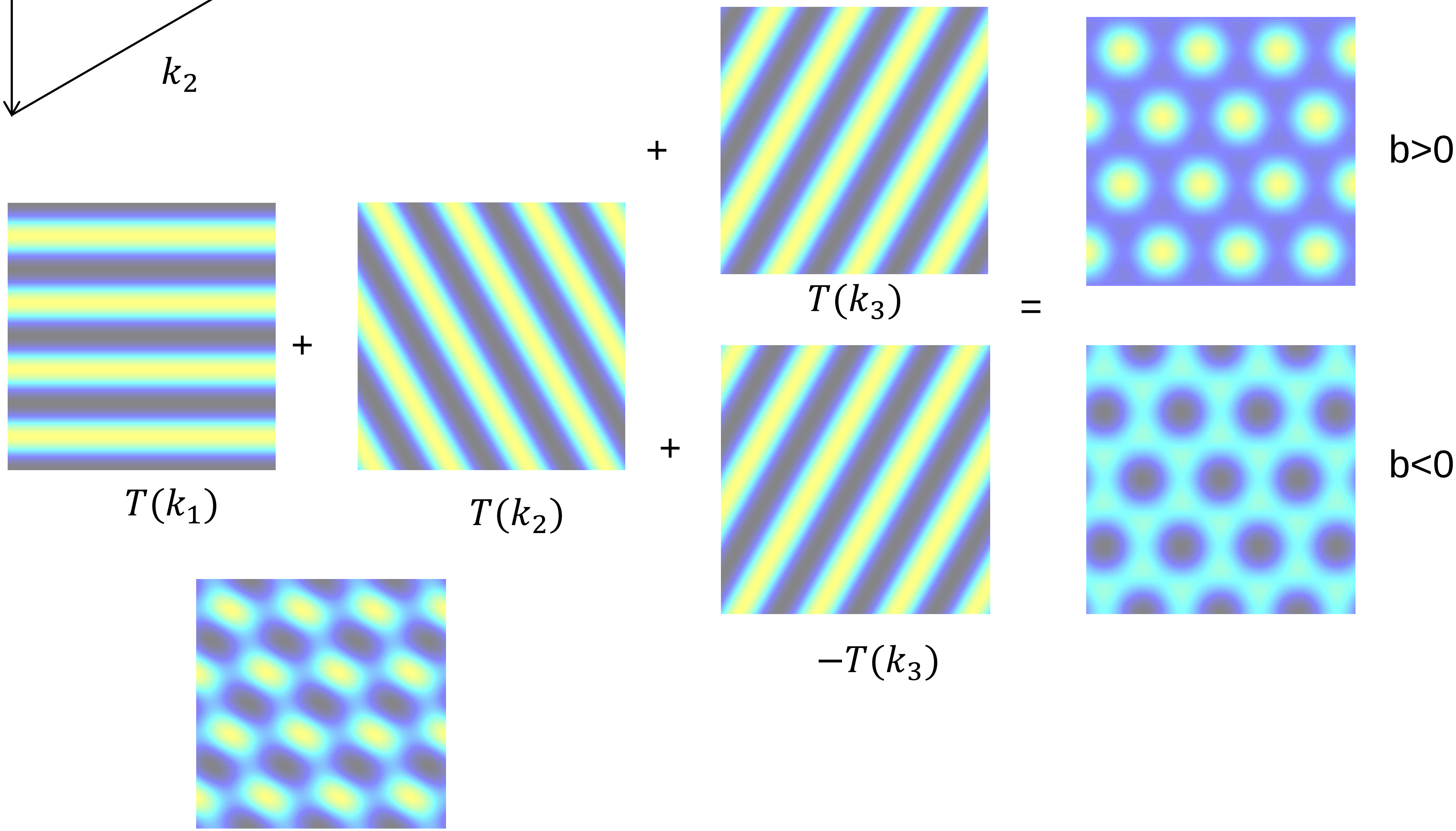
$$\langle \Theta(\mathbf{l}_1)\Theta(\mathbf{l}_2)\Theta(\mathbf{l}_3)\Theta(\mathbf{l}_4)\rangle_C = \frac{1}{2}\int \frac{d^2\mathbf{L}}{(2\pi)^2}\delta(\mathbf{l}_1 + \mathbf{l}_2 + \mathbf{L})\delta(\mathbf{l}_3 + \mathbf{l}_4 - \mathbf{L})\mathbb{T}^{(\ell_1\ell_2)}_{(\ell_3\ell_4)}(L) + \text{perms.}$$

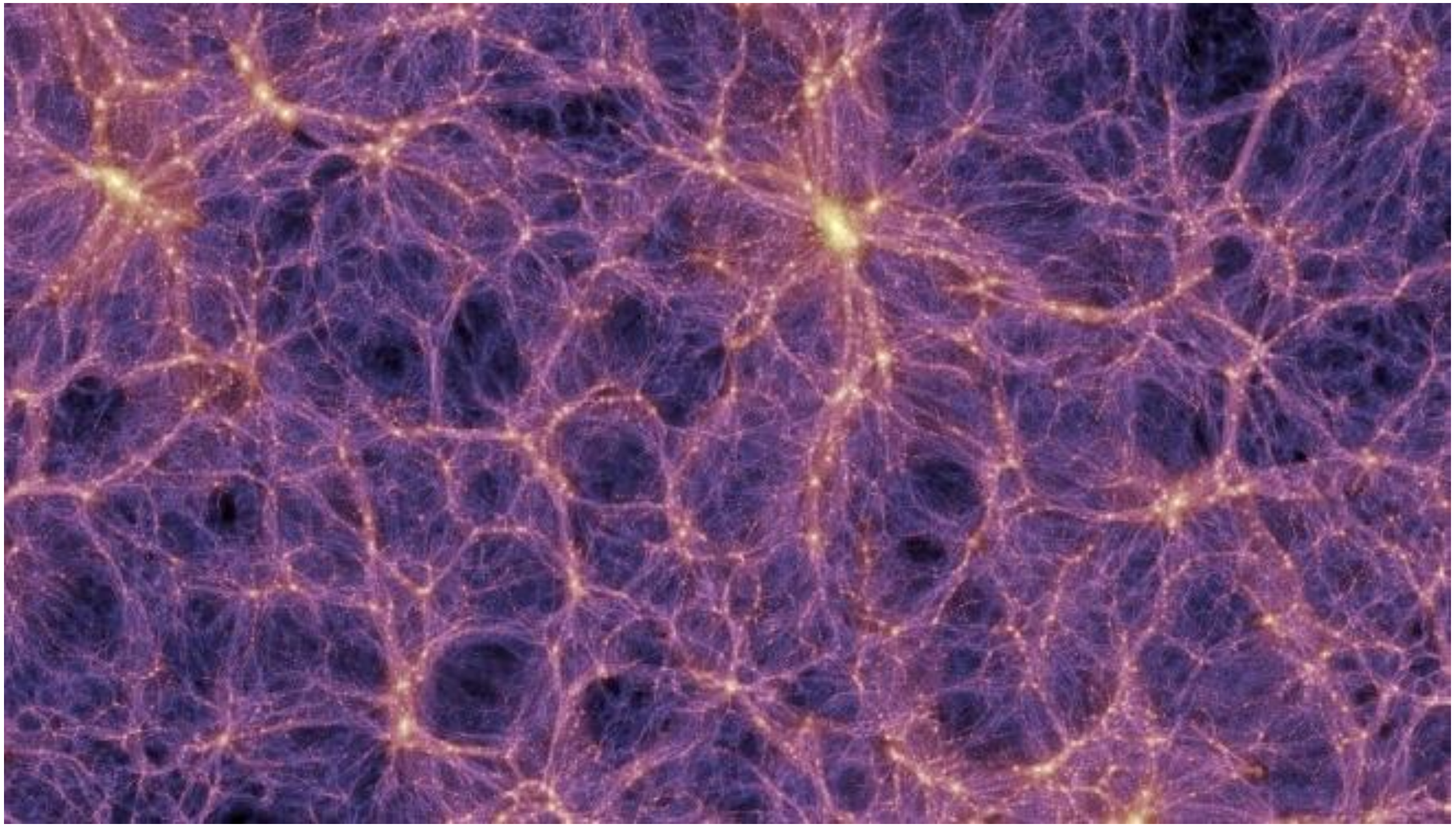


N-spectra…

Equilateral $k_1 + k_2 + k_3 = 0, |k_1| = |k_2| = |k_3|$

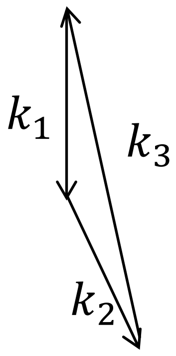


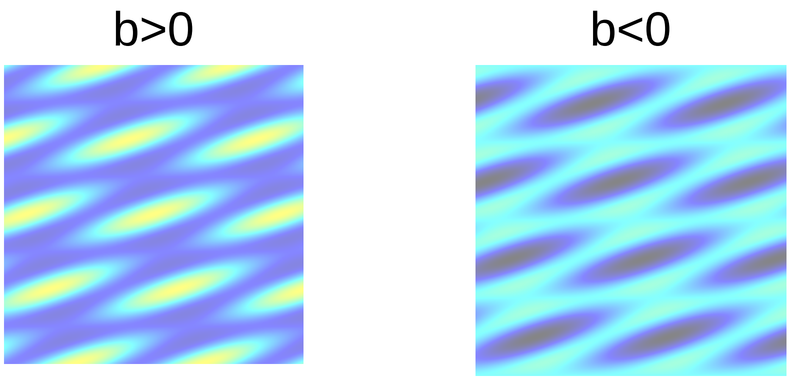




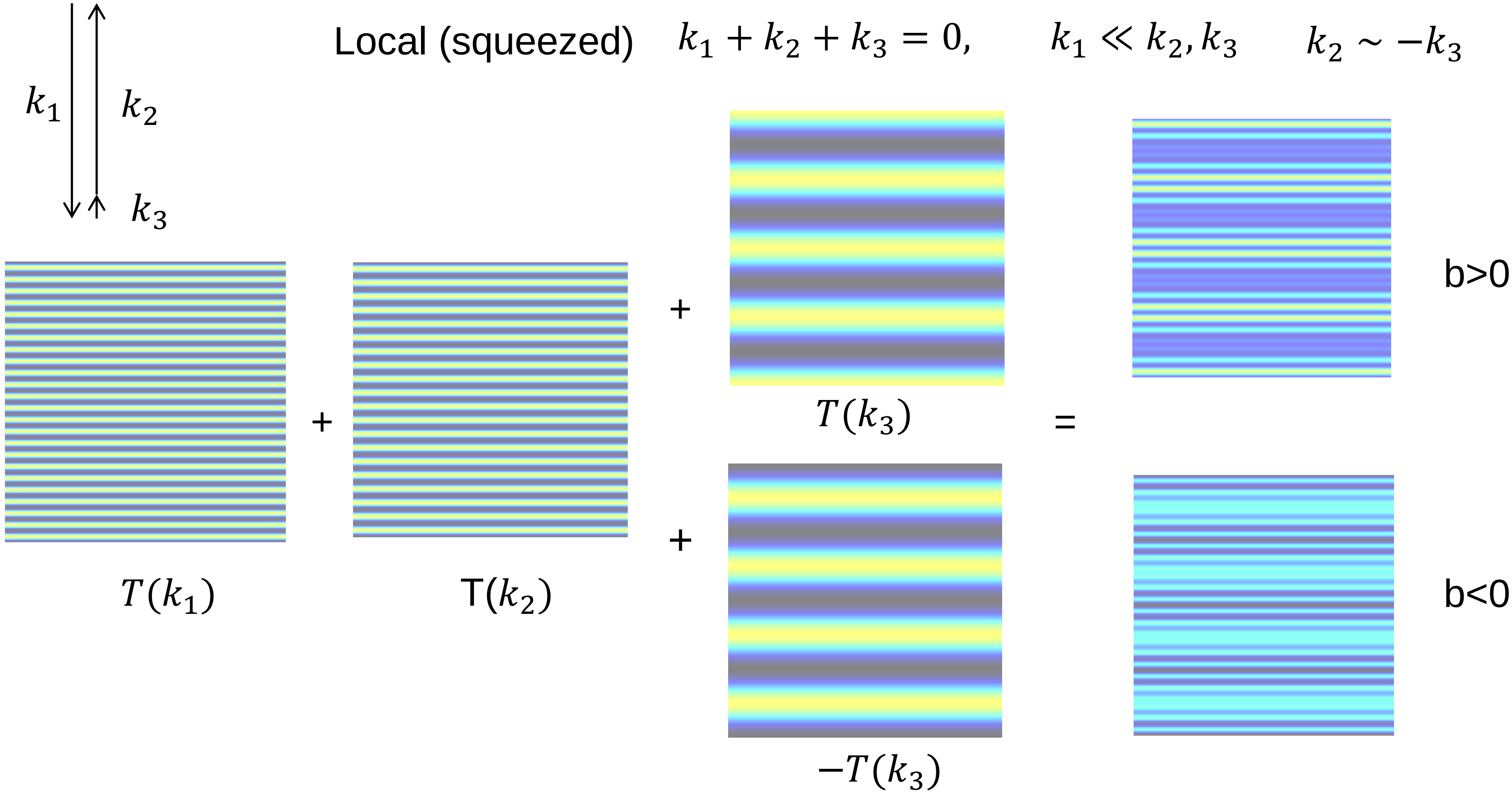
Millennium simulation

Near-equilateral to flattened:





Local (squeezed) $k_1 + k_2 + k_3 = 0,\quad k_1 \ll k_2, k_3 \quad k_2 \sim -k_3$



Squeezed bispectrum is a *correlation* of small-scale power with large-scale modes

How do you get it?

Local primordial spatial modulation

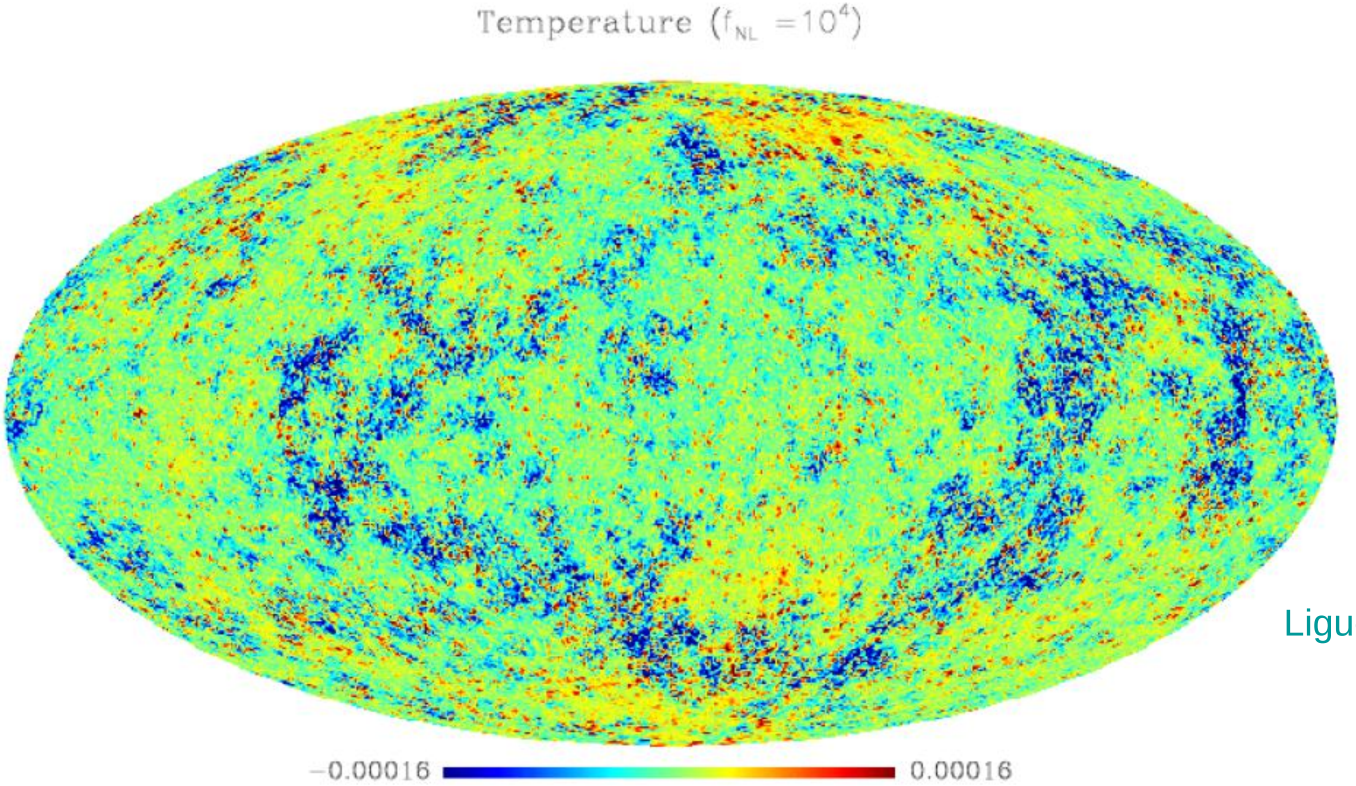
$$\chi(\mathbf{x}) = \chi_0(\mathbf{x})[1 + \phi(\mathbf{x})]$$

Large-scale modulating field (small)

Gaussian and statistically homogeneous

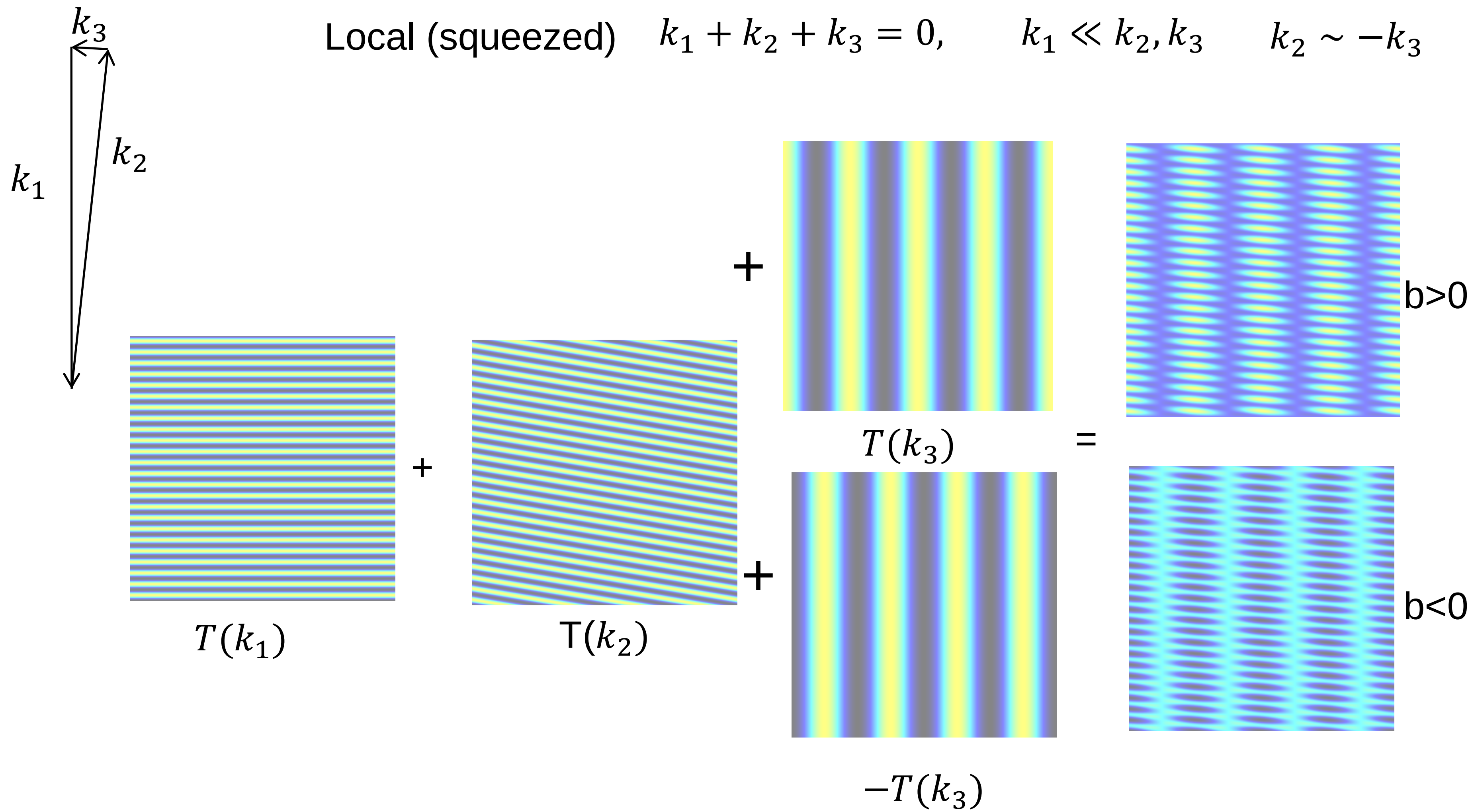
Bispectrum if modulation correlated to χ_0

e.g. $\chi = \chi_0(1 + f_{NL}\chi_0)$



Liguori et al 2007

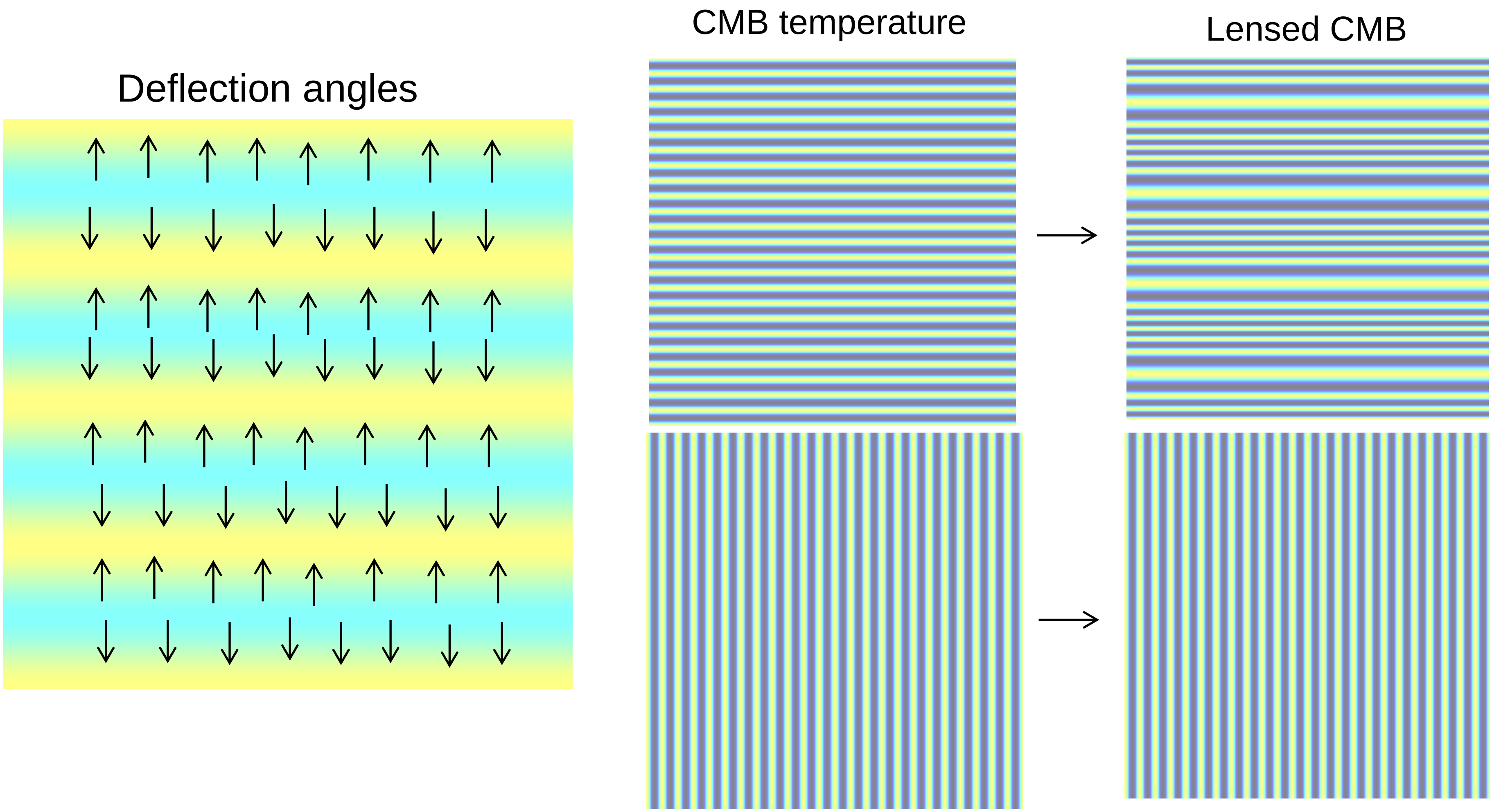
Local (squeezed) $k_1 + k_2 + k_3 = 0,$ $k_1 \ll k_2, k_3$ $k_2 \sim -k_3$



Possible direction-dependent modulation.

Local modulations (e.g. f_{NL}) are isotropic, but e.g. CMB lensing is not

Why is lensing anisotropic?



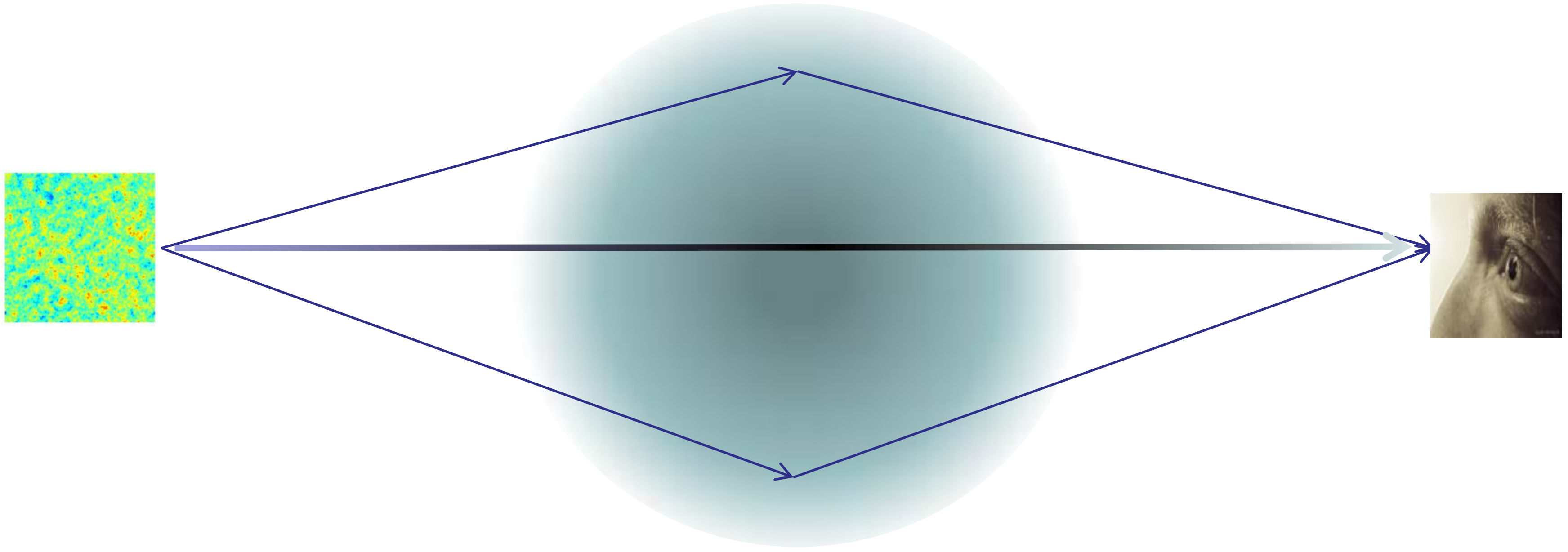
Modulation depends on relative orientation

$\Rightarrow$ anisotropic ψTT bispectrum

Anisotropic ψTT bispectrum $\Rightarrow TTT$ bispectrum if ψ and T are correlated

Is there a correlation between large-scale lenses and the CMB temperature?

$$\Delta T_{\mathrm{ISW}}(\hat{\mathbf{n}}) = 2 \int_0^{\chi_*} \mathrm{d}\chi \, \dot{\Psi}(\chi \hat{\mathbf{n}}; \eta_0 - \chi)$$

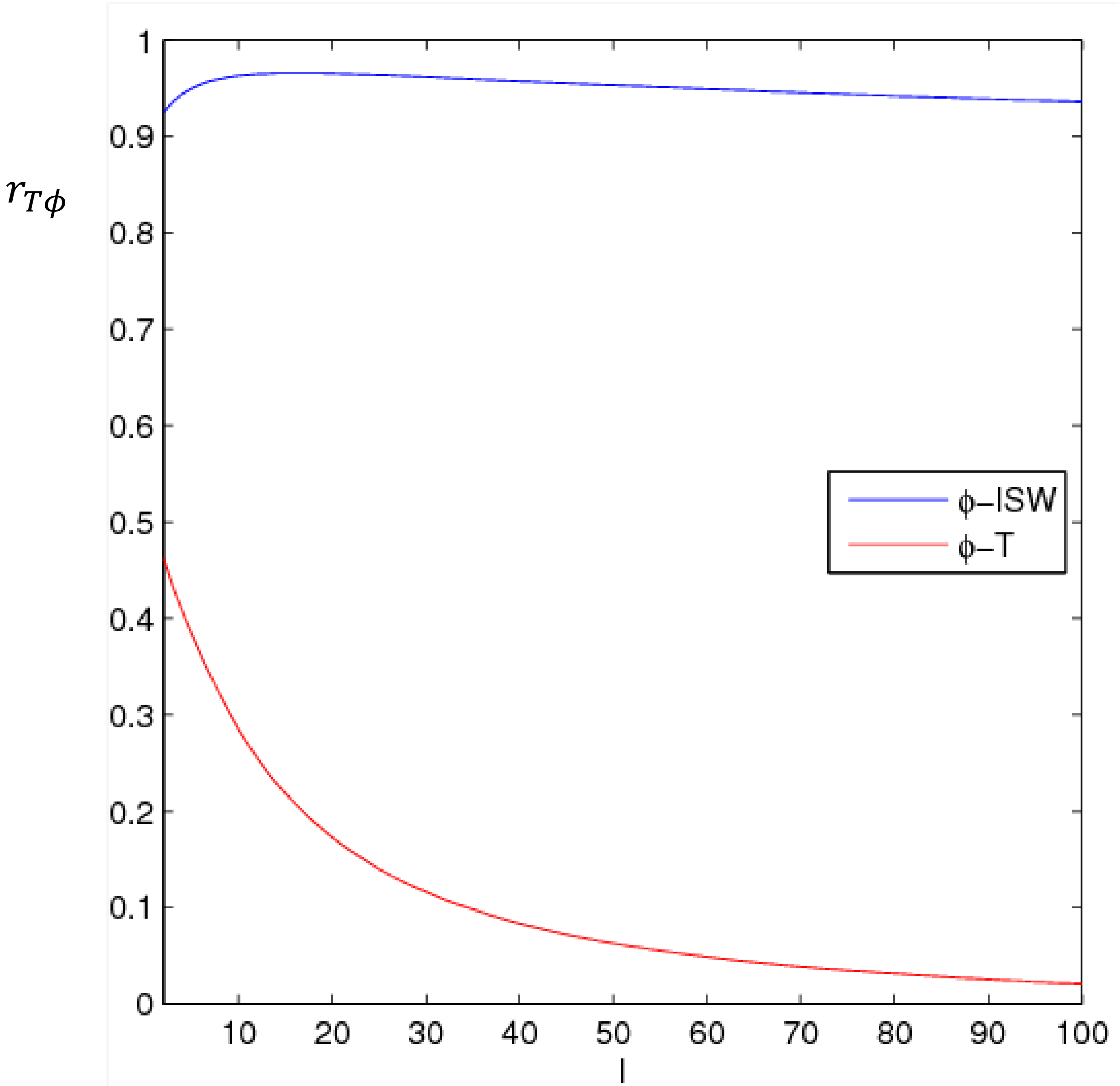


Overdensity: magnification correlated with positive Integrated Sachs-Wolfe (net blueshift)

Underdensity: demagnification correlated with negative Integrated Sachs-Wolfe (net redshift)

(small-scales: also SZ , Rees-Sciama..)

Large-scale $T - \psi$ correlation



CMB polarization

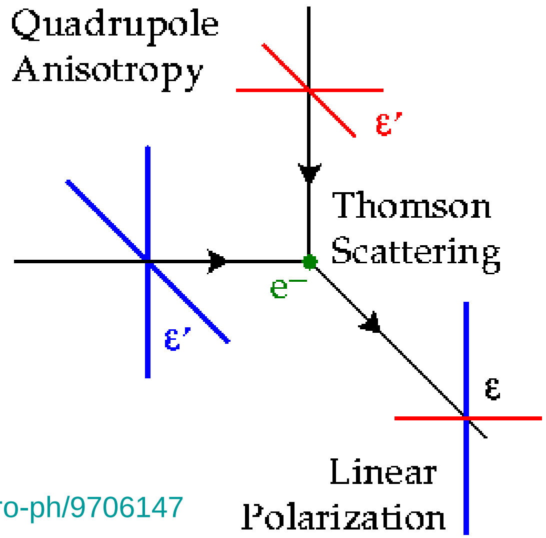
General full-sky bispectrum: $\mathbf{a}_{lm} = (T_{lm}, E_{lm}, B_{lm})^T$

$$B^{ijk}_{l_1 l_2 l_3} = \sum_{m_1 m_2 m_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \langle a^i_{l_1 m_1} a^j_{l_2 m_2} a^k_{l_3 m_3} \rangle$$

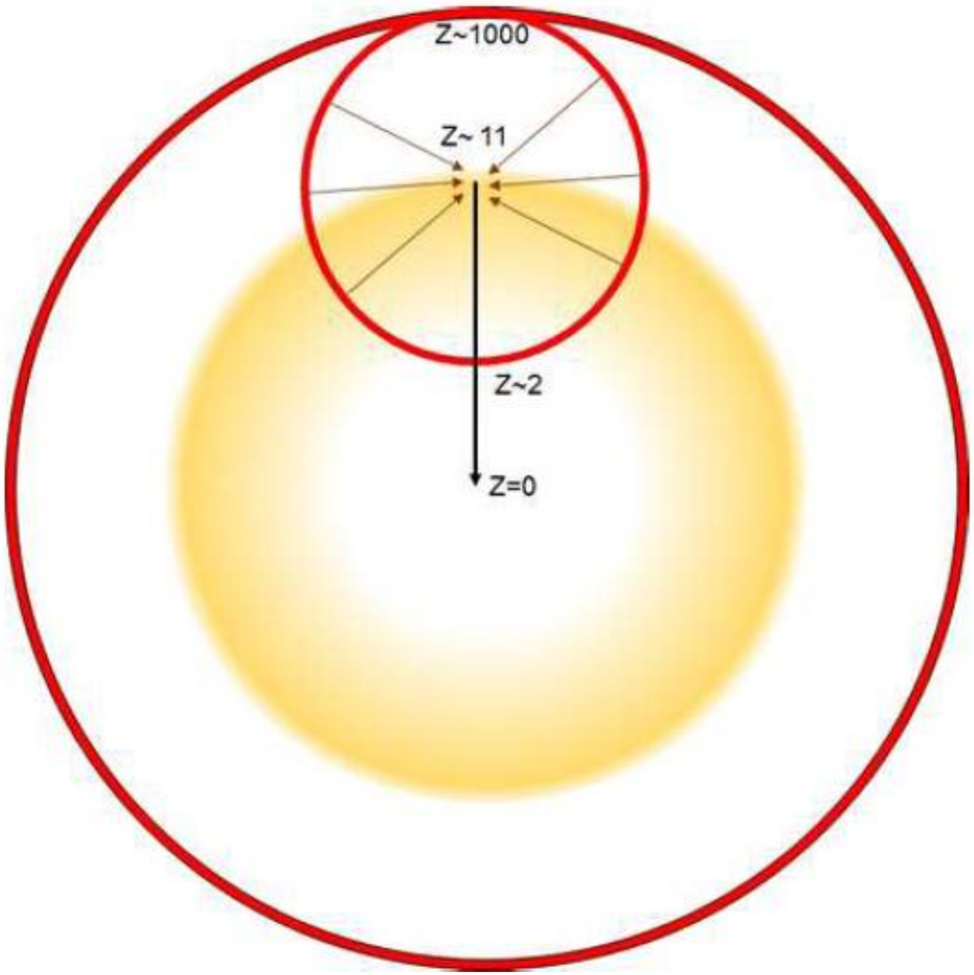
$$\approx F^{s_k}_{l_3 l_1 l_2} C^{a^i \psi}_{l_1} \tilde{C}^{a^j a^k}_{l_2} + i F^{-s_k}_{l_3 l_1 l_2} C^{a^i \psi}_{l_1} \tilde{C}^{a^j \bar{a}^k}_{l_2}$$ + perms

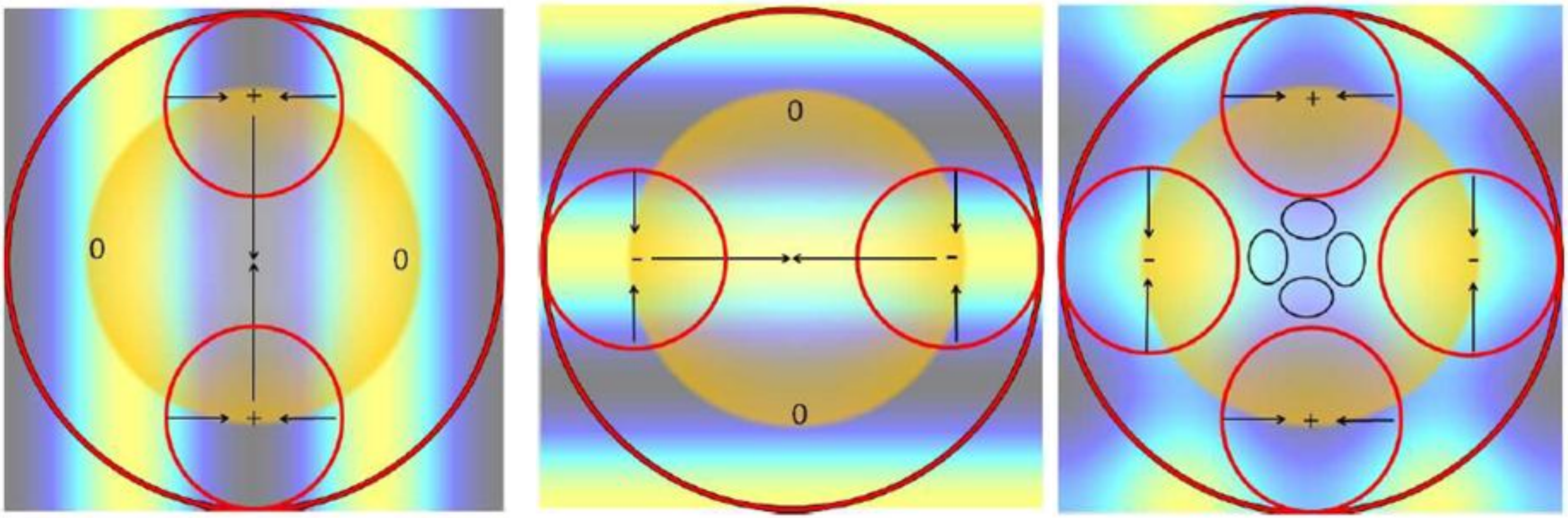
Is the polarization correlated to the large-scale lenses? $C_l^{E\psi} = ?$

Yes! Significant large-scale correlation due to reionization



Hu astro-ph/9706147

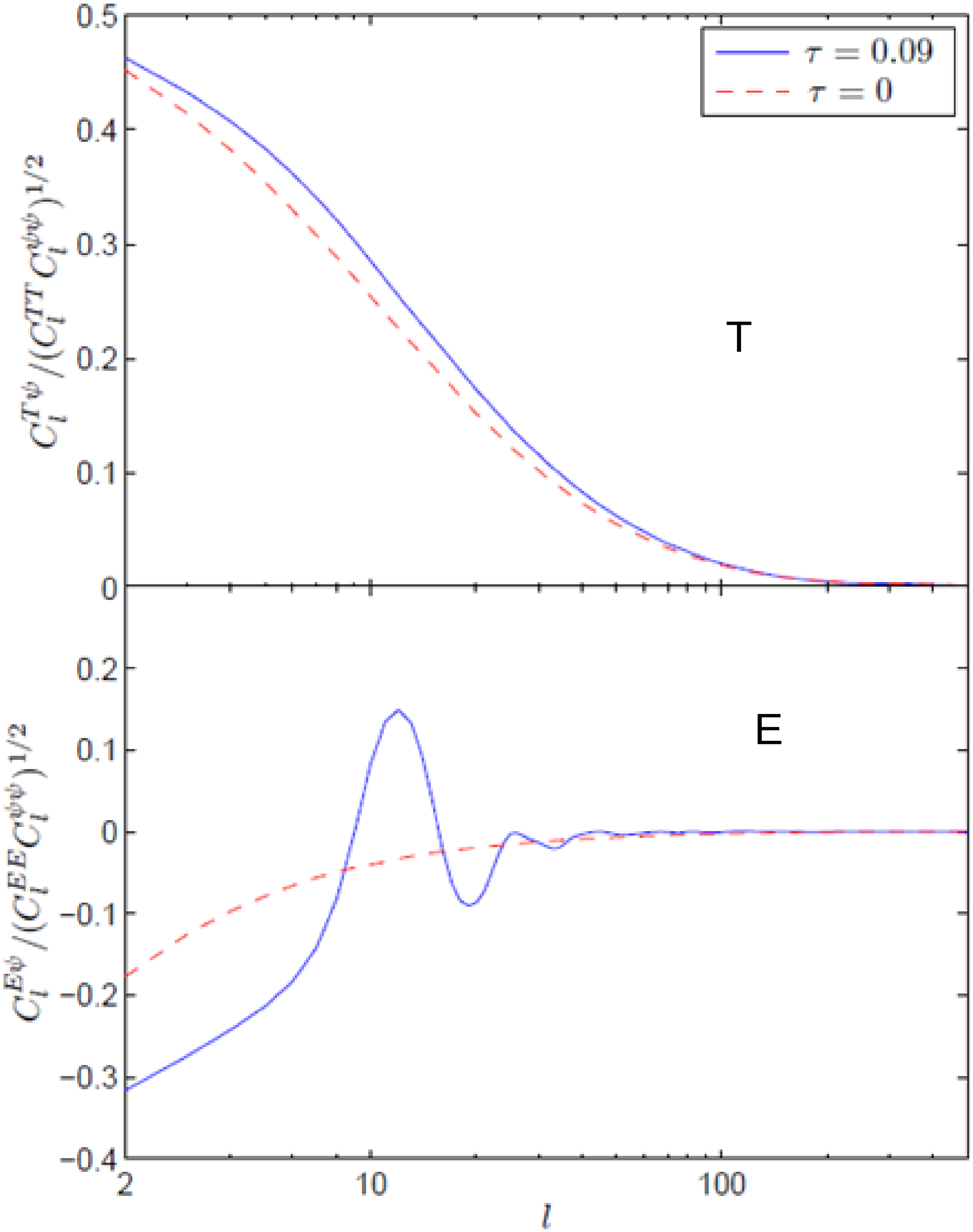




+
0
0
+
0
-
-
0
+
-
-
+

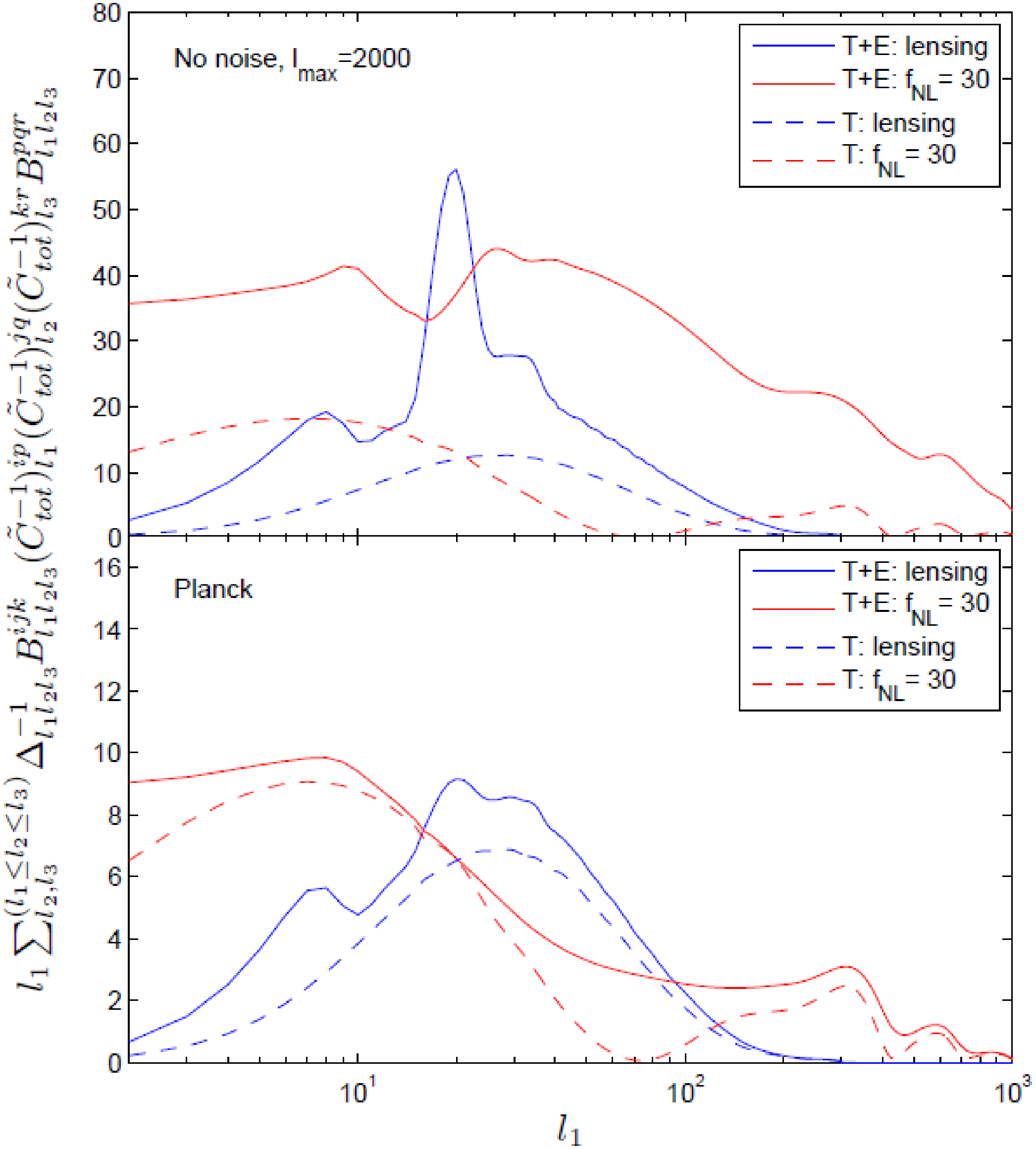
Lensing potential correlations give bispectra $\propto C^{T\psi}, C^{E\psi}$

Cosmic variance:
$C^{T\psi} \sim 7\sigma, C^{E\psi} \sim 2.5\sigma$



Signal to noise

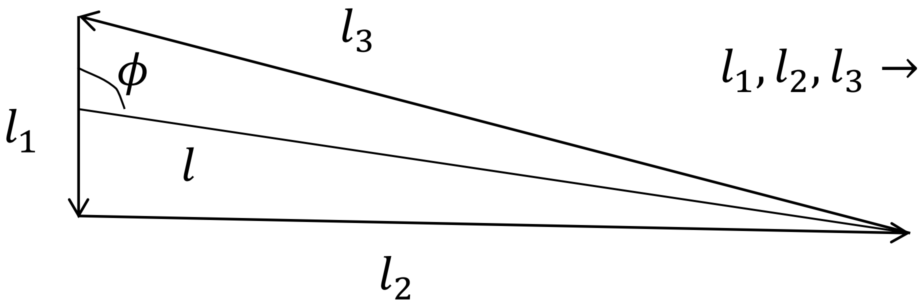
Contributions to Fisher inverse variance for $b_{l_1 l_2 l_3} = 0$



If lensing neglected, gives bias $f_{NL} \sim 9$ for Planck

How do we distinguish lensing from primordial?

Shape decomposition of squeezed triangles



$$l_1, l_2, l_3 \to l_1, l, \phi$$

$$b_{l_1 l_2 l_3} = \sum_m b^m_{l_1 l} e^{im\phi}$$

Local isotropic modulations: $m = 0$

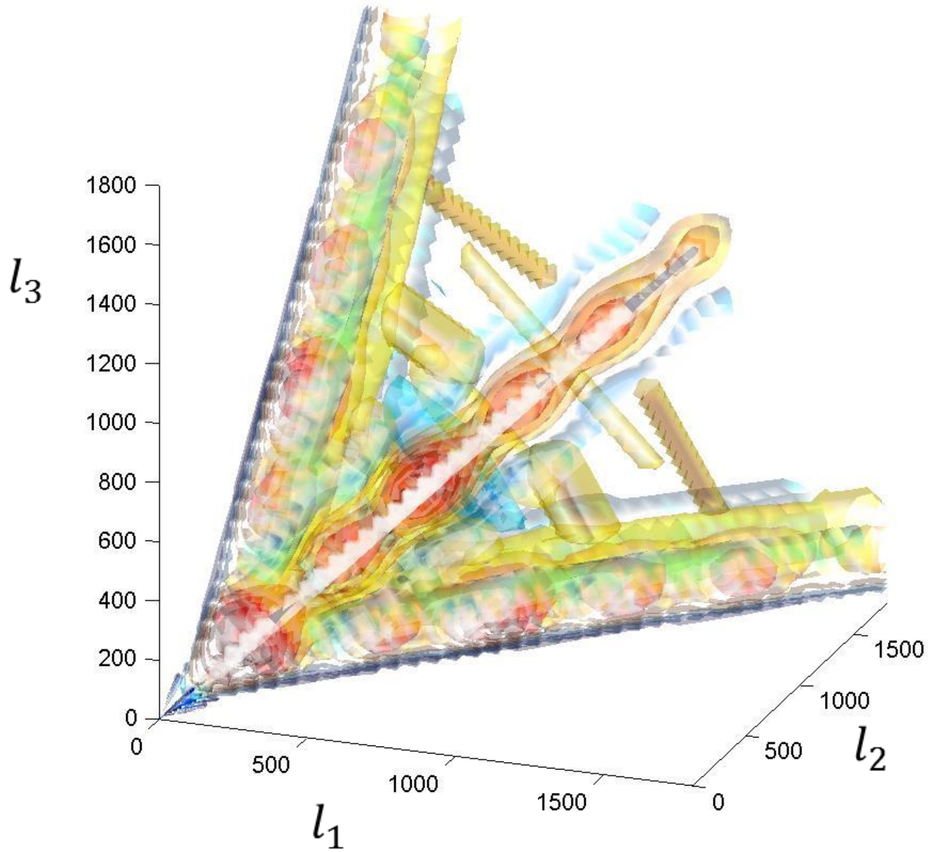
CMB lensing: $m = 0$ + $m = 2$

Looks like $f_{NL} \sim 9$ Orthogonal to f_{NL}

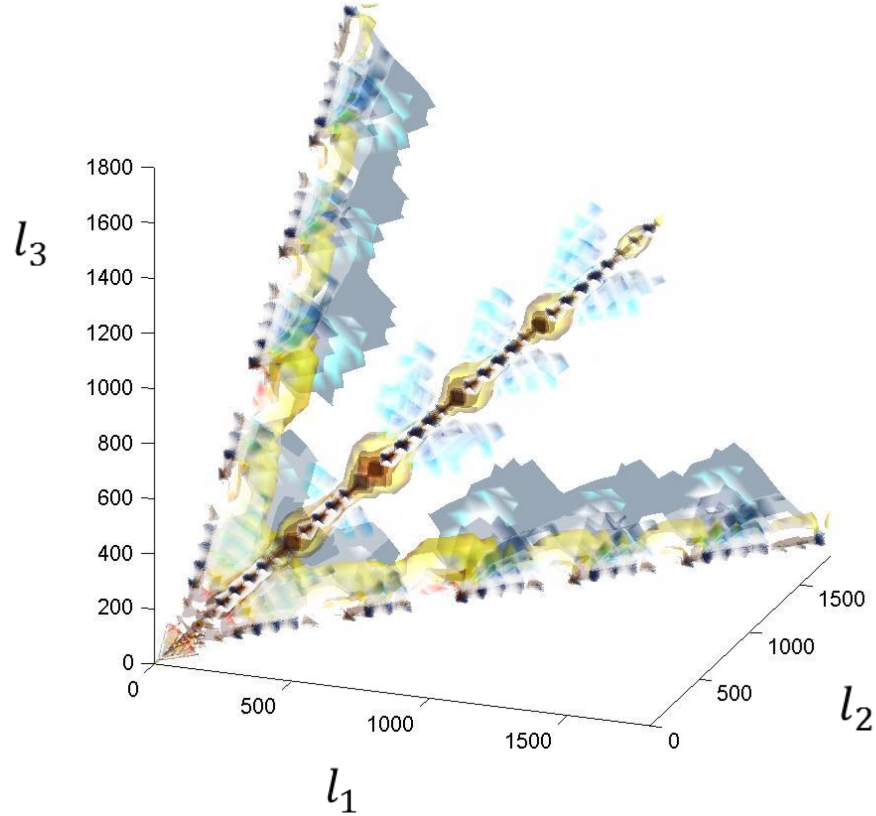
Angular dependence can be used to isolate and subtract different signals

- also l_1 dependence and phase of l dependence is distinctive

$b_{l_1 l_2 l_3}$



Local f_{NL}



CMB lensing

Lewis, Challinor & Hanson 2010

Reconstructing the modulation field

Marginalized over (unobservable) lensing field:

$$T \sim \int P(T,\psi)d\psi$$

- Non-Gaussian statistically isotropic temperature distribution
- Bispectrum + significant connected 4-point function

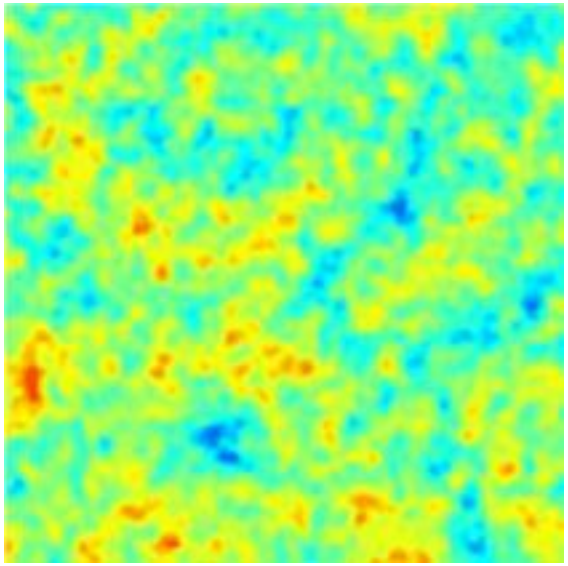
For a given lensing field :

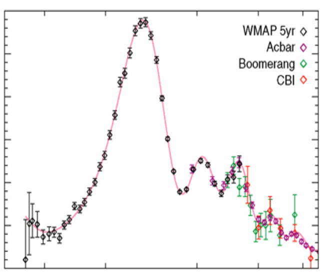
$$T \sim P(T|\psi)$$

- Anisotropic Gaussian temperature distribution
- Different parts of the sky magnified and demagnified
- Re-construct the actual lensing field – infer ψ

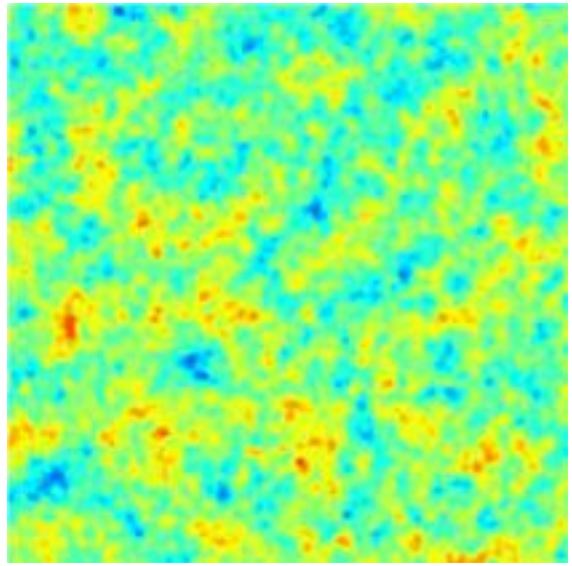
Anisotropy estimators – reconstruct the ‘modulating’ field

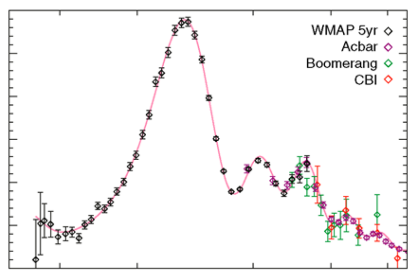
Magnified



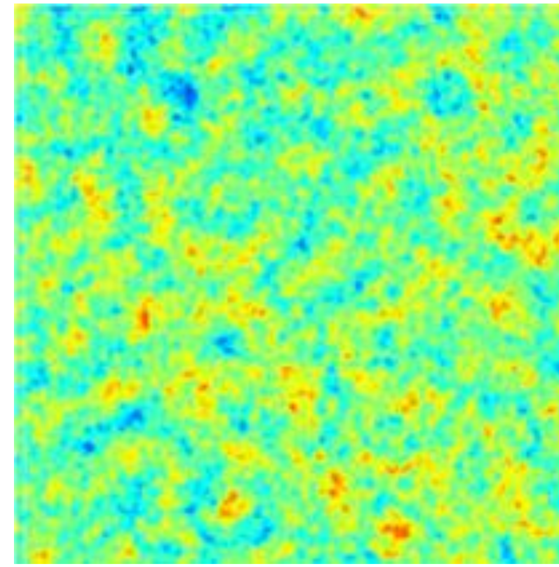


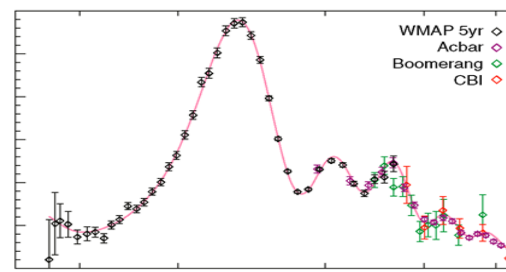
Unlensed





Demagnified

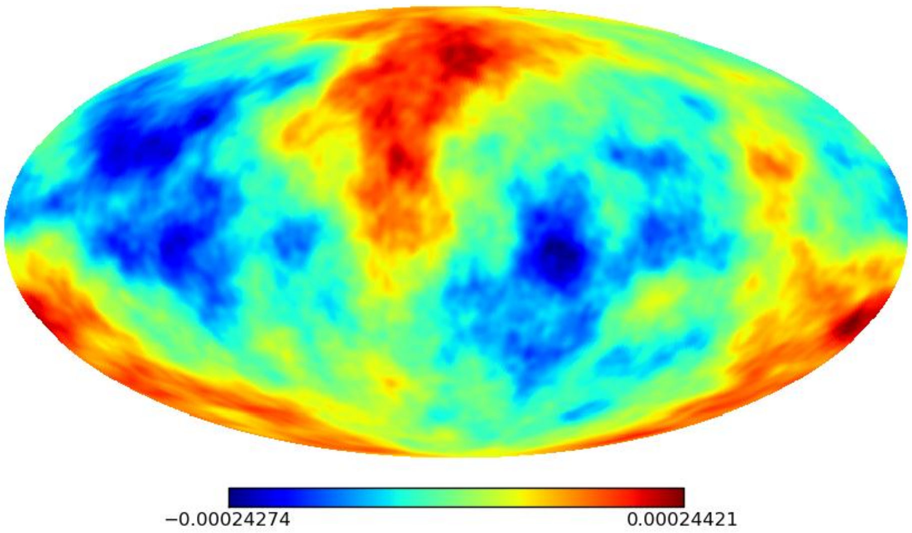




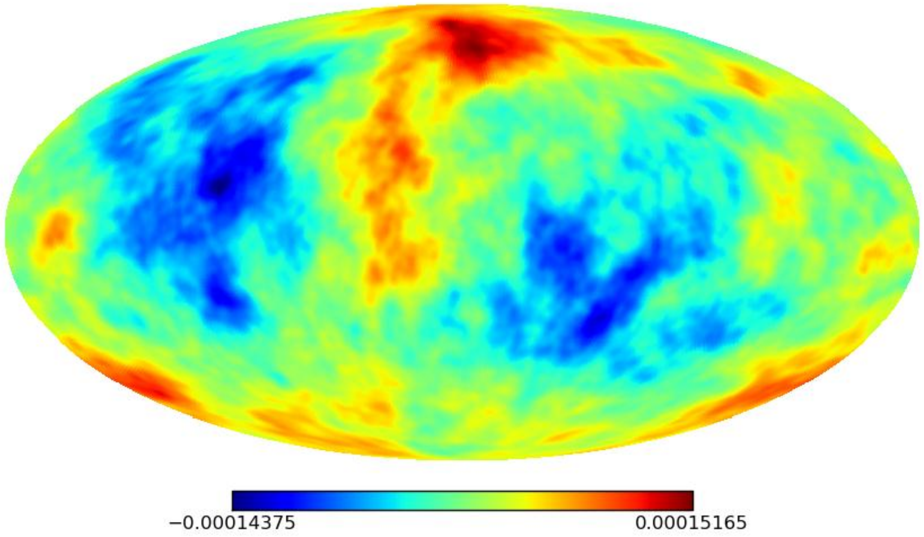
⇒Quadratic Maximum Likelihood Estimator for $\psi \sim g_{\mathrm{linear}}(T,T)$

Can reconstruct CMB lensing potential, ψ_{lm}

True (simulated)



Reconstructed (Planck noise, Wiener filtered)



(Credit: Duncan Hanson)

From reconstructed ψ_{lm} can estimate lensing power spectrum $C_l^{\psi\psi}$

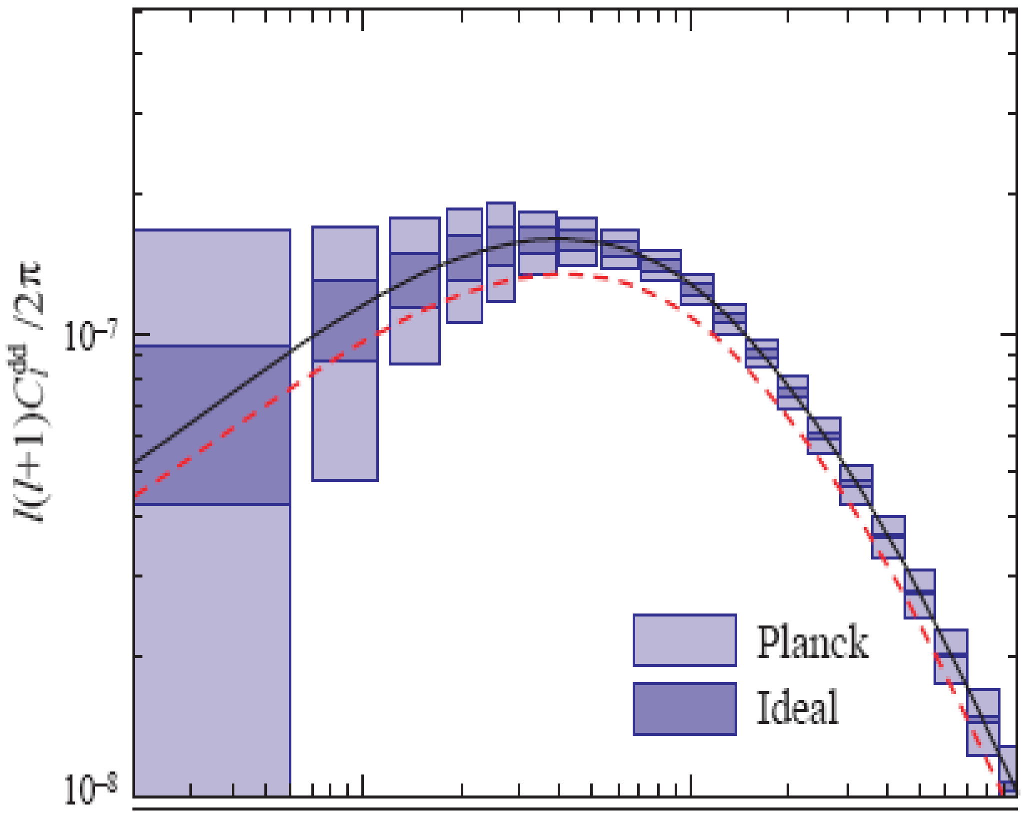


FIG. 3. CMB lensing power spectra for the fiducial $w = -1$ model (solid) and the degenerate $w = -2/3$ model (dashed) of Fig. 1. Boxes represent 1σ errors on band powers assuming the Planck and ideal experiments of Tab. I. Top: deflection power spectra. Bottom: cross correlation of deflection and temperature fields.

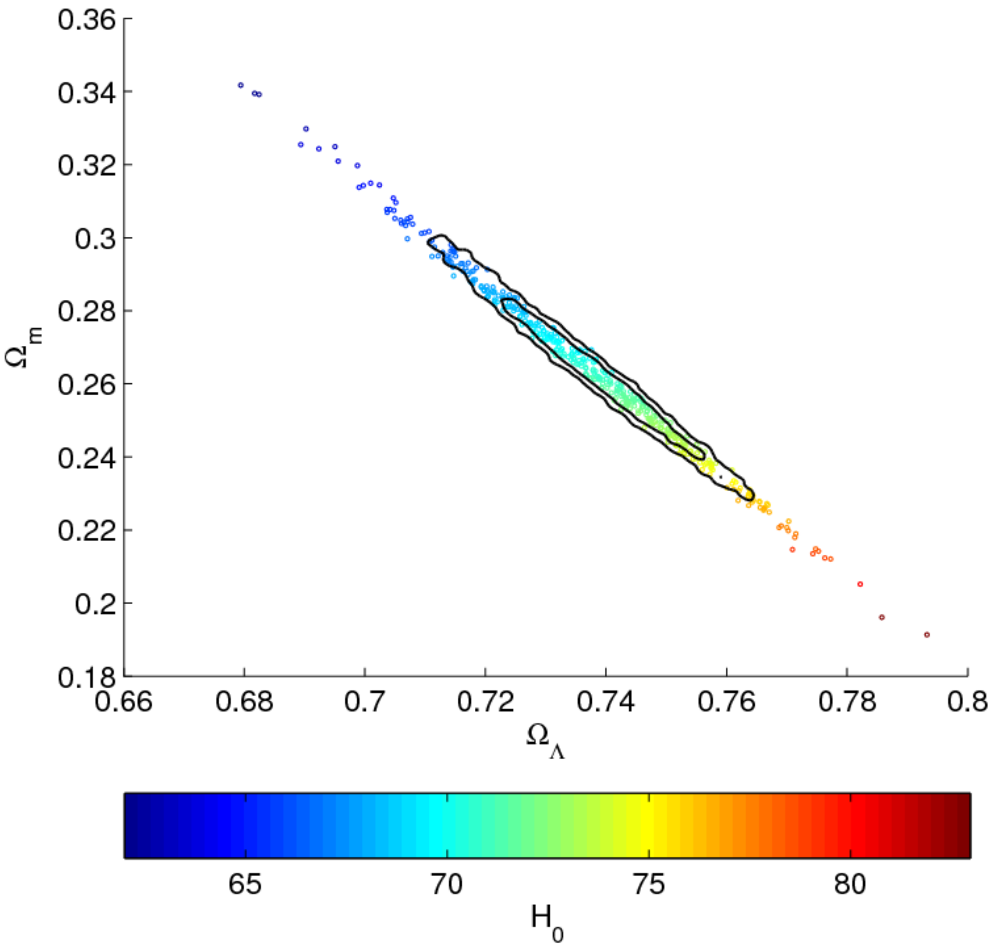
Hu: astro-ph/0108090

What does an estimate of $C_l^{\psi\psi}$ do for us?

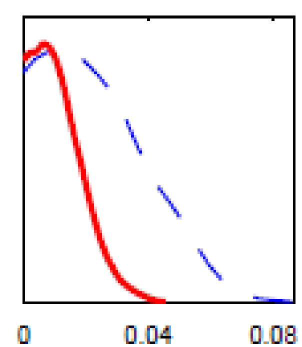
Probe 0.5<~ z <~ 6: depends on geometry and matter power spectrum

Can break degeneracies in the linear CMB power spectrum

- Better constraints on neutrino mass, dark energy, Ω_K, …



Neutrino mass fraction with and without lensing (Planck only)



Perotto et al. 2006

$$\Delta \sum m_\nu \sim 0.1 eV \ (1\sigma)$$

Reconstructed $\psi \sim \ T\ T$

Hence:

Bispectrum measured by $C_l^{\mathrm{T}\psi} = \langle \mathrm{T}_{lm}^* \ \psi_{lm} \rangle$

Trispectrum measured by $C_l^{\psi\psi} = \langle {\psi_{lm}}^* \psi_{lm} \rangle$

$\left(C_l^{T\psi}\right)^2 < C_l^{TT} C_l^{\psi\psi}$ ⇒ “bispectrum < trispectrum” (*in some sense*)

- general property of squeezed non-Gaussianity

What non-Gaussianity does a large-scale modulation give you?

$$\chi(\mathbf{x}) = \chi_0(\mathbf{x})[1 + \phi(\mathbf{x})]$$

Gives squeezed non-Gaussianity. Small ϕ:

$$\langle\chi\chi\chi\rangle \sim \langle\chi_0\chi_0\chi_0\phi\rangle + \cdots \qquad \sim P_{\chi_0\chi_0}P_{\chi_0\phi}$$

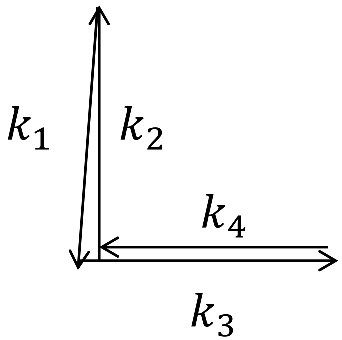
$$\langle\chi\chi\chi\chi\rangle \sim \langle\chi_0\chi_0\chi_0\chi_0\rangle + \langle\chi_0\chi_0\chi_0\chi_0\phi\phi\rangle + \cdots$$

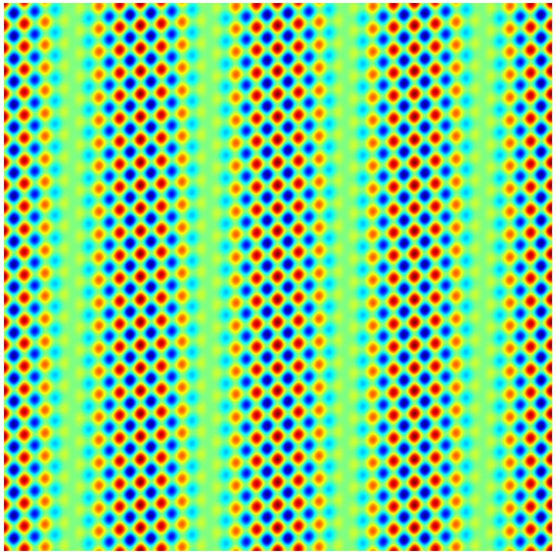
$$\sim (Gaussian) + \ P_{\chi_0\chi_0}P_{\chi_0\chi_0}P_{\phi\phi}$$

Since $P_{\chi_0\phi}^2 \leq P_{\chi_0\chi_0}P_{\phi\phi}$ $\qquad P_{\chi_0\chi_0}\langle\chi\chi\chi\chi\rangle_{squeezed} \geq \langle\chi\chi\chi\rangle_{squeezed}^2$

In conventional definitions $\tau_{NL} \geq \left(\frac{6f_{NL}}{5}\right)^2$ (also L by L if quasi local)

Diagonal squeezed trispectra $|k_1| \sim |k_2|, |k_3| \sim |k_4|, |k_1 + k_2| = |k_3 + k_4| \ll |k_2|, |k_3|$





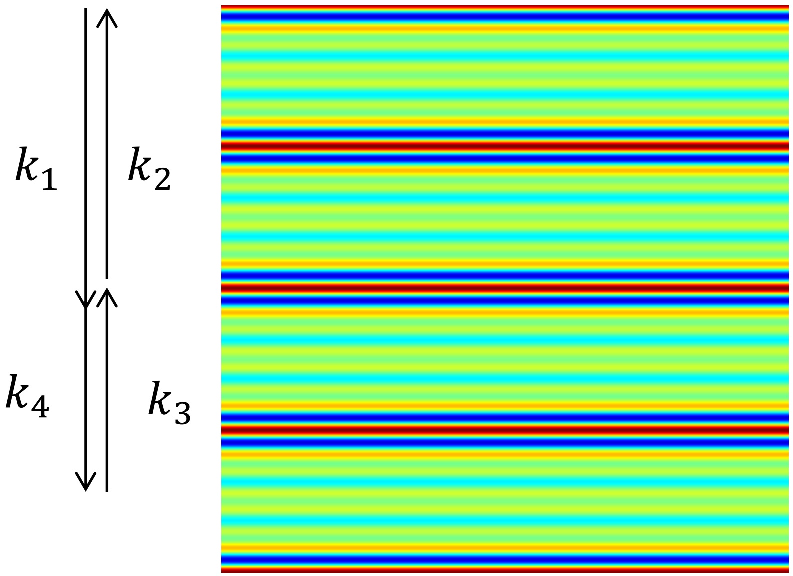
Trispectrum = power spectrum of modulation

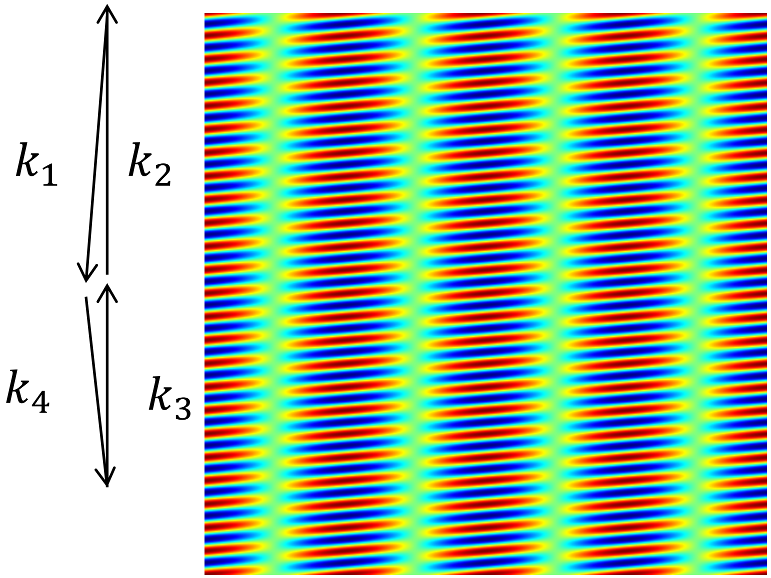
e.g. $\chi = \chi_0(1 + f_{NL}\chi_0)$

$\tau_{NL} \sim f_{NL}^2$

or $\chi = \chi_0(1 + \phi)$

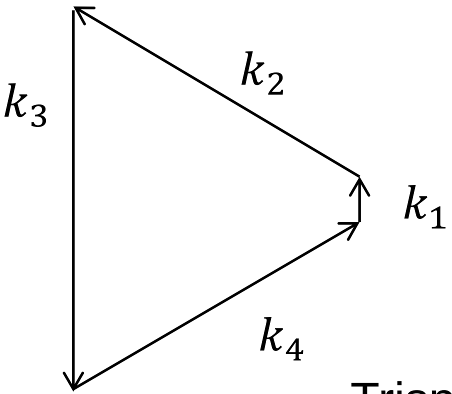
(any correlation, $\tau_{NL} > f_{NL}^2$)





One-leg squeezed trispectra

$|k_1| \ll |k_2| \sim |k_3| \sim |k_4|$

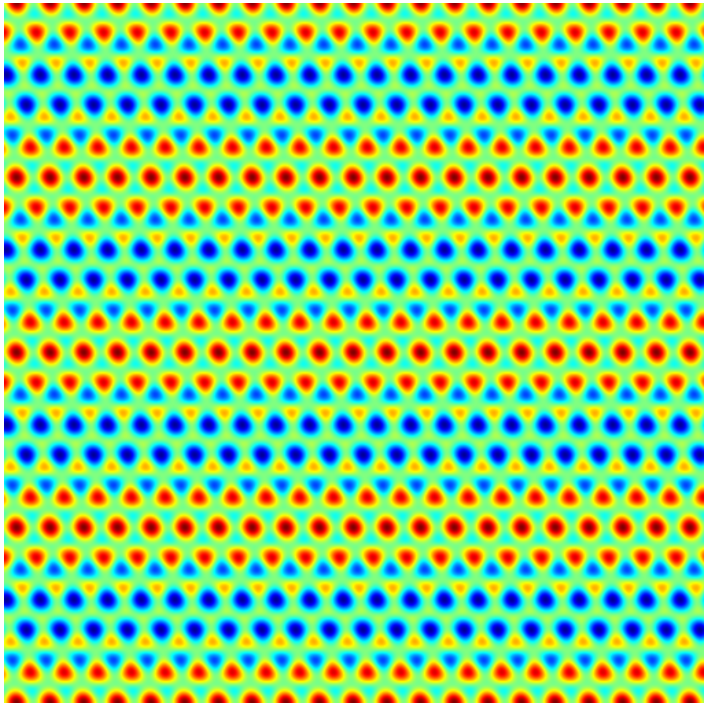


Correlated modulation of equilateral bispectrum

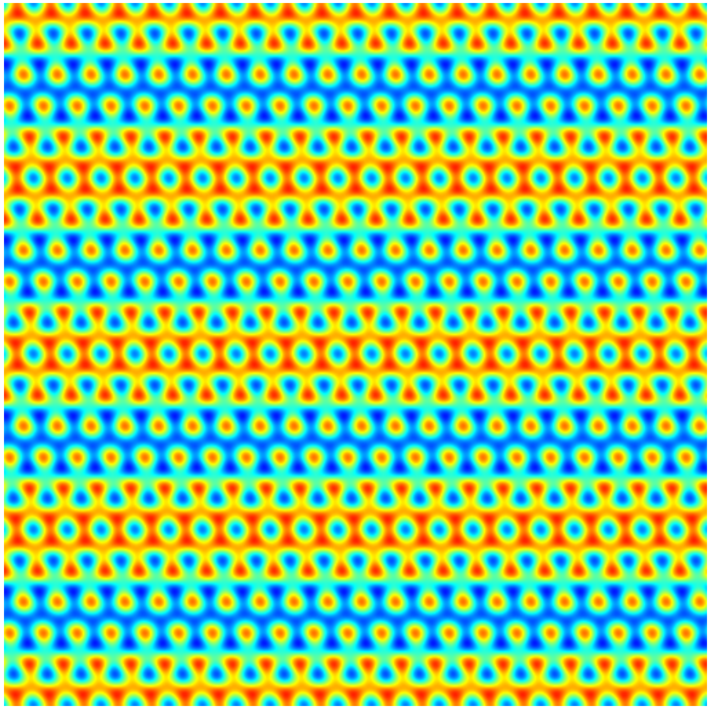
e.g. from $\chi = \chi_0(1 + g_{NL}\chi_0^2)$ [***no*** diagonal squeezed]

Trispectrum>0

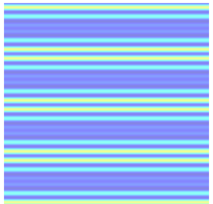
also from $\chi = \chi_0(1 + f_{NL}\chi_0)$ [***plus*** diagonal squeezed]



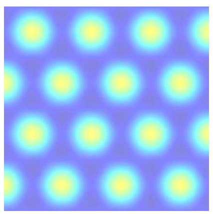
Trispectrum<0



Hunters' guide to the non-Gaussianity zoo



Bispectrum



Squeezed

Isotropic (f_{NL}) Local modulations $\chi = \chi_0(1+\phi)$, $P_{\chi_0 \phi} \neq 0$

CMB lensing magnification

Anisotropic

CMB lensing

Non-standard anisotropic inflation

Bispectrum = power modulation correlated to temperature $(\chi)(\chi\chi)$

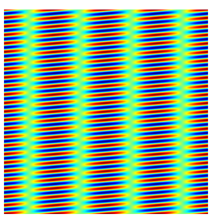
Equilateral-flattened

Dynamically generated

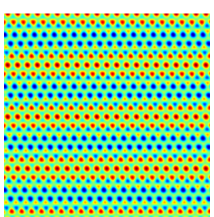
e.g. non-linear structure growth

inflation before horizon exit (e.g. $c_s \ll 1$)

Bispectrum = concentrated hot spots in areas of milder cold (or vice versa)



Trispectrum



Diagonal squeezed

Isotropic(τ_{NL}) Local modulations $\chi = \chi_0(1+\phi)$ (any $P_{\chi_0 \phi}$)

CMB lensing magnification

Anisotropic

CMB lensing

Anisotropic primordial power spectrum

Non-standard anisotropic inflation

Gravitational-wave modulation (small)

Trispectrum = power spectrum of power modulation, $(\chi\chi)(\chi\chi)$

One-leg squeezed (small: g_{NL} either sign, or τ_{NL} positive)

Trispectrum = bispectrum modulation correlated to temperature $(\chi)(\chi\chi\chi)$

…

Conclusions

- CMB is still by far the cleanest probe of early-universe physics and cosmological parameters; Planck analysis underway

- CMB lensing very important: changes power spectra and generates B modes

- Non-Gaussianities. Many can be studied by reconstructing modulations with quadratic estimators.

    - CMB Lensing – definitely there

        - Large anisotropic bispectrum, must be modelled but not a problem

        - Large trispectrum – reconstruct the lensing potential

$\longrightarrow$ Break primary CMB degeneracies, improve parameters