CMB Lensing and other non-Gaussianities

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Lewis, Challinor & Hanson arXiv:1101.2234
Evolution of the universe

(almost) uniform 2.726K blackbody

Dipole (local motion)

O$(10^{-5})$ perturbations (+galaxy)

Observations: the microwave sky today

Source: NASA/WMAP Science Team
Can we predict the primordial perturbations?

- Maybe..

Quantum Mechanics
  “waves in a box”

Inflation
  make $>10^{30}$ times bigger

After inflation
  Huge size, amplitude $\sim 10^{-5}$
CMB temperature

End of inflation

Last scattering surface

gravity+ pressure+ diffusion
Observed CMB temperature power spectrum

Observations

Constrain theory of early universe + evolution parameters and geometry

WMAP team
Planck and the future

High sensitivity and resolution
CMB temperature and polarization

14 May 2009
Parameters. WMAP4 vs Planck

FIG 2.18.—Forecasts of 1 and 2σ contour regions for various cosmological parameters when the spectral index is allowed to run. Blue contours show forecasts for WMAP after 4 years of observation and red contours show results for Planck after 1 year of observations. The curves show marginalized posterior distributions for each parameter.
e.g. Geometry: curvature

We see:

flat

θ

closed

θ
or is it just closer??

We see:

Degeneracies between parameters
Need other information to break remaining degeneracies
Constrain *combinations* of parameters accurately

Assume Flat, $w = -1$

$$\left(\frac{\Omega_m}{0.254}\right) \left(\frac{h}{0.72}\right)^{3.15} = 1.00 \pm 0.03$$

$$\left(\frac{\Omega_m}{0.254}\right) \left(\frac{h}{0.72}\right)^{-3.15} = 1.03 \pm 0.23$$

Use other data to break remaining degeneracies

Planck forecast
CMB Lensing

Last scattering surface

Inhomogeneous universe - photons deflected

Observer
Lensing order of magnitudes

Newtonian argument: $\beta = 2\,\Psi$
General Relativity: $\beta = 4\,\Psi$ \quad (\beta \ll 1)

Potentials linear and approx Gaussian: $\Psi \sim 2 \times 10^{-5}$
\[ \beta \sim 10^{-4} \]

Characteristic size from peak of matter power spectrum \sim 300\text{Mpc}
Comoving distance to last scattering surface \sim 14000 \text{Mpc}

pass through \sim 50 \text{lumps} \quad \text{total deflection} \sim 50^{1/2} \times 10^{-4} \quad \sim 2 \text{arcminutes}

(ignores angular factors, correlation, etc.)
So why does it matter?

• 2arcmin: $ell \sim 3000$
  
  - On small scales CMB is very smooth so lensing dominates the linear signal

• Deflection angles coherent over $300/(14000/2) \sim 2^\circ$
  
  - comparable to CMB scales
  
  - expect $2\text{arcmin}/60\text{arcmin} \sim 3\%$ effect on main CMB acoustic peaks
In detail, lensed temperature depends on deflection angle:

\[
\tilde{T}(\hat{n}) = T(\hat{n}') = T(\hat{n} + \alpha)
\]

\[
\alpha = \delta \theta = -2 \int_0^{\chi^*} d\chi \frac{f_K(\chi^* - \chi)}{f_K(\chi^*)} \nabla_\perp \Psi(\chi \hat{n}; \eta_0 - \chi)
\]

**Lensing Potential**

Deflection angle on sky given in terms of lensing potential \( \alpha = \nabla \psi \)

\[
\psi(\hat{n}) = -2 \int_0^{\chi^*} d\chi \Psi(\chi \hat{n}; \eta_0 - \chi) \frac{f_K(\chi^* - \chi)}{f_K(\chi^*) f_K(\chi)}
\]

\[
\tilde{X}(n) = X(n') = X(n + \nabla \psi(n))
\]
Deflections $O(10^{-3})$, but coherent on degree scales $\Rightarrow$ important!
Easily simulated assuming Gaussian fields
- just re-map points using Gaussian realisations of CMB and potential
Lensing effect on CMB temperature power spectra

CAMB’s 0.1% calculation; [http://camb.info](http://camb.info) : Challinor & Lewis 2005, astro-ph/0502425
Why lensing is important

- Known effect, small but significant amplitude ($\sim 10^{-3}$)
- Modifies the power spectra on small-scales ($\sim 10^{-2}$)
- Lensing of E gives B-mode polarization (confusion for tensors/strings)
- Anisotropy/non-Gaussianities…. 
Beyond the power spectrum

Magnified

Unlensed

Demagnified
Beyond Gaussianity – general possibilities

Flat sky approximation:

$$\Theta(x) = \frac{1}{2\pi} \int d^2 l \, \Theta(l) e^{ix \cdot l}$$

($$\Theta = T$$)

Gaussian + statistical isotropy

$$\langle \Theta(l_1) \Theta(l_2) \rangle = \delta(l_1 + l_2) C_l$$

- power spectrum encodes all the information
- modes with different wavenumber are independent

Higher-point correlations

Gaussian: can be written in terms of $$C_l$$

Non-Gaussian: non-zero connected $$n$$-point functions
Flat sky approximation: \[ \langle \Theta(l_1)\Theta(l_2)\Theta(l_3) \rangle = \frac{1}{2\pi} \delta(l_1 + l_2 + l_3) b_{l_1l_2l_3} \]

If you know \( \Theta(l_1), \Theta(l_2) \), sign of \( b_{l_1l_2l_3} \) tells you which sign of \( \Theta(l_3) \) is more likely

Trispectrum

\[
\langle \Theta(l_1)\Theta(l_2)\Theta(l_3)\Theta(l_4) \rangle = (2\pi)^{-2} \delta(l_1 + l_2 + l_3 + l_4) T(l_1, l_2, l_3, l_4)
\]

\[
\langle \Theta(l_1)\Theta(l_2)\Theta(l_3)\Theta(l_4) \rangle = \frac{1}{2} \int \frac{d^2 L}{(2\pi)^2} \delta(l_1 + l_2 + L) \delta(l_3 + l_4 - L) T^{(\ell_1\ell_2)}_{(\ell_3\ell_4)}(L) + \text{perms.}
\]
Equilateral \( k_1 + k_2 + k_3 = 0, |k_1| = |k_2| = |k_3| \)
Millennium simulation
Near-equilateral to flattened:

$\begin{align*}
&b > 0 \\
&b < 0
\end{align*}$
Squeezed bispectrum is a *correlation* of small-scale power with large-scale modes.
How do you get it?

Local primordial spatial modulation

\[ \chi(x) = \chi_0(x)[1 + \phi(x)] \]

- Gaussian and statistically homogeneous
- Large-scale modulating field (small)
- Bispectrum if modulation correlated to \( \chi_0 \)
e.g. $\chi = \chi_0 (1 + f_{NL} \chi_0)$
Local (squeezed) \[ k_1 + k_2 + k_3 = 0, \quad k_1 \ll k_2, k_3 \quad k_2 \sim -k_3 \]

Possible direction-dependent modulation.

Local modulations (e.g. \( f_{NL} \)) are isotropic, but e.g. CMB lensing is not
Why is lensing anisotropic?

Deflection angles

CMB temperature

Lensed CMB

Modulation depends on relative orientation

⇒ anisotropic $\psi TT$ bispectrum
Anisotropic $\psi TT$ bispectrum $\Rightarrow TTT$ bispectrum if $\psi$ and $T$ are correlated

Is there a correlation between large-scale lenses and the CMB temperature?
\[ \Delta T_{\text{ISW}}(\hat{n}) = 2 \int_0^{\chi^*} d\chi \Psi(\chi \hat{n}; \eta_0 - \chi). \]

Overdensity: magnification correlated with positive Integrated Sachs-Wolfe (net blueshift)

Underdensity: demagnification correlated with negative Integrated Sachs-Wolfe (net redshift)

(small-scales: also SZ, Rees-Sciama..)
Large-scale $T - \psi$ correlation

$r_{T\phi}$

$I$
CMB polarization

General full-sky bispectrum:

$$a_{lm} = (T_{lm}, E_{lm}, B_{lm})^T$$

$$B_{l_1 l_2 l_3}^{ijk} = \sum_{m_1 m_2 m_3} \left( \begin{array}{ccc} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{array} \right) \langle a_{l_1 m_1}^i a_{l_2 m_2}^j a_{l_3 m_3}^k \rangle$$

$$\approx F_{l_3 l_1 l_2}^{s_k} C_{l_1}^{a^i} \tilde{C}_{l_2}^{a^j a^k} + i F_{l_3 l_1 l_2}^{-s_k} C_{l_1}^{a^i} \tilde{C}_{l_2}^{a^j \tilde{a}^k} + \text{perms}$$

Is the polarization correlated to the large-scale lenses? $C_l^{E \psi} = ?$
Yes! Significant large-scale correlation due to reionization

Quadrupole Anisotropy

Thomson Scattering

Linear Polarization

Hu astro-ph/9706147
Lensing potential correlations give bispectra $\propto C^{T\psi}, C^{E\psi}$

Cosmic variance:
$C^{T\psi} \sim 7\sigma, C^{E\psi} \sim 2.5\sigma$
Contributions to Fisher inverse variance for $b_{l_1l_2l_3} = 0$

If lensing neglected, gives bias $f_{NL} \sim 9$ for Planck

How do we distinguish lensing from primordial?
Shape decomposition of squeezed triangles

\[ l_1, l_2, l_3 \rightarrow l_1, l, \phi \]

\[ b_{l_1l_2l_3} = \sum_m b^m_{l_1l} e^{im\phi} \]

Local isotropic modulations: \( m = 0 \)

CMB lensing: \( m = 0 \) + \( m = 2 \)

Looks like \( f_{NL} \sim 9 \) Orthogonal to \( f_{NL} \)

Angular dependence can be used to isolate and subtract different signals
- also \( l_1 \) dependence and phase of \( l \) dependence is distinctive
Reconstructing the modulation field

Marginalized over (unobservable) lensing field:

\[ T \sim \int P(T, \psi) d\psi \]

- Non-Gaussian statistically isotropic temperature distribution
- Bispectrum + significant connected 4-point function

For a given lensing field:

\[ T \sim P(T|\psi) \]

- Anisotropic Gaussian temperature distribution
- Different parts of the sky magnified and demagnified
- Re-construct the actual lensing field – infer \( \psi \)
Anisotropy estimators – reconstruct the ‘modulating’ field

Magnified

Unlensed

Demagnified

⇒ Quadratic Maximum Likelihood Estimator for $\psi \sim g_{\text{linear}}(T, T)$
Can reconstruct CMB lensing potential, $\psi_{lm}$

True (simulated)

Reconstructed (Planck noise, Wiener filtered)

(Credit: Duncan Hanson)
From reconstructed \( \psi_{lm} \) can estimate lensing power spectrum \( C_l^{\psi\psi} \)

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**FIG. 3.** CMB lensing power spectra for the fiducial \( w = -1 \) model (solid) and the degenerate \( w = -2/3 \) model (dashed) of Fig. 1. Boxes represent 1\( \sigma \) errors on band powers assuming the Planck and ideal experiments of Tab. 1. Top: deflection power spectra. Bottom: cross correlation of deflection and temperature fields.
What does an estimate of $C_l^{\psi\psi}$ do for us?

Probe $0.5 < z < 6$: depends on geometry and matter power spectrum

Can break degeneracies in the linear CMB power spectrum

- Better constraints on neutrino mass, dark energy, $\Omega_K$, ...

Neutrino mass fraction with and without lensing (Planck only)

$\Delta \Sigma m_\nu \sim 0.1eV (1\sigma)$
Reconstructed $\psi \sim T T$

Hence:

- **Bispectrum measured by** $C_{l}^{T\psi} = \langle T_{lm}^{*} \psi_{lm} \rangle$
- **Trispectrum measured by** $C_{l}^{\psi\psi} = \langle \psi_{lm}^{*} \psi_{lm} \rangle$

\[
\left( C_{l}^{T\psi} \right)^{2} < C_{l}^{TT} C_{l}^{\psi\psi} \Rightarrow \text{“bispectrum < trispectrum” (in some sense)}
\]

- general property of squeezed non-Gaussianity
What non-Gaussianity does a large-scale modulation give you?

\[ \chi(x) = \chi_0(x)[1 + \phi(x)] \]

Gives squeezed non-Gaussianity. Small \( \phi \):

\[ \langle \chi \chi \chi \rangle \sim \langle \chi_0 \chi_0 \chi_0 \phi \rangle + \cdots \sim P_{\chi_0 \chi_0} P_{\chi_0 \phi} \]

\[ \langle \chi \chi \chi \chi \rangle \sim \langle \chi_0 \chi_0 \chi_0 \chi_0 \rangle + \langle \chi_0 \chi_0 \chi_0 \chi_0 \phi \phi \rangle + \cdots \sim (Gaussian) + P_{\chi_0 \chi_0} P_{\chi_0 \chi_0} P_{\phi \phi} \]

Since \( P_{\chi_0 \phi}^2 \leq P_{\chi_0 \chi_0} P_{\phi \phi} \) \quad \( P_{\chi_0 \chi_0} \langle \chi \chi \chi \chi \rangle_{\text{squeezed}} \geq \langle \chi \chi \chi \rangle_{\text{squeezed}}^2 \)

In conventional definitions \( \tau_{NL} \geq \left( \frac{6f_{NL}}{5} \right)^2 \) (also L by L if quasi local)
Diagonal squeezed trispectra

|\k_1| \sim |k_2|, |k_3| \sim |k_4|, |k_1 + k_2| = |k_3 + k_4| \ll |k_2|, |k_3|

Trispectrum = power spectrum of modulation

e.g. \chi = \chi_0(1 + f_{NL}\chi_0)

\tau_{NL} \sim f_{NL}^2

or \chi = \chi_0(1 + \phi)

(any correlation, \tau_{NL} > f_{NL}^2)
One-leg squeezed trispectra

\[ |k_1| \ll |k_2| \sim |k_3| \sim |k_4| \]

Correlated modulation of equilaterial bispectrum

e.g. from \( \chi = \chi_0 (1 + g_{NL} \chi_0^2) \) [no diagonal squeezed]

also from \( \chi = \chi_0 (1 + f_{NL} \chi_0) \) [plus diagonal squeezed]
Hunters’ guide to the non-Gaussianity zoo

Bispectrum

- Squeezed
  - Isotropic ($f_{NL}$)
    - Local modulations $\chi = \chi_0 (1 + \phi)$, $P_{\chi_0 \phi} \neq 0$
    - CMB lensing magnification

- Anisotropic
  - CMB lensing
  - Non-standard anisotropic inflation

  Bispectrum = power modulation correlated to temperature $(\chi)(\chi'\chi')$

  Equilateral-flattened

  Dynamically generated
  e.g. non-linear structure growth
  inflation before horizon exit (e.g. $c_s \ll 1$)

  Bispectrum = concentrated hot spots in areas of milder cold (or vice versa)

Trispectrum

- Diagonal squeezed
  - Isotropic ($\tau_{NL}$)
    - Local modulations $\chi = \chi_0 (1 + \phi)$ (any $P_{\chi_0 \phi}$)
    - CMB lensing magnification

- Anisotropic
  - CMB lensing
  - Anisotropic primordial power spectrum
  - Non-standard anisotropic inflation
  - Gravitational-wave modulation (small)

  Trispectrum = power spectrum of power modulation, $(\chi)(\chi'\chi')$

  One-leg squeezed (small: $g_{NL}$ either sign, or $\tau_{NL}$ positive)

  Trispectrum = bispectrum modulation correlated to temperature $(\chi)(\chi'\chi'\chi')$
Conclusions

- CMB is still by far the cleanest probe of early-universe physics and cosmological parameters; Planck analysis underway

- CMB lensing very important: changes power spectra and generates B modes

- Non-Gaussianities. Many can be studied by reconstructing modulations with quadratic estimators.

  - CMB Lensing – definitely there
    - Large anisotropic bispectrum, must be modelled but not a problem
    - Large trispectrum – reconstruct the lensing potential

  → Break primary CMB degeneracies, improve parameters