CMB statistical anisotropy and the lensing bispectrum

Hanson & Lewis: 0908.0963
Hanson, Lewis & Challinor: 1003.0198

Lewis, Challinor & Hanson: in prep

Antony Lewis
http://cosmologist.info/
Evolution of the universe

- Proton
- Neutron
- Photon
- Helium nucleus
- Helium atom
- Hydrogen atom
- First stars
- Early galaxies
- Modern galaxies

Opaque

Transparent

Observations: the microwave sky today

(almost) uniform 2.726K blackbody

Dipole (local motion)

O(10^-5) perturbations (+galaxy)

Source: NASA/WMAP Science Team
Can we predict the primordial perturbations?

- Maybe..

Quantum Mechanics
“waves in a box”

Inflation
make $>10^{30}$ times bigger

After inflation
Huge size, amplitude $\sim 10^{-5}$
CMB temperature

End of inflation

Last scattering surface

gravity+ pressure+ diffusion
Observed CMB temperature power spectrum

Constrain theory of early universe + evolution parameters and geometry

Hinshaw et al.
The Vanilla Universe Assumptions

- Translation invariance - statistical homogeneity
  (observers see the same things on average after spatial translation)

- Rotational invariance - statistical isotropy
  (observations at a point the same under sky rotation on average)

- Primordial adiabatic nearly scale-invariant Gaussian fluctuations filling a flat universe

Statistically isotropic CMB with Gaussian fluctuations and smooth power spectrum
WMAP spice - not so vanilla?

Low quadrupole?

Alignments?

Tegmark et al.

Quadrupole

Octopole
Cold spot?

Eriksen et al, Hansen et al.

Power asymmetry?

Cruz et al, 0901.1986

+Non-Gaussianity?... +....?
Gaussian statistical anisotropy

- CMB lensing
- Power asymmetries
- Anisotropic primordial power
- Spatially-modulated primordial power
- Non-Gaussianity

+ various systematics, anisotropic noise, beam effects, ...
Gaussian anisotropic models

\[ -\mathcal{L}(\hat{\Theta}|h) = \frac{1}{2} \hat{\Theta}^\dagger (C^{\hat{\Theta}\hat{\Theta}})^{-1} \hat{\Theta} + \frac{1}{2} \ln \det(C^{\hat{\Theta}\hat{\Theta}}) \]

Or is it a statistically isotropic non-Gaussian model??
Example: CMB lensing

\[ \alpha = -2 \int_0^{x^*} d\chi \frac{f_K(x^* - \chi)}{f_K(x^*)} \nabla_\perp \Psi(x_0; \eta_0 - \chi) \]

\[ \tilde{T}(\hat{n}) = T(\hat{n}') = T(\hat{n} + \hat{\alpha}) \]
For a given lensing field:

\[ T \sim P(T|\psi) \]

- Anisotropic Gaussian temperature distribution
- Different parts of the sky magnified and demagnified
- Re-construct the actual lensing field – infer \( \psi \)

Or marginalized over lensing fields:

\[ T \sim \int P(T,\psi)d\psi \]

- Non-Gaussian statistically isotropic temperature distribution
- Significant connected 4-point function
- Excess variance to anisotropic-looking realizations
- Lensed temperature power spectrum
Anisotropy estimators

\[- \mathcal{L}(\hat{\Theta} \mid h) = \frac{1}{2} \hat{\Theta}^\dagger (C^{\hat{\Theta} \hat{\Theta}})^{-1} \hat{\Theta} + \frac{1}{2} \ln \det(C^{\hat{\Theta} \hat{\Theta}}) \]

Maximum likelihood:

\[
\frac{\delta \mathcal{L}}{\delta h^\dagger} = -\frac{1}{2} \hat{\Theta}^\dagger (C^{\hat{\Theta} \hat{\Theta}})^{-1} \frac{\delta C^{\hat{\Theta} \hat{\Theta}}}{\delta h^\dagger} (C^{\hat{\Theta} \hat{\Theta}})^{-1} \hat{\Theta} + \frac{1}{2} \text{Tr} \left[ (C^{\hat{\Theta} \hat{\Theta}})^{-1} \frac{\delta C^{\hat{\Theta} \hat{\Theta}}}{\delta h^\dagger} \right] = 0
\]

First iteration solution: Quadratic Maximum Likelihood (QML)

\[
\hat{h} = \mathcal{F}^{-1}[\tilde{h} - \langle \tilde{h} \rangle].
\]

\[
\tilde{h} = \mathcal{H}_0 = \frac{1}{2} \hat{\Theta}^\dagger \frac{\delta C^{\hat{\Theta} \hat{\Theta}}}{\delta h^\dagger} \hat{\Theta} = \frac{1}{2} \sum_{l_m, l'_m} \left[ \frac{\delta C_{l_m, l'_m}}{\delta h^\dagger} \right] \hat{\Theta}_{l_m}^* \hat{\Theta}_{l'_m},
\]

\[
\hat{\Theta} = (C^{\hat{\Theta} \hat{\Theta}})^{-1} |_0 \hat{\Theta}
\]
Reconstruction recipe

\[ \Theta = (C^\Theta \Theta)^{-1} \big|_0 \Theta \] (sets to zero in sky cut)

Inverse variance filter

Make filtered maps

\[ F_1 = \left[ \sum_{l_1 m_1} \Theta_{l_1 m_1} Y_{l_1 m_1} \right] \]

\[ F_2 = \left[ \sum_{l_2 m_2} C_{l_2} \Theta_{l_2 m_2} Y_{l_2 m_2} \right] \]

Simulate * many times to calculate \( \langle \hat{h} \rangle \) (accounts for anisotropic noise/sky cut)

\[ \hat{h} = \mathcal{F}^{-1} [\tilde{h} - \langle \tilde{h} \rangle] \]

\( \mathcal{F} \) Approximated or from sims

Quadratic estimator
For lensing get generalization of Okamoto & Hu 2003 estimators for anisotropic noise/partial sky

True (simulated)

Reconstructed (Planck noise, Wiener filtered)

- Constrain curvature, dark energy, neutrino mass...

Fig. 5 Power spectrum of the errors in the gradient part of the reconstructed deflection field for two full-sky experiments. Left: an approximation to the Planck satellite, with $\sigma(T) = 27\mu K$-arcmin. Right: a version of the proposed CMBPol mission, with $\sigma(T) = 1\mu K$-arcmin. For both experiments, we assume a Gaussian beam of FWHM 7 arcmin and $Q$ and $U$ noise levels equal to $\sqrt{2}\sigma(T)$. The curves are the theory deflection power (solid black), reconstruction errors from temperature alone (red dot-dashed), polarization alone (green dashed), temperature and polarization together (black dashed), and the Fisher limit (black dotted). For Planck, the Fisher limit is saturated.
Unexpected signals?..

Sky modulation?

Popular modulation model:

$$\Theta_f(\hat{n}) = [1 + f(\hat{n})] \Theta^i_f(\hat{n})$$

QML estimator for f:

$$\tilde{h}^f_{lm} = \int dΩ Y^*_{lm} \left[ \sum_{l_1m_1} \tilde{\Theta}_{l_1m_1} Y_{l_1m_1} \right] \left[ \sum_{l_2m_2} C_{l_2} \tilde{\Theta}_{l_2m_2} Y_{l_2m_2} \right]$$

$$\ell_{max} = 25$$

WMAP power reconstruction
(V band, KQ85 mask, foreground cleaned; reconstruction smoothed to 10 degrees)

Following Eriksen et al, WMAP, etc..
$\ell_{\text{max}} = 64$

$\ell_{\text{max}} = 100$

+ peak of QML dipole
Dipole amplitude as function of $l_{\text{max}}$

- Only ~1% modulation allowed on small scales
- Consistent with Hirata 2009
  - Very small observed anisotropy in quasar distribution
Unexpected signals?..

**Primordial power spectrum anisotropy**

Look for direction-dependence in primordial power spectrum:

\[
\langle \chi_0(k)\chi_0^*(k') \rangle = (2\pi)^3 \delta(k-k') P_\chi(k)
\]

Simple case:

\[
P_\chi(k) = P_\chi(k)[1 + a(k)g(\hat{k})]
\]

e.g.

* Ackerman et.al. astro-ph/0701357
* Gumrukcuoglu et al 0707.4179

Anisotropic covariance:

\[
C_{l_1m_1l_2m_2} = \int d^3k P_\chi(k) \Delta_{l_1}(k) \Delta_{l_2}(k) Y_{l_1m_1}^*(\hat{k}) Y_{l_2m_2}(\hat{k})
\]
Reconstruct $g(k)$

QML estimator:

$$\tilde{h}_{lm}^g = \frac{1}{2} \int d\Omega Y^*_{lm} \sum_{l_1l_2} i^{l_1-l_2} C_{l_1l_2}$$

$$\times \left[ \sum_{m_1} \bar{\Theta}_{l_1m_1} Y_{l_1m_1} \right] \left[ \sum_{m_2} \bar{\Theta}_{l_2m_2} Y_{l_2m_2} \right]$$

Many-sigma quadrupole primordial power anisotropy??
Direction close to ecliptic!
Also varies with frequency and detector.
Could it be systematics? - beam asymmetries? uncorrected in WMAP maps

Check with analytic model of scan strategy

\[
\tilde{\Theta}(\Omega_p) = \sum_s w(\Omega_p, -s) \left[ \sum_{lm} B_{ls} \Theta_{lm} s Y_{lm}(\Omega_p) \right]
\]

Scan strategy

Beam shape multipoles

\[
w(\Omega_p, -s) = \sum_{i \in p} e^{-is\alpha_i} / H_p \\
\quad = v(\Omega_p, s) / v(\Omega_p, 0)
\]

\[
v(\Omega_p, s) = \sum_{i \in p} e^{is\alpha_i}
\]

WMAP model

1. a beam at an angle \(\theta_b\) to the satellite spin axis, which rotates with period \(\tau_s\);

2. a precession at an angle \(\theta_p\) to the anti-solar direction, with period \(\tau_p\); and

3. a continuous repointing of the anti-solar direction as the observer orbits the sun.

\[
[v(\Omega_p, s)]_{lm} = \delta_{m0} K P_l(0) P_l(\cos \theta_p) s Y_{l0}(\theta_b, 0)
\]

Monte Carlo with subtraction of mean field analytic model of beam asymmetries

No detection..

\[ |g_{2M}| < 0.07 \text{ at 95\% confidence.} \]

Consistent with Pullen et al 2010 constraint from large-scale structure 1003.0673
Bispectrum non-Gaussianity

- Local ‘squeezed’ models: small scale power correlated with large-scale temperature
- Considering large-scale modes to be fixed, expect power anisotropy

\[
\Psi = \Psi_0 + f_{NL} \Psi_0^2 = \Psi_0(1 + f_{NL} \Psi_0)
\]

Liguori et al 2007

e.g. Local primordial non-Gaussianity

Unexpected signals?..
Write general quadratic anisotropy estimator:

\[
6X_{lm} \equiv \sum_{l_1 m_1, l_2 m_2} B_{ll_1 l_2} (-1)^{m_1} \begin{pmatrix} l & l_1 & l_2 \\ m & -m_1 & m_2 \end{pmatrix} \bar{\Theta}_{l_1 m_1} \bar{\Theta}^*_{l_2 m_2}
\]

\[
= \int d\Omega Y^*_l \times \sum_{l_1 l_2} b_{ll_1 l_2} \left[ \sum_{m_1} \bar{\Theta}_{l_1 m_1} Y_{l_1 m_1} \right] \left[ \sum_{m_2} \bar{\Theta}_{l_2 m_2} Y_{l_2 m_2} \right]
\]

Bispectrum estimators are basically the cross-correlation of an anisotropy estimator with the temperature

\[
\mathcal{E} = \frac{1}{F_{\mathcal{E}}} \bar{\Theta}^\dagger (X - 3\langle X \rangle),
\]

In harmonic space

\[
\mathcal{E} = \frac{1}{6F_{\mathcal{E}}} \sum_{l_1 m_1} B_{l_1 l_2 l_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} 
\times \left[ \bar{\Theta}_{l_1 m_1} \bar{\Theta}_{l_2 m_2} \bar{\Theta}_{l_3 m_3} - 3C^{-1}_{l_1 m_1 l_2 m_2} \bar{\Theta}_{l_3 m_3} \right].
\]

CMB lensing bispectrum

Lensed temperature depends on deflection angle:

\[ \tilde{T}(\hat{n}) = T(\hat{n}') = T(\hat{n} + \alpha) \]

\[ \alpha = \delta \theta = -2 \int_0^{\chi^*} d\chi \frac{f_K(\chi^* - \chi)}{f_K(\chi^*)} \nabla_{\perp} \Psi(\chi \hat{n}; \eta_0 - \chi) \]

Lensing Potential

Deflection angle on sky given in terms of lensing potential

\[ \alpha = \nabla \psi \]

\[ \psi(\hat{n}) = -2 \int_0^{\chi^*} d\chi \Psi(\chi \hat{n}; \eta_0 - \chi) \frac{f_K(\chi^* - \chi)}{f_K(\chi^*) f_K(\chi)} \]

\[ \tilde{X}(n) = X(n') = X(n + \nabla \psi(n)) \]
LensPix sky simulation code:
http://cosmologist.info/lenspix
Lewis 2005, Hammimeche & Lewis 2008
Bispectrum as statistical anisotropy correlation

Lensing by fixed $\psi$ field introduced statistical anisotropy

Construct QML estimator for $\psi$ (following Hu and Okomoto 2003)

$$\langle \tilde{T}(l_2)\tilde{T}(l_1 - l_2) \rangle_T \propto \psi(l_1)$$

Bispectrum measures cross-correlation of quadratic estimator for $\psi$ with the large-scale temperature

For squeezed triangles, $l_1 \ll l_2, l_3$,

$$\tilde{T}(l_1) \sim T(l_1) \text{ and } \langle \tilde{T}(l_2)\tilde{T}(l_3) \rangle_T \propto \psi(l_1)$$

$$\langle \tilde{T}(l_1)\tilde{T}(l_2)\tilde{T}(l_3) \rangle \sim \langle T(l_1)\psi(l_1) \rangle \sim C_{l_1}^{\psi T}$$
Why is there a correlation between large-scale lenses and the temperature?

( small-scales: also SZ, Rees-Sciama.. )

\[ \Delta T_{\text{ISW}}(\hat{n}) = 2 \int_0^{\chi^*} d\chi \dot{\Psi}(\chi \hat{n}; \eta_0 - \chi). \]

Overdensity: magnification correlated with positive Integrated Sachs-Wolfe (net blueshift)
Underdensity: demagnification correlated with negative Integrated Sachs-Wolfe (net redshift)
Accurate bispectrum calculation

Assume Gaussian fields. Non-perturbative result:

\[
\langle T(l_1)\tilde{T}(l_2)\tilde{T}(l_3) \rangle = C_{l_1}^{T\psi} \left\langle \frac{\delta}{\delta \psi(l_1)^*} \left( \tilde{T}(l_2)\tilde{T}(l_3) \right) \right\rangle
\]

Use \( \tilde{T}(x) = T(x + \nabla \psi) \)

\[
\frac{\delta}{\delta \psi(l_1)^*} \tilde{T}(l) = -\frac{i}{2\pi} l_1 \cdot \nabla T(l + l_1).
\]

\[
\langle T(l_1)\tilde{T}(l_2)\tilde{T}(l_3) \rangle = -\frac{i}{2\pi} C_{l_1}^{T\psi} l_1 \cdot \left\langle \nabla \tilde{T}(l_1 + l_2)\tilde{T}(l_3) \right\rangle + (l_2 \leftrightarrow l_3)
\]

\[
= -\frac{1}{2\pi} \delta(l_1 + l_2 + l_3) C_{l_1}^{T\psi} \left[ (l_1 \cdot l_2)\tilde{C}_{l_2}^{T\nabla T} + (l_1 \cdot l_3)\tilde{C}_{l_3}^{T\nabla T} \right]
\]

\( \sim \) Lensed temperature power spectrum
Lensing bispectrum depends on changes in the small-scale lensed power

\[ b_{l_1l_2l_3} \approx -C_{l_1}^{TT} \left[ (l_1 \cdot l_2) \tilde{C}_{l_2}^{TT} + (l_1 \cdot l_3) \tilde{C}_{l_3}^{TT} \right] \]

\[ \approx l_1^2 C_{l_1}^{T\psi} \left[ \frac{(l_1 \cdot l_2)^2}{l_1^2 l_2^2} \frac{d\tilde{C}_l^{TT}}{d\ln l} \bigg|_{l_2} + \tilde{C}_{l_2}^{TT} \right]. \]

\[ (l_1 + l_2 + l_3 = 0) \]

- Quite large signal. Expect \( \sim 5\sigma \) with Planck. Cosmic variance \( \sim 7\sigma \).

- Using lensed power spectra important at 5-20% level: leading-order result (using unlensed spectra) not accurate enough.
If lensing is neglected get bias $\Delta f_{NL} \sim 9$ on primordial local models with Planck (see e.g. Hanson et al 0905.4732, Mangilli 0906.2317)

BUT:

- Lensing bispectrum depends on power difference: has phase shift compared to any adiabatic primordial bispectrum (and different scale dependence)

- Lensing bispectrum is strongly scale dependent (small ISW for larger $l_1$)

- Lensing bispectrum depends on shape of squeezed triangle ($l_1 \cdot l_2$ factor)
Lensing bispectrum also squeezed triangles but quite distinctive

Temperature bispectrum correlation with local $f_{NL} \sim 30\%$: in null hypothesis can measure amplitude using optimized estimator and accurately subtract from $f_{NL}$ estimator
CMB polarization

General full-sky bispectrum:

\[ a_{lm} = (T_{lm}, E_{lm}, B_{lm})^T \]

\[ B_{l_1 l_2 l_3}^{ijk} = \sum_{m_1 m_2 m_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \langle a_{l_1 m_1}^i a_{l_2 m_2}^j a_{l_3 m_3}^k \rangle \]

\[ \approx F_{l_3 l_1 l_2}^{s_k} C_{l_1}^{a^i \psi} \tilde{C}_{l_2}^{a^j a^k} + i F_{l_3 l_1 l_2}^{-s_k} C_{l_1}^{a^i \psi} \tilde{C}_{l_2}^{a^j \tilde{a}^k} + \text{perms} \]

Is the polarization correlated? \( C_l^{E\psi} = ? \)
Yes! Significant large-scale correlation due to reionization

Hu astro-ph/9706147
Lensing potential correlation power spectra

Cosmic variance: $C^{T\psi} \sim 7\sigma, C^{E\psi} \sim 2.5\sigma$
$l_1 = 4$

$\ell_1 = 50$

$f_{NL} = 30$
Also parity odd bispectra, TEB etc. (compared to sims)

\[ l_1 = 4 \]

\[ l_1 = 50 \]
Signal to Noise

Signal quite large, so cosmic variance important as well as noise

\[
[F^{(l_1)} - 1]_{ij} = [F_{l_1}^{\text{lens, lens} - 1}]_{ij} + \frac{[C_{l_1}^{a \times \psi} C_{l_1}^{a \times \psi} + C_{l_1}^{a \times a} C_{l_1}^{a \times \psi}]}{(2l_1 + 1)C_{l_1}^{a \times \psi} C_{l_1}^{a \times \psi}}
\]

FIG. 10: The Fisher detection significance of the CMB lensing bispectrum as a function of \( l_{\text{max}} \) for no noise and Planck, using just the temperature bispectrum or using all the T and E-polarization bispectra. The dotted lines show the (incorrect) results obtained if the signal contribution to the variance is neglected: all results are bounded by the cosmic variance detection limit on a measurement of the low-\( l \) cross-correlation spectra \( C_{l}^{T, \psi} \) and \( C_{l}^{E, \psi} \).
Signal to noise

Contributions to Fisher inverse variance

Lensing signal peaks around $l_1 \sim 30$
- trade-off between size of signal and number of modes

- Cosmic variance limits simply determined by cosmic variance detection limits on $C^T_\ell \psi$ and $C^E_\ell \psi$

Planck $\sim 5\sigma$; Cosmic Variance $\sim 9\sigma$
Conclusions

• Can easily test for and reconstruct many forms of Gaussian statistical anisotropy
  - Fast nearly-optimal Quadratic Maximum Likelihood estimators
  - Strong detection of ‘primordial power anisotropy’ in WMAP – but actually explained by beam asymmetries
  - Lensing signal expected and can reconstruct lensing potential (learn about cosmological parameters)

• CMB lensing bispectrum is significant
  - Temperature bispectrum from ISW-$\psi$ correlation
  - Also E-$\psi$ correlation ($\sim 2.5\sigma$ cosmic variance limit)
  - Distinctive phase and scale-dependence

Should be detected by Planck
  - Potential confusion with local $f_{NL}$ but contribution easily distinguished/subtracted
  - Also SZ correlation on smaller scales, but frequency dependent; other terms includes Rees Sciama ($f_{NL} \sim 1$)

• Public codes available:
  $C_l^{E\psi}$, $C_l^{T\psi}$, $C_l^{\psi\psi}$, Local $f_{NL}$ and lensing bispectrum in CAMB update: http://camb.info