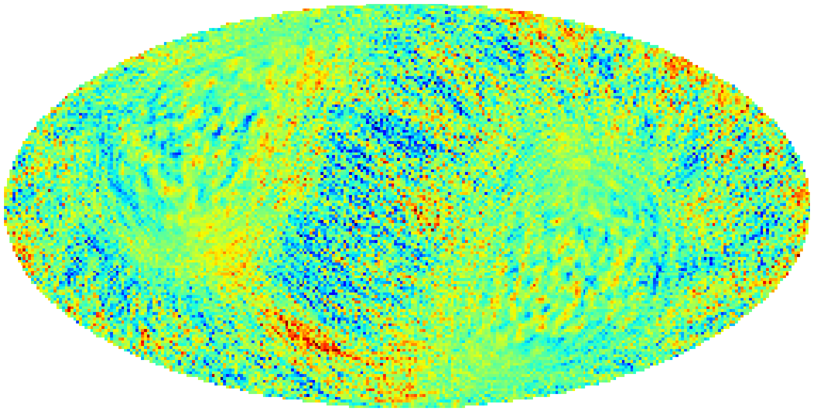
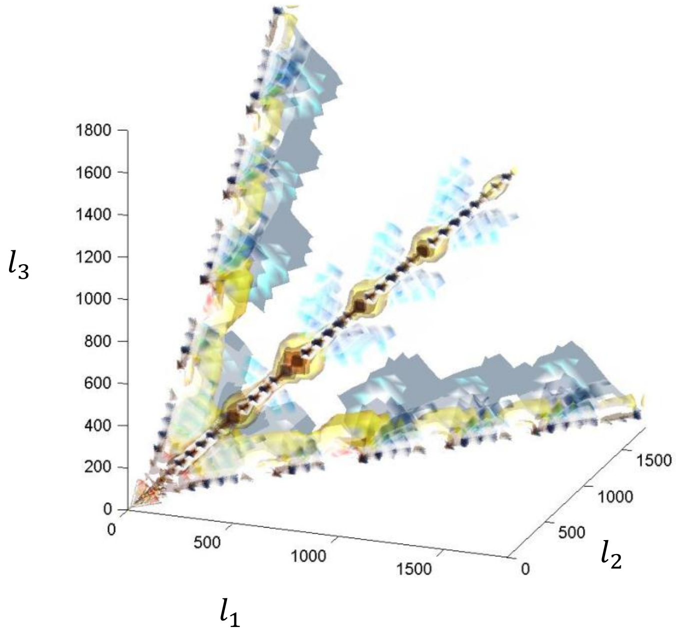
CMB statistical anisotropy and the lensing bispectrum





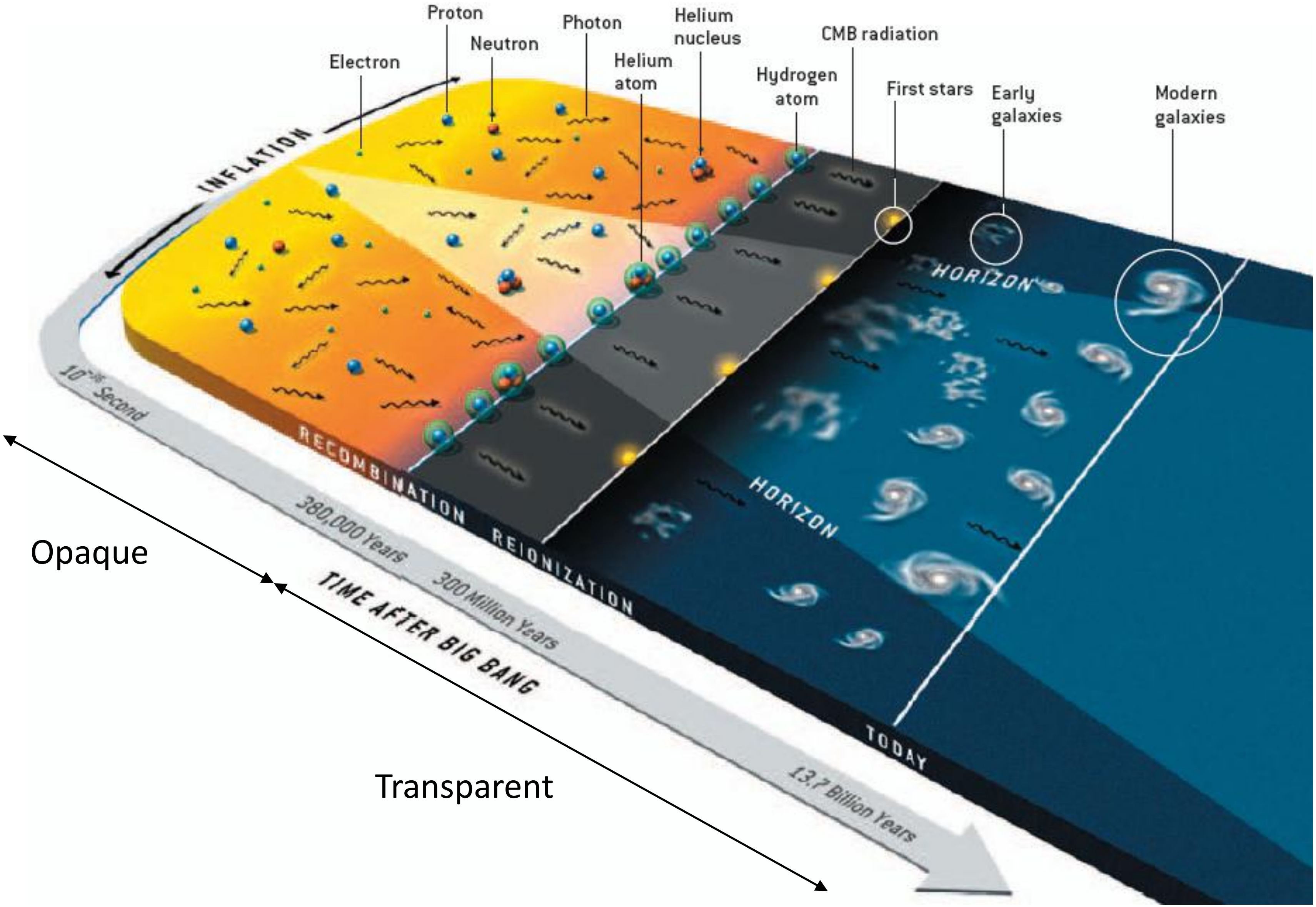
Hanson & Lewis: 0908.0963
Hanson, Lewis & Challinor: 1003.0198

Lewis, Challinor & Hanson: *in prep*

University of Sussex

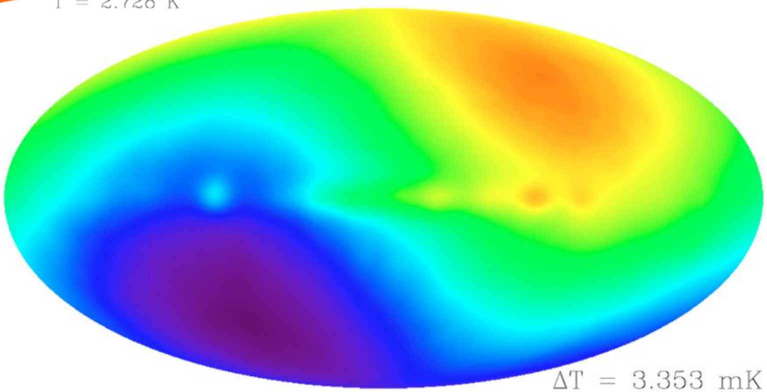
Antony Lewis
http://cosmologist.info/

Evolution of the universe



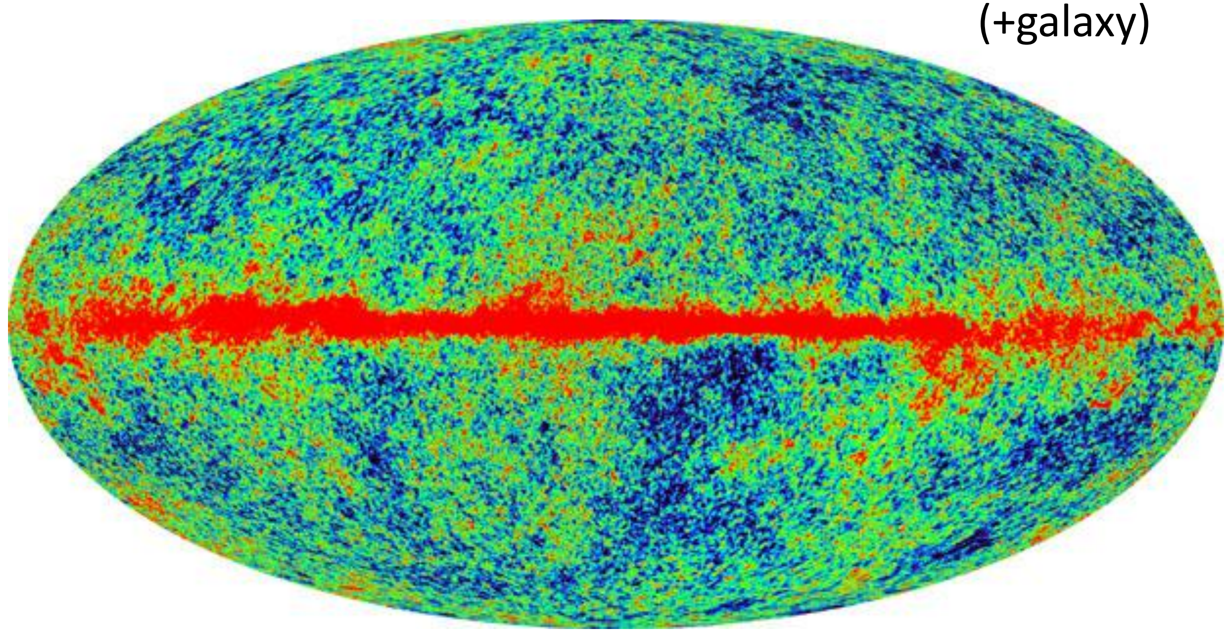
Hu & White, Sci. Am., 290 44 (2004)

(almost) uniform 2.726K blackbody



Dipole (local motion)

O(10^{-5}) perturbations
(+galaxy)

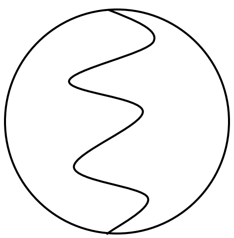


Observations: the microwave sky today

Source: NASA/WMAP Science Team

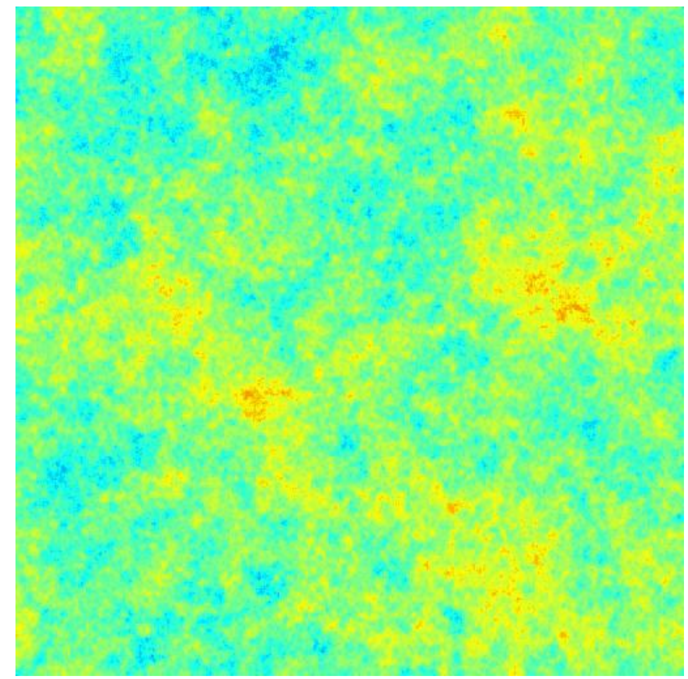
Can we predict the primordial perturbations?

- Maybe..



Quantum Mechanics
"waves in a box"

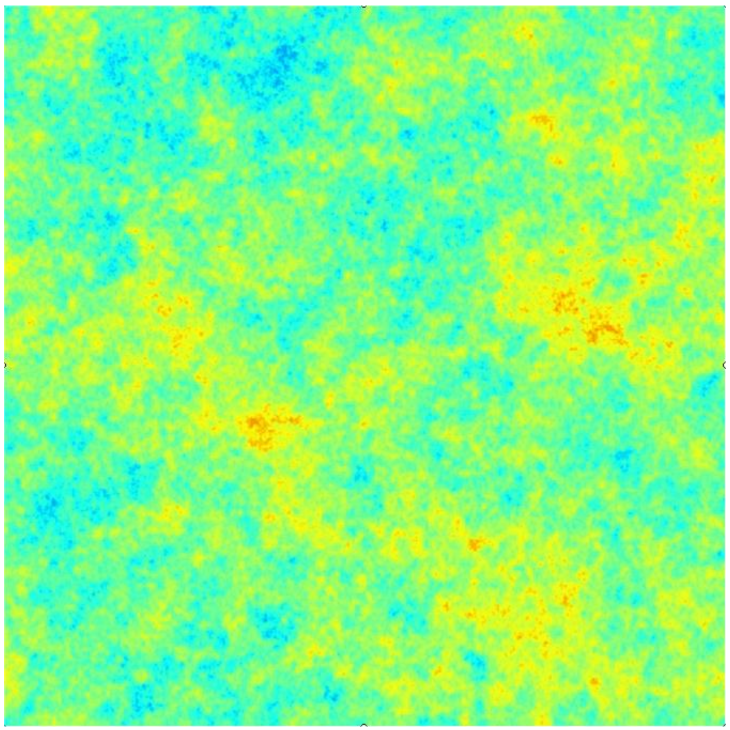
Inflation
make >10^{30} times bigger



After inflation
Huge size, amplitude ~ 10^{-5}

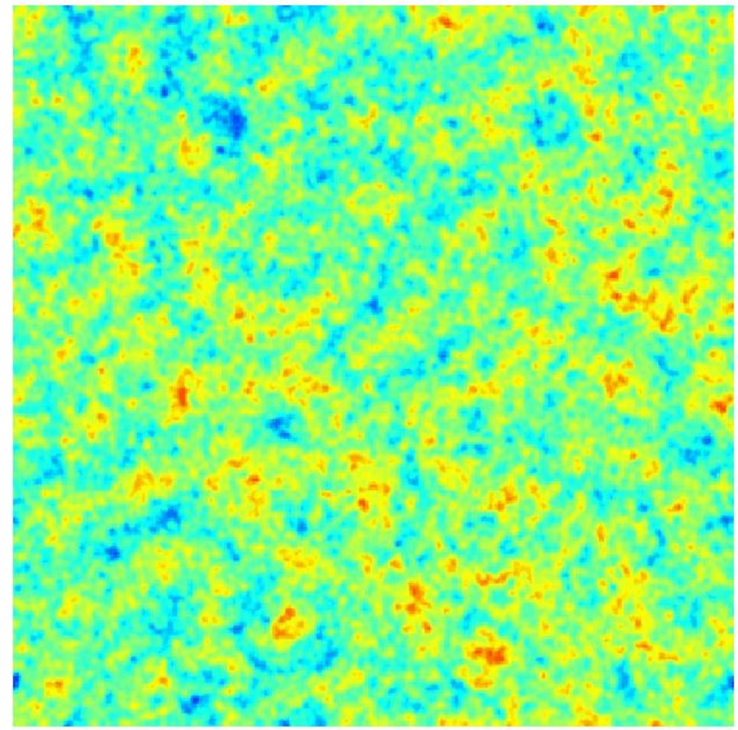
CMB temperature

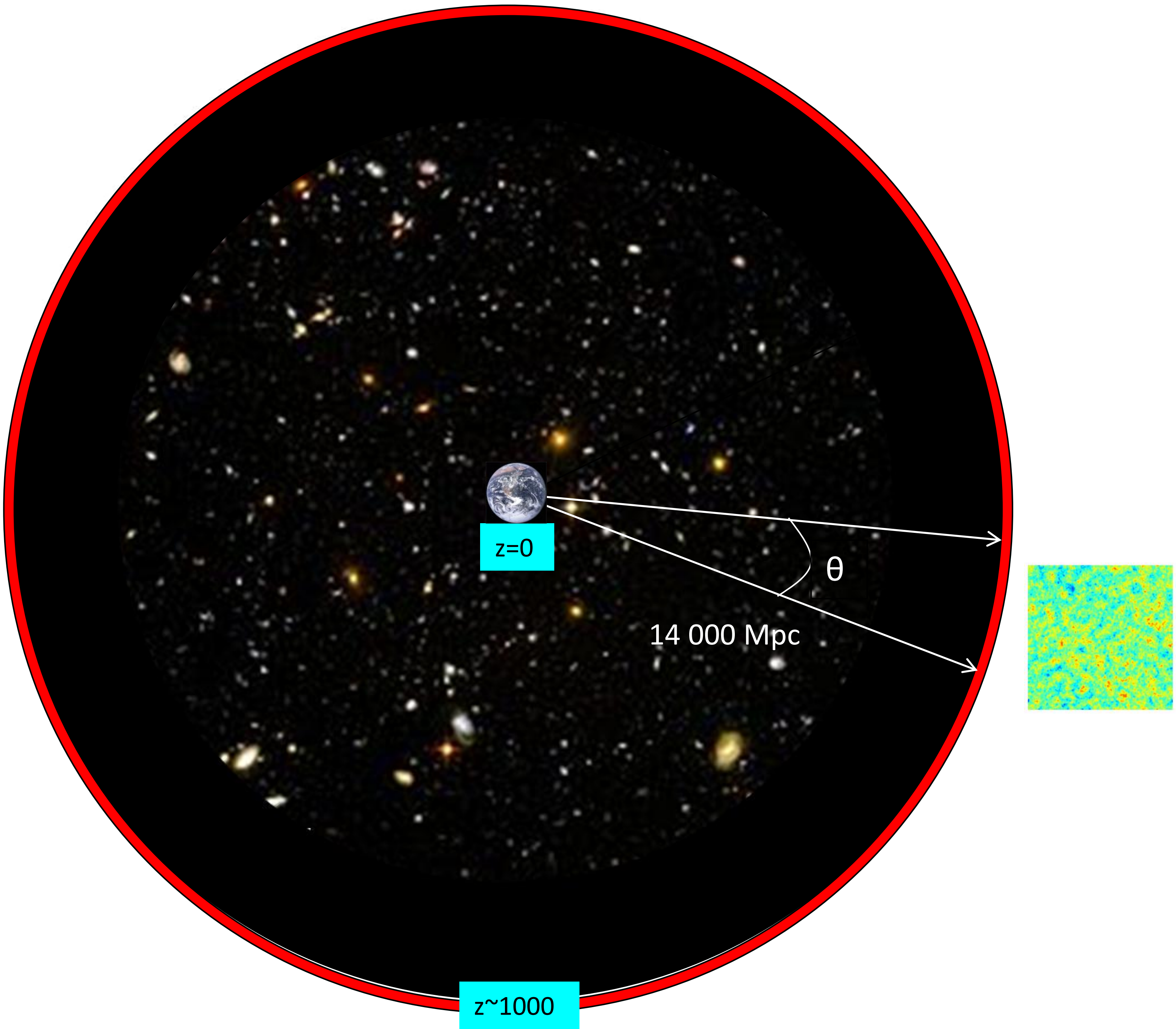
End of inflation



gravity+
pressure+
diffusion

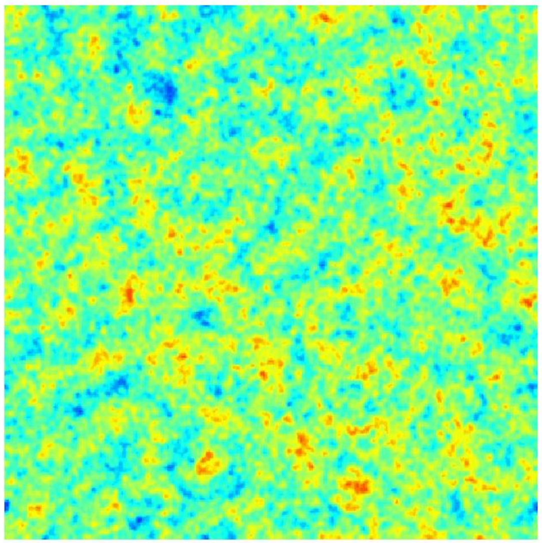
Last scattering surface

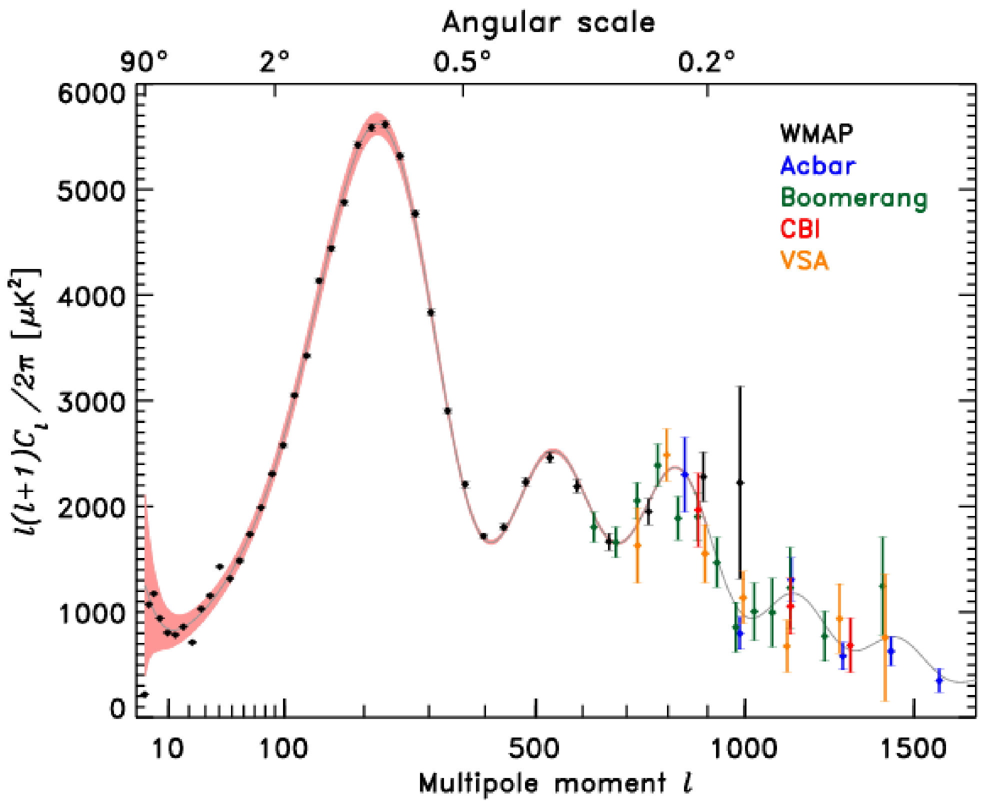




z=0
θ
14 000 Mpc
z~1000

Observed CMB temperature power spectrum





Hinshaw et al.

Observations → **Constrain theory of early universe + evolution parameters and geometry**

The Vanilla Universe Assumptions

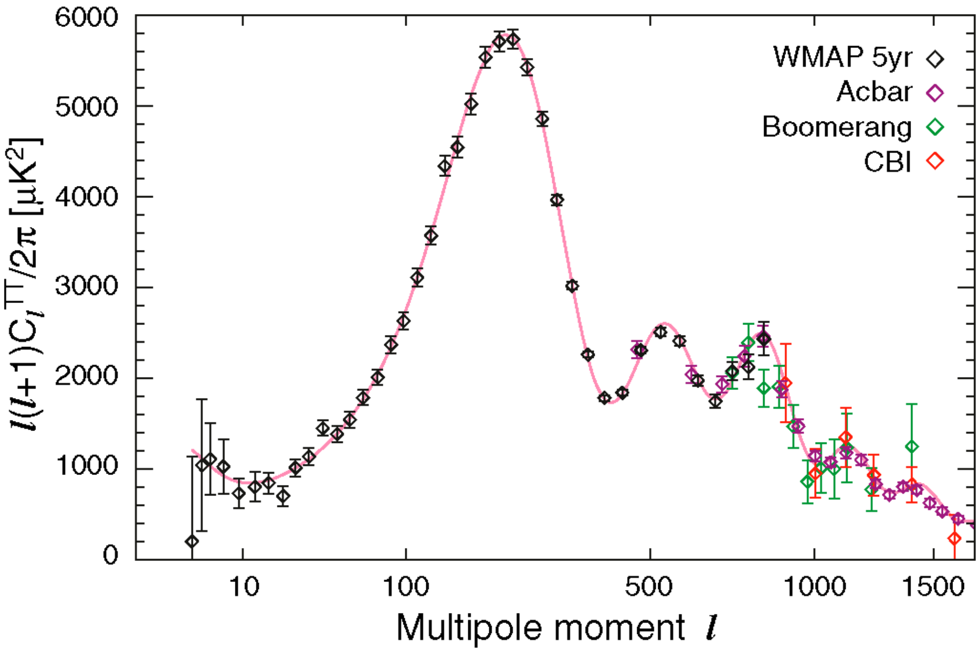
- Translation invariance - statistical homogeneity
  (observers see the same things on average after spatial translation)

- Rotational invariance - statistical isotropy
  (observations at a point the same under sky rotation on average)

- Primordial adiabatic nearly scale-invariant Gaussian fluctuations filling a flat universe

Statistically isotropic CMB with Gaussian fluctuations and smooth power spectrum

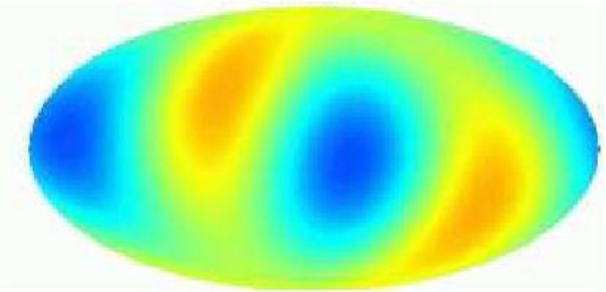
WMAP spice - not so vanilla?

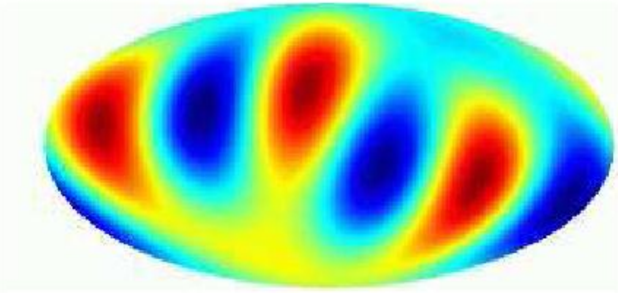


Low quadrupole?

WMAP team

Alignments?



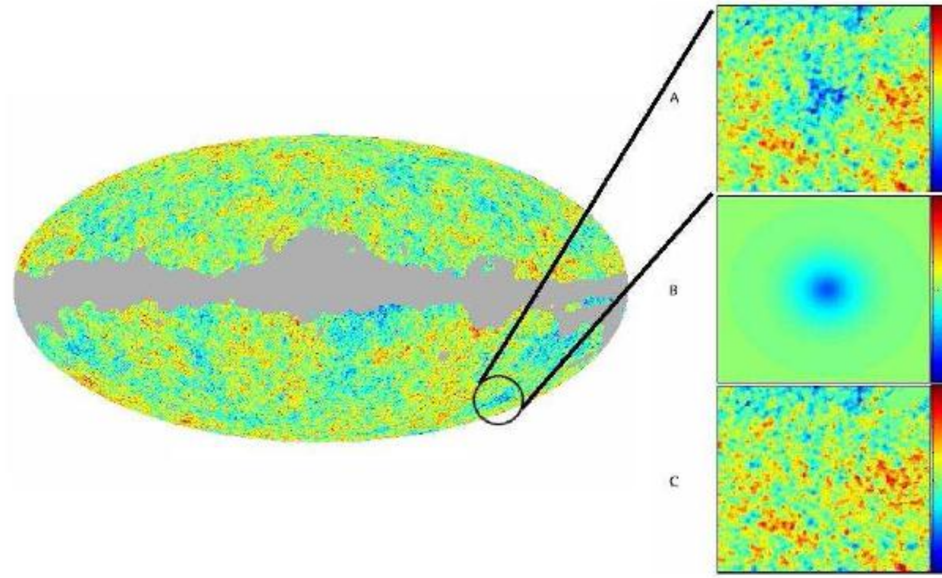


Tegmark et al.

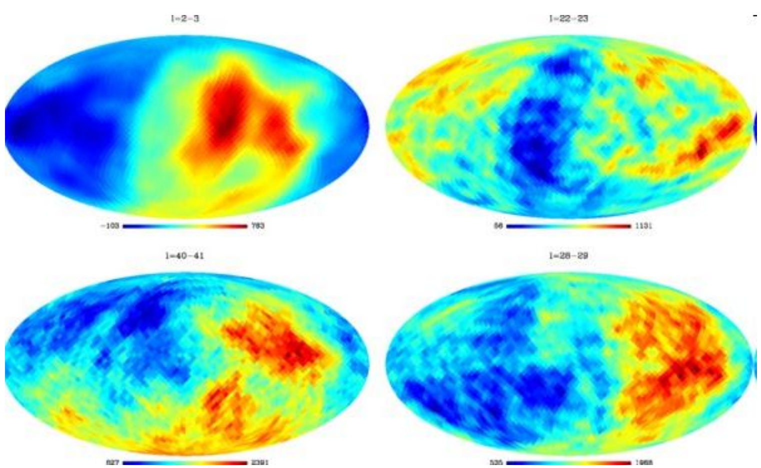
Quadrupole

Octopole

Cold spot?



Cruz et al, 0901.1986



Power asymmetry?

Eriksen et al, Hansen et al.

+Non-Gaussianity?... +....?

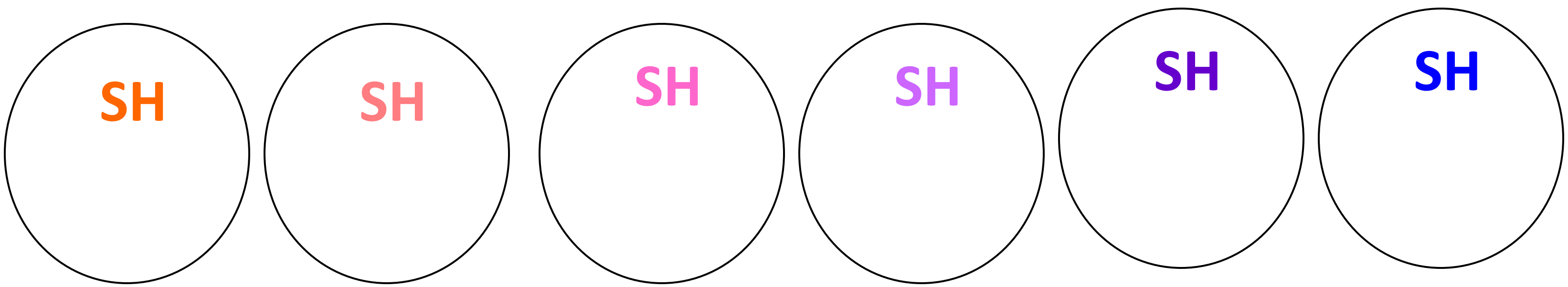
Gaussian statistical anisotropy

- CMB lensing
- Power asymmetries
- Anisotropic primordial power
- Spatially-modulated primordial power
- Non-Gaussianity

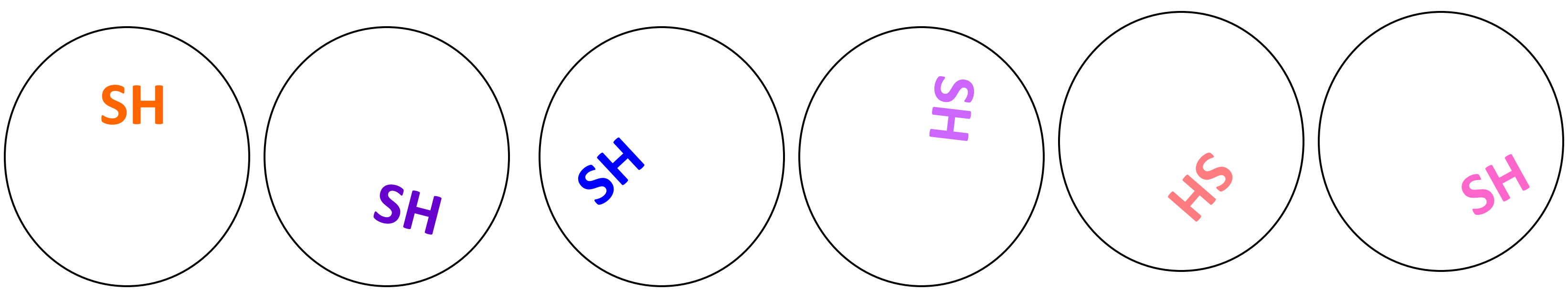
+ various systematics, anisotropic noise, beam effects, …

Gaussian anisotropic models

$$-\mathcal{L}(\hat{\Theta}|\mathrm{h}) = \frac{1}{2}\hat{\Theta}^{\dagger}(C^{\hat{\Theta}\hat{\Theta}})^{-1}\hat{\Theta} + \frac{1}{2}\ln\det(C^{\hat{\Theta}\hat{\Theta}})$$



Or is it a statistically isotropic non-Gaussian model??

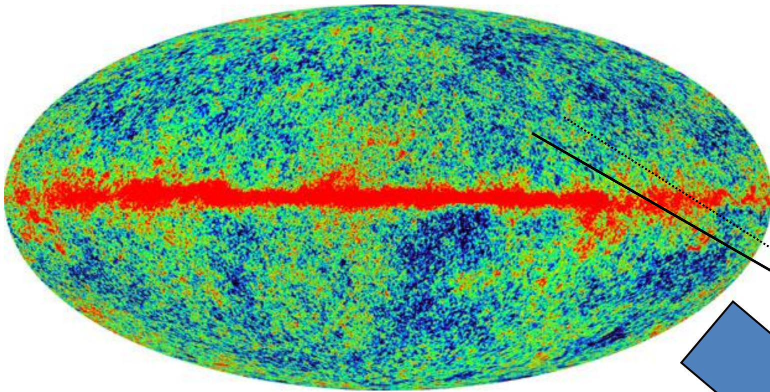


Expected signal..

Example: CMB lensing

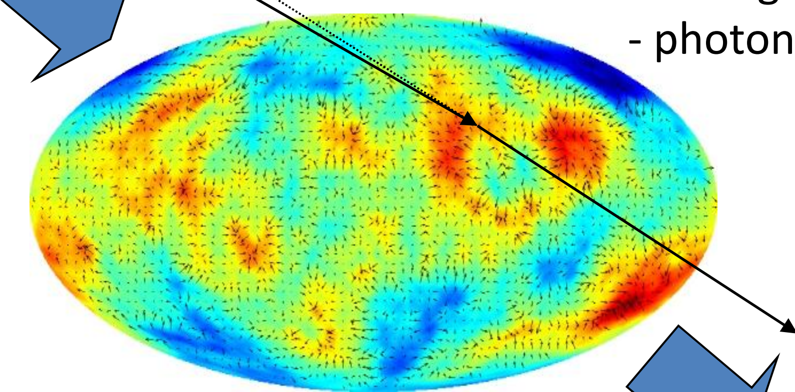
Last scattering surface

$$\boldsymbol{\alpha} = -2 \int_0^{\chi_*} \mathrm{d}\chi \frac{f_K(\chi_* - \chi)}{f_K(\chi_*)} \boldsymbol{\nabla}_\perp \Psi(\chi \hat{\mathbf{n}}; \eta_0 - \chi)$$



Gaussian LSS

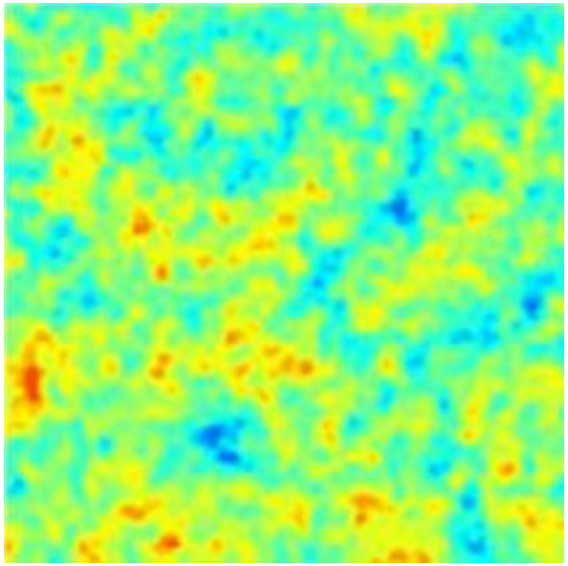
Inhomogeneous universe
- photons deflected

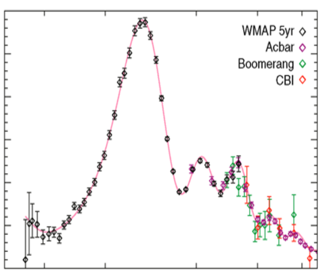


Observer

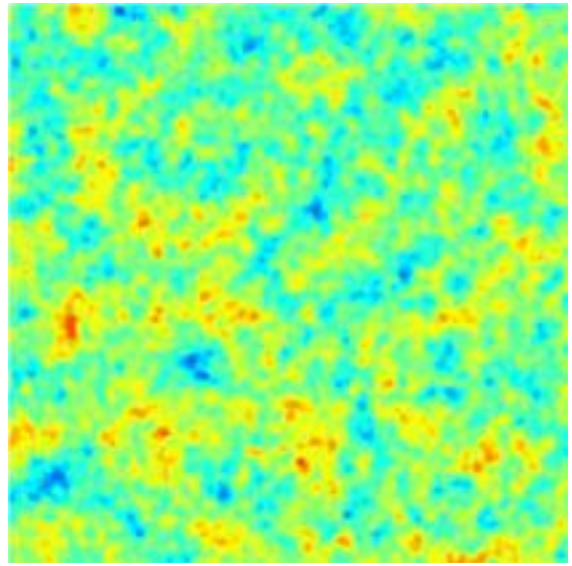
$$\tilde{T}(\hat{\mathbf{n}}) = T(\hat{\mathbf{n}}') = T(\hat{\mathbf{n}} + \boldsymbol{\alpha})$$

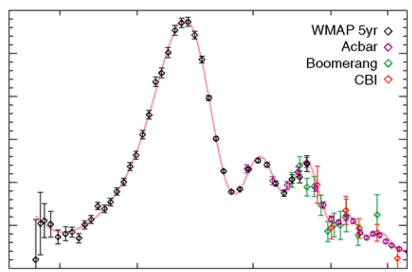
Magnified



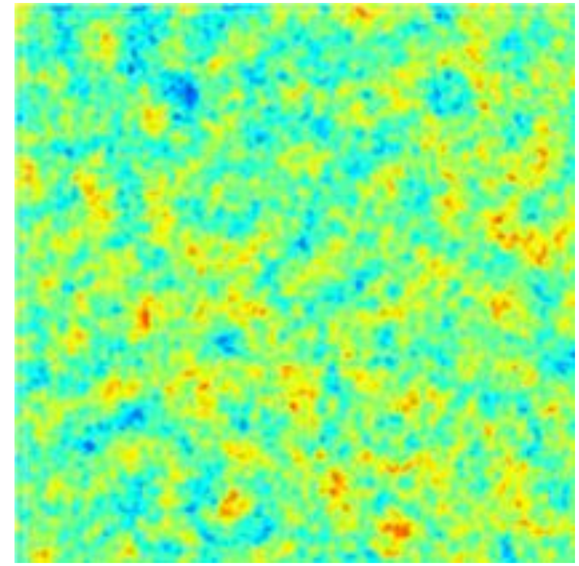


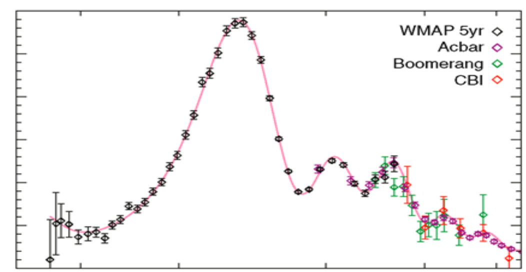
Unlensed





Demagnified





For a given lensing field :

$$T \sim P(T|\psi)$$

- Anisotropic Gaussian temperature distribution

- Different parts of the sky magnified and demagnified
- Re-construct the actual lensing field – infer ψ

Or marginalized over lensing fields:

$$T \sim \int P(T,\psi)d\psi$$

-Non-Gaussian statistically isotropic temperature distribution

- Significant connected 4-point function
- Excess variance to anisotropic-looking realizations
- Lensed temperature power spectrum

Anisotropy estimators

$$-\mathcal{L}(\hat{\Theta}|\mathrm{h}) = \frac{1}{2}\hat{\Theta}^{\dagger}(C^{\hat{\Theta}\hat{\Theta}})^{-1}\hat{\Theta} + \frac{1}{2}\ln\det(C^{\hat{\Theta}\hat{\Theta}})$$

Maximum likelihood:

$$\frac{\delta\mathcal{L}}{\delta\mathrm{h}^{\dagger}} = -\frac{1}{2}\hat{\Theta}^{\dagger}(C^{\hat{\Theta}\hat{\Theta}})^{-1}\frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta\mathrm{h}^{\dagger}}(C^{\hat{\Theta}\hat{\Theta}})^{-1}\hat{\Theta} \quad +\frac{1}{2}\mathrm{Tr}\left[(C^{\hat{\Theta}\hat{\Theta}})^{-1}\frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta\mathrm{h}^{\dagger}}\right] = 0$$

First iteration solution: Quadratic Maximum Likelihood (QML)

$$\hat{\mathrm{h}} = \mathcal{F}^{-1}[\tilde{\mathrm{h}} - \langle\tilde{\mathrm{h}}\rangle].$$

$$\tilde{\mathrm{h}} = \mathcal{H}_0 = \frac{1}{2}\bar{\Theta}^{\dagger}\frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta\mathrm{h}^{\dagger}}\bar{\Theta}$$

$$= \frac{1}{2}\sum_{lm,l'm'}\left[\frac{\delta C^{\hat{\Theta}\hat{\Theta}}_{lm,\ l'm'}}{\delta\mathrm{h}^{\dagger}}\right]\bar{\Theta}^{*}_{lm}\bar{\Theta}_{l'm'},$$

$$\bar{\Theta} = (C^{\hat{\Theta}\hat{\Theta}})^{-1}|_0\hat{\Theta}$$

Reconstruction recipe

$\hat{\Theta}$

Inverse variance filter

$$\bar{\Theta} = (C^{\hat{\Theta}\hat{\Theta}})^{-1}|_0 \hat{\Theta}$$ (sets to zero in sky cut)

Make filtered maps

*

F_1 = $\left[\sum_{l_1 m_1}^{l_{\max}} \bar{\Theta}_{l_1 m_1} Y_{l_1 m_1}\right]$

F_2 = $\left[\sum_{l_2 m_2}^{l_{\max}} C_{l_2} \bar{\Theta}_{l_2 m_2} Y_{l_2 m_2}\right]$

$$\tilde{h}^f_{lm} = \int d\Omega Y^*_{lm} \, \mathbf{F_1} \, \mathbf{F_2}$$ Quadratic estimator

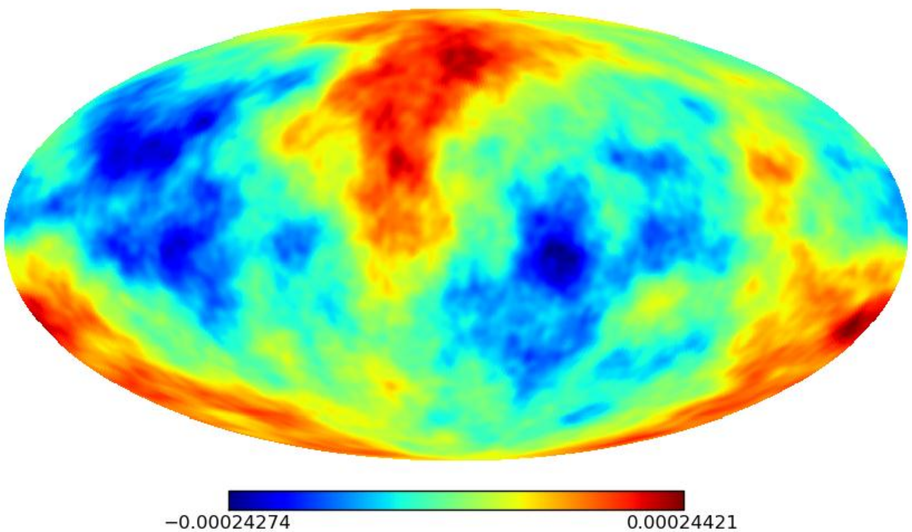
Simulate * many times to calculate $\langle \tilde{\mathrm{h}} \rangle$ (accounts for anisotropic noise/sky cut)

$$\hat{\mathrm{h}} = \mathcal{F}^{-1}[\tilde{\mathrm{h}} - \langle \tilde{\mathrm{h}} \rangle]$$

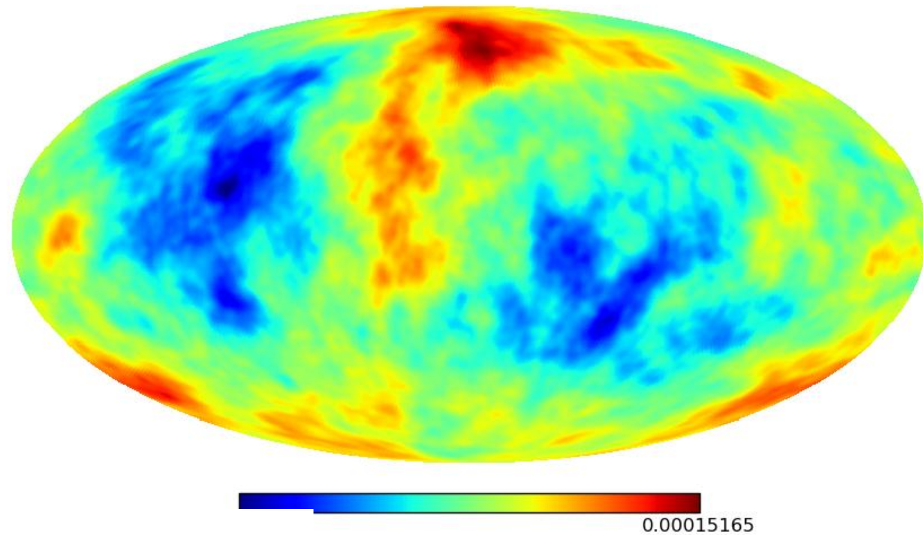
$\mathcal{F}$ Approximated or from sims

For lensing get generalization of Okamoto & Hu 2003 estimators for anisotropic noise/partial sky

True (simulated)



Reconstructed (Planck noise, Wiener filtered)



(thanks Duncan!)

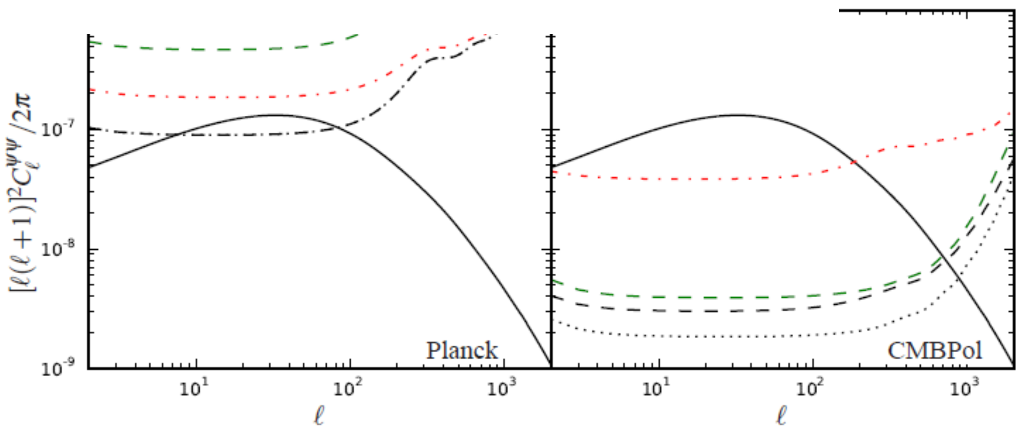


Fig. 5 Power spectrum of the errors in the gradient part of the reconstructed deflection field for two full-sky experiments. Left: an approximation to the Planck satellite, with $\sigma(T) = 27\,\mu$K-arcmin. Right: a version of the proposed CMBPol mission, with $\sigma(T) = 1\,\mu$K-arcmin. For both experiments, we assume a Gaussian beam of FWHM 7 arcmin and Q and U noise levels equal to $\sqrt{2}\sigma(T)$. The curves are the theory deflection power (solid black), reconstruction errors from temperature alone (red dot-dashed), polarization alone (green dashed), temperature and polarization together (black dashed), and the Fisher limit (black dotted). For Planck, the Fisher limit is saturated.

- Constrain curvature, dark energy, neutrino mass…

Hanson, Challinor & Lewis **0911.0612**

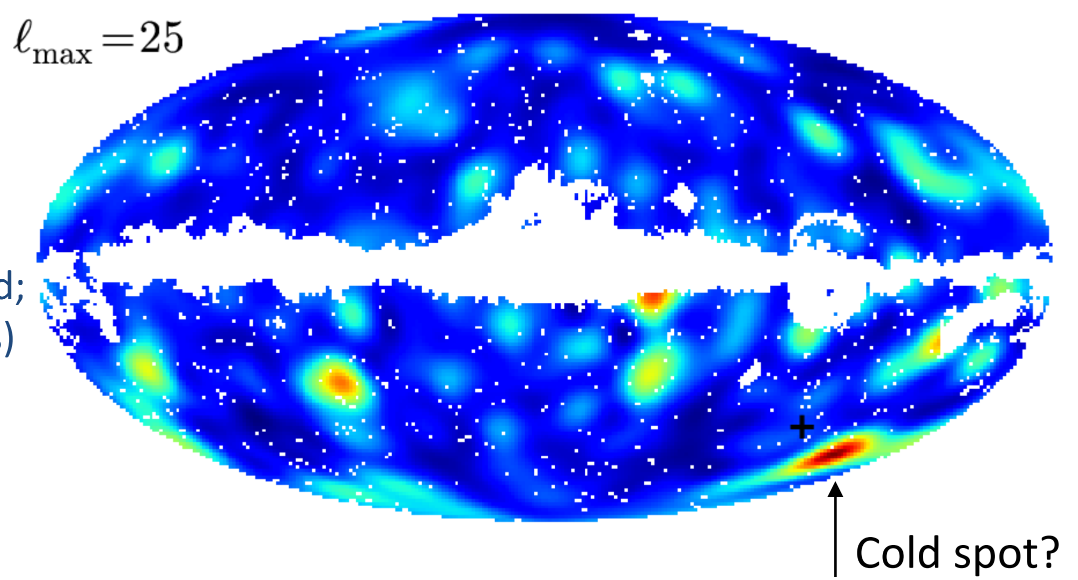
Unexpected signals?..

Sky modulation?

Popular modulation model: $$\Theta_f(\hat{\mathbf{n}}) = [1 + f(\hat{\mathbf{n}})]\Theta^i_f(\hat{\mathbf{n}})$$

QML estimator for f:

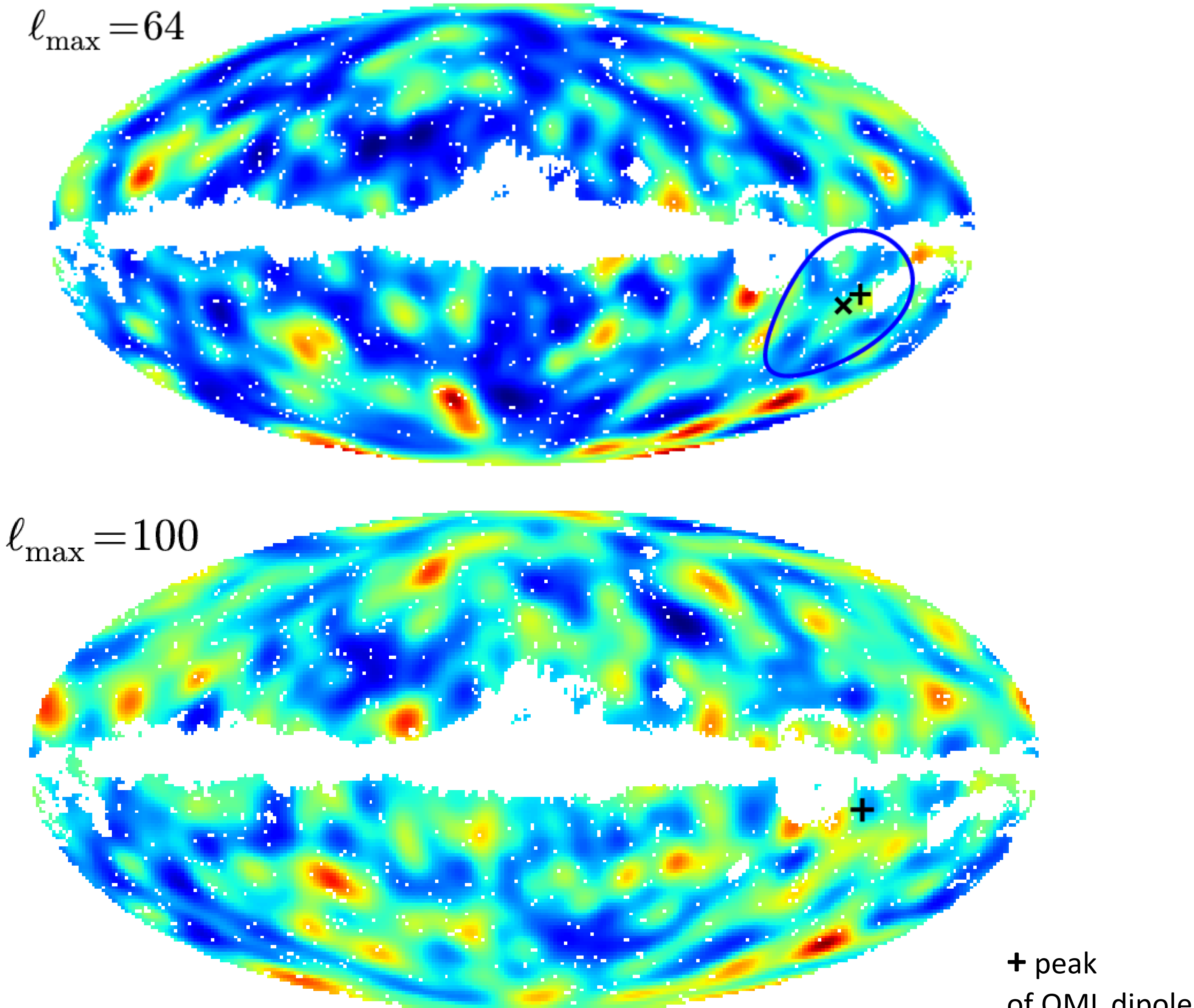
$$\tilde{h}^f_{lm} = \int d\Omega Y^*_{lm} \left[\sum_{l_1 m_1}^{l_{\max}} \bar{\Theta}_{l_1 m_1} Y_{l_1 m_1}\right] \left[\sum_{l_2 m_2}^{l_{\max}} C_{l_2} \bar{\Theta}_{l_2 m_2} Y_{l_2 m_2}\right]$$



WMAP power reconstruction

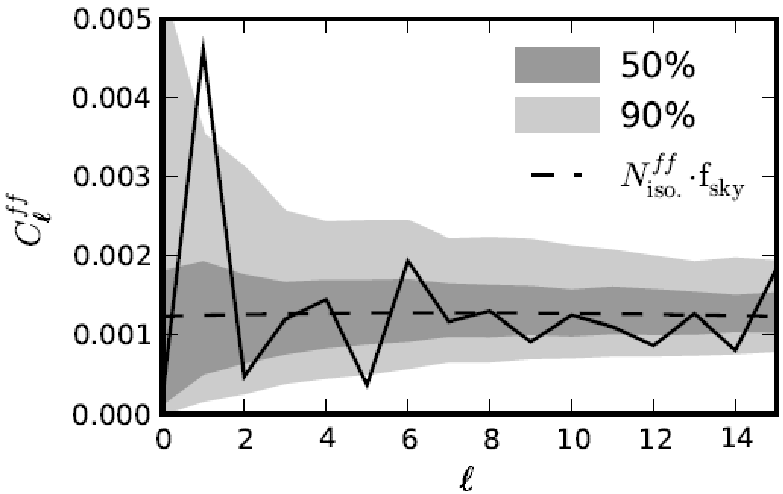
(V band, KQ85 mask, foreground cleaned; reconstruction smoothed to 10 degrees)

Following Eriksen et al, WMAP, etc..

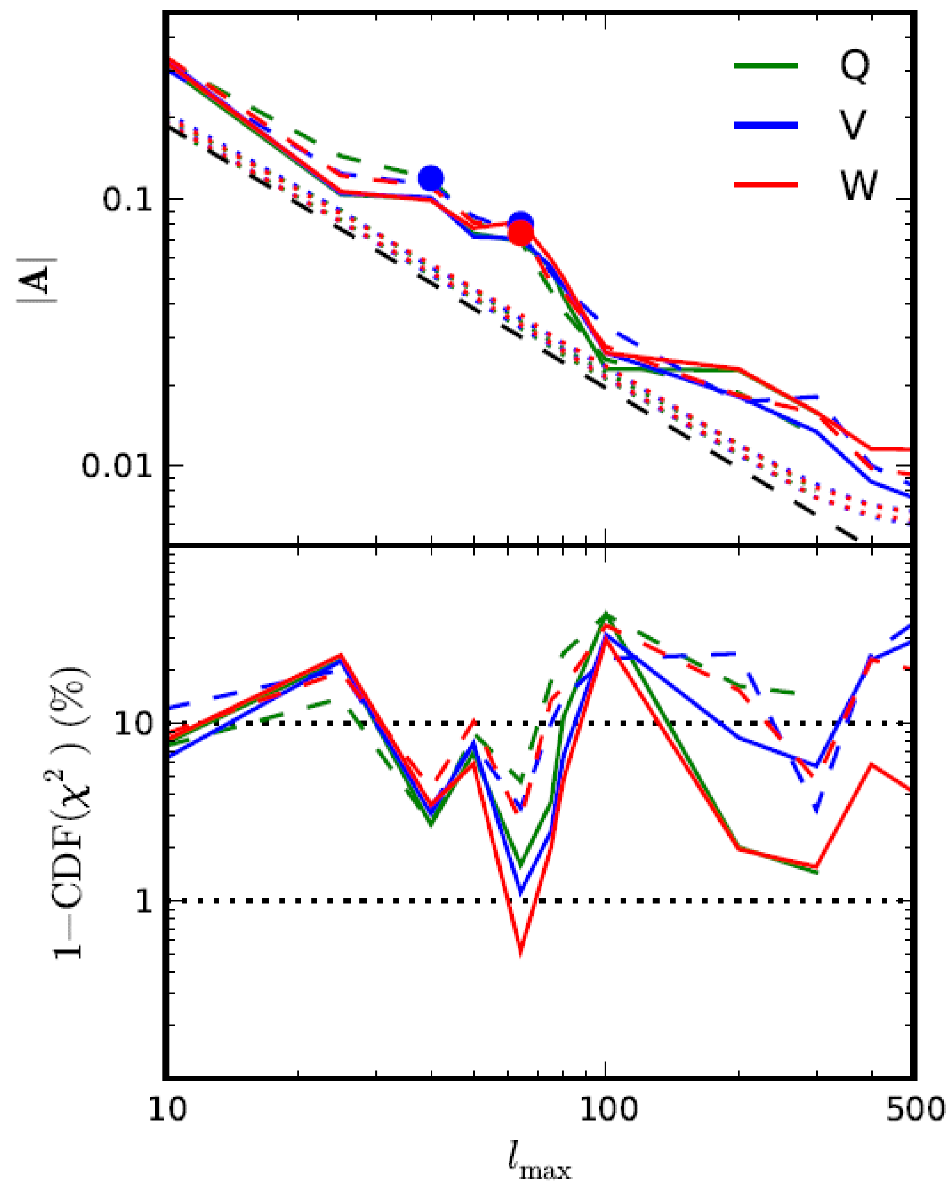


$\ell_{\max} = 64$
$\ell_{\max} = 100$
+ peak
of QML dipole

Modulation power spectrum l_{max}=64



Dipole amplitude as function of l_{max}



Only ~1% modulation allowed on small scales

Consistent with Hirata 2009
- Very small observed anisotropy in quasar distribution

Unexpected signals?..

Primordial power spectrum anisotropy

Look for direction-dependence in primordial power spectrum:

$$\langle \chi_0(\mathbf{k}) \chi_0^*(\mathbf{k}') \rangle = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') P_\chi(\mathbf{k})$$

Simple case:

$$\mathcal{P}_\chi(\mathbf{k}) = \mathcal{P}_\chi(k)[1 + a(k) g(\hat{\mathbf{k}})]$$

e.g.
Ackerman et.al. astro-ph/0701357
Gumrukcuoglu et al 0707.4179

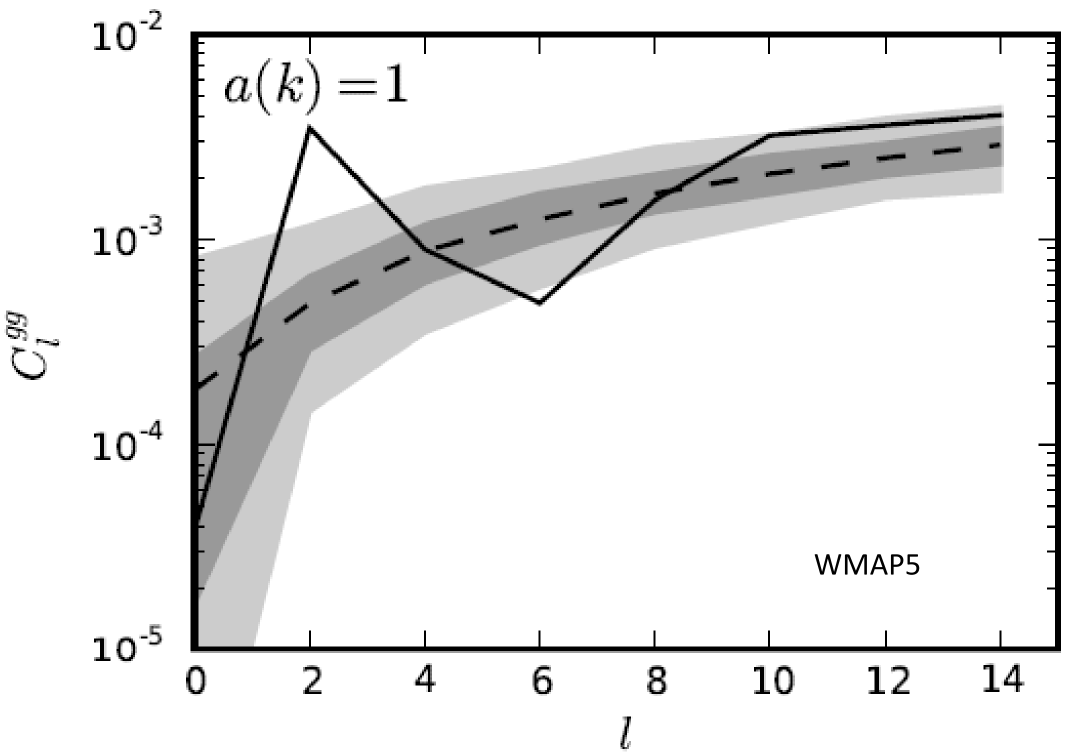
Anisotropic covariance:

$$C_{l_1 m_1 l_2 m_2} = i^{l_1 - l_2} \frac{\pi}{2} \int \mathrm{d}^3\mathbf{k} P_\chi(\mathbf{k}) \Delta_{l_1}(k) \Delta_{l_2}(k) Y^*_{l_1 m_1}(\hat{\mathbf{k}}) Y_{l_2 m_2}(\hat{\mathbf{k}})$$

Reconstruct g(k)

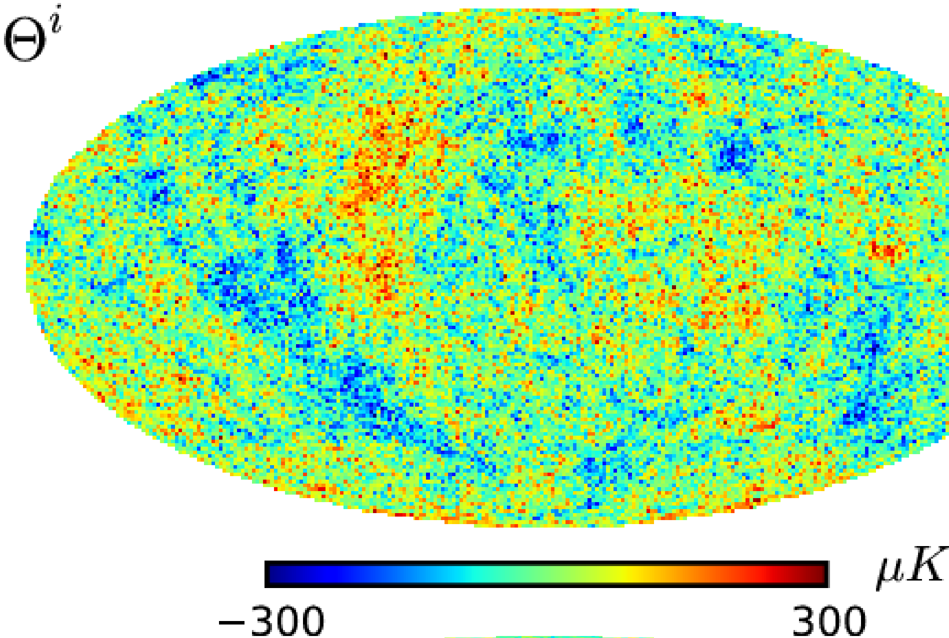
QML estimator:

$$\tilde{h}^g_{lm} = \frac{1}{2}\int d\Omega Y^*_{lm} \sum_{l_1 l_2} i^{l_1 - l_2} C_{l_1 l_2} \times \left[\sum_{m_1} \bar{\Theta}_{l_1 m_1} Y_{l_1 m_1}\right]\left[\sum_{m_2} \bar{\Theta}_{l_2 m_2} Y_{l_2 m_2}\right]$$

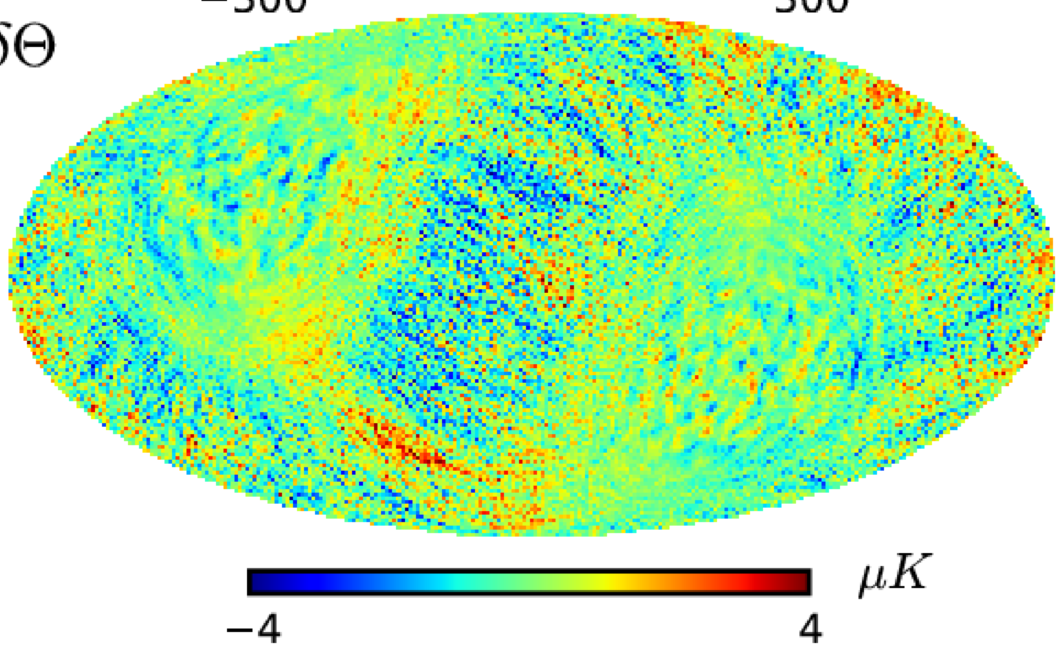


Many-sigma quadrupole primordial power anisotropy??

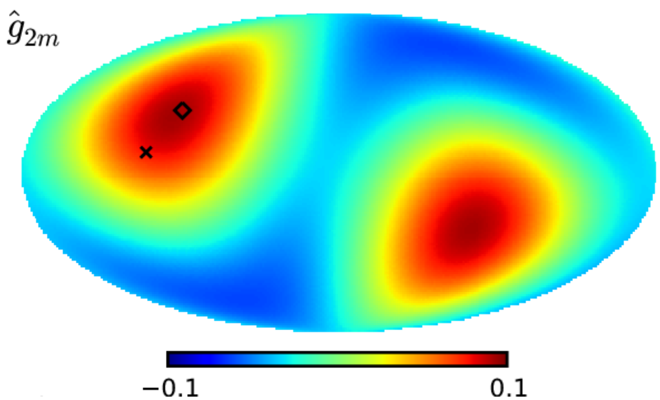
Θ^i



$\delta\Theta$



$\hat{g}_{2m}$



Direction close to ecliptic!
Also varies with frequency and detector.

Could it be systematics? - beam asymmetries? uncorrected in WMAP maps

Check with analytic model of scan strategy

$$\tilde{\Theta}(\Omega_p) = \sum_s w(\Omega_p, -s) \left[\sum_{lm} B_{ls} \Theta_{lm} \, {}_sY_{lm}(\Omega_p) \right]$$

Scan strategy

Beam shape multipoles

$$w(\Omega_p, -s) = \sum_{i \in p} e^{-is\alpha_i} / H_p$$
$$= v(\Omega_p, s) / v(\Omega_p, 0)$$

$$v(\Omega_p, s) = \sum_{i \in p} e^{is\alpha_i}$$

WMAP model

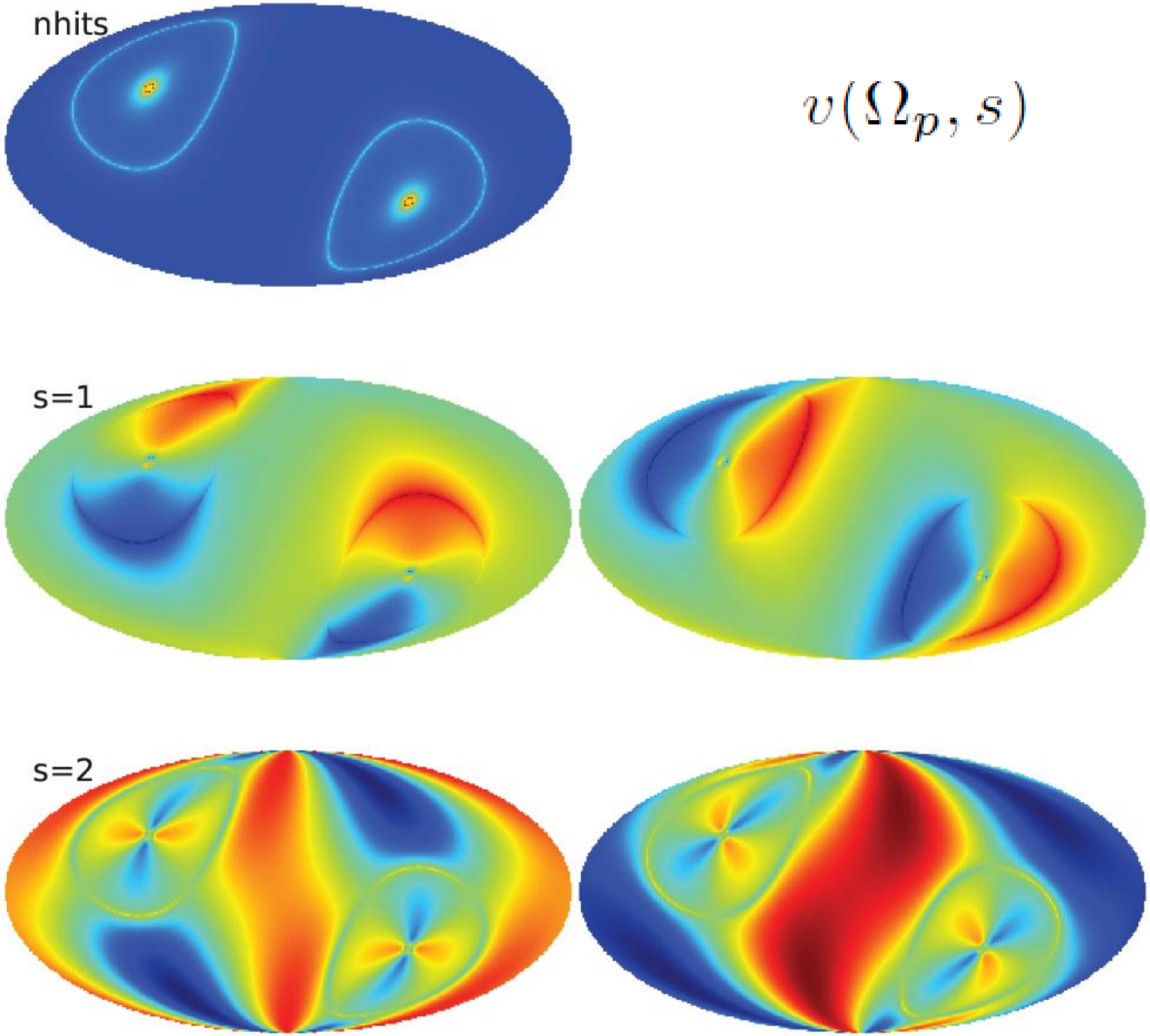
Hirata et al astro-ph/0406004.

(1) a beam at an angle θ_b to the satellite spin axis, which rotates with period τ_s;

(2) a precession at an angle θ_p to the anti-solar direction, with period τ_p; and

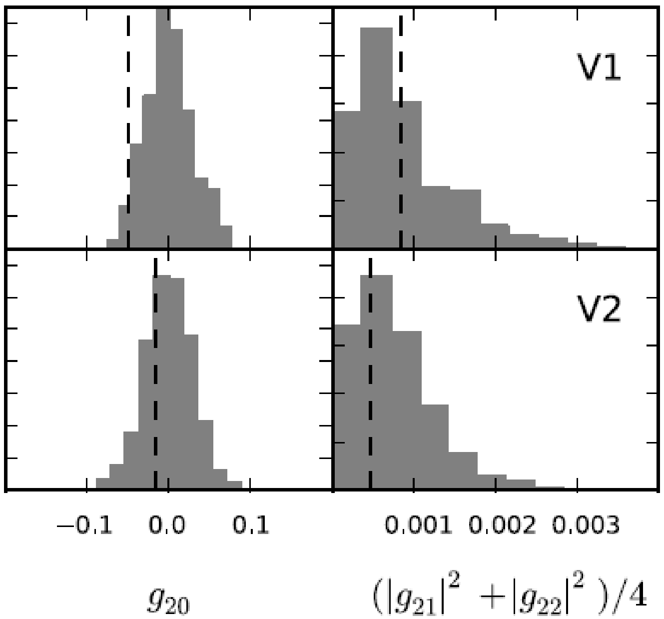
(3) a continuous repointing of the anti-solar direction as the observer orbits the sun.

$$\longrightarrow \quad [v(\Omega_p, s)]_{lm} = \delta_{m0} K P_l(0) P_l(\cos\theta_p) \, {}_sY_{l0}(\theta_b, 0)$$



nhits
$v(\Omega_p, s)$
s=1
s=2

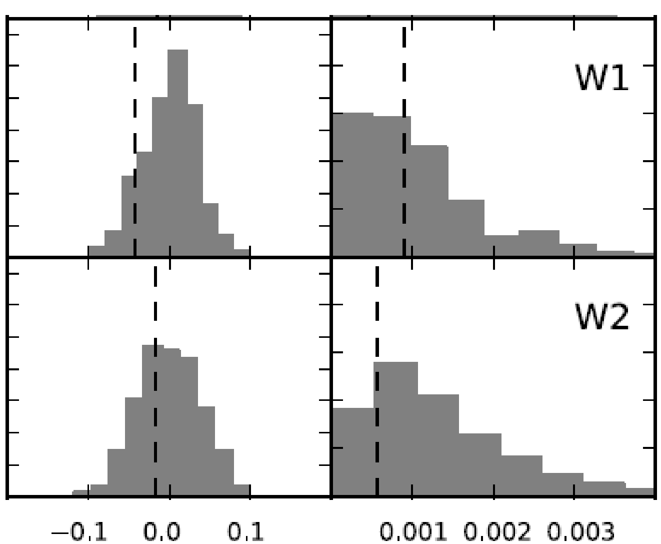
Monte Carlo with subtraction of mean field analytic model of beam asymmetries

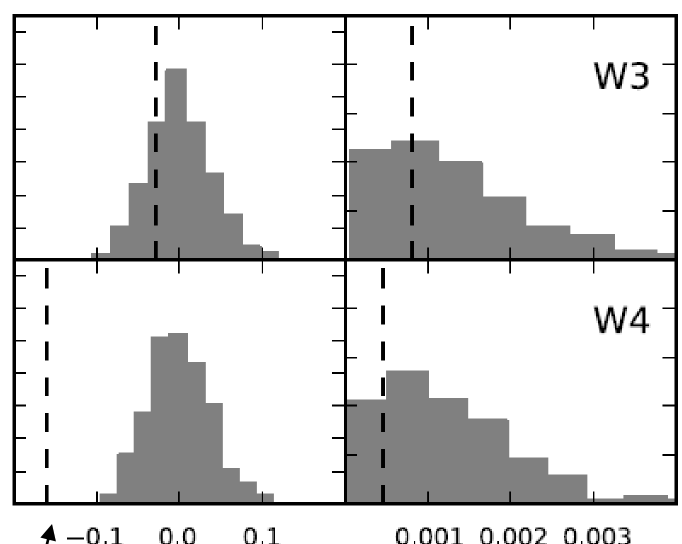


No detection..

$|g_{2M}| < 0.07$ at 95% confidence.

Consistent with Pullen et al 2010
constraint from large-scale structure [1003.0673]



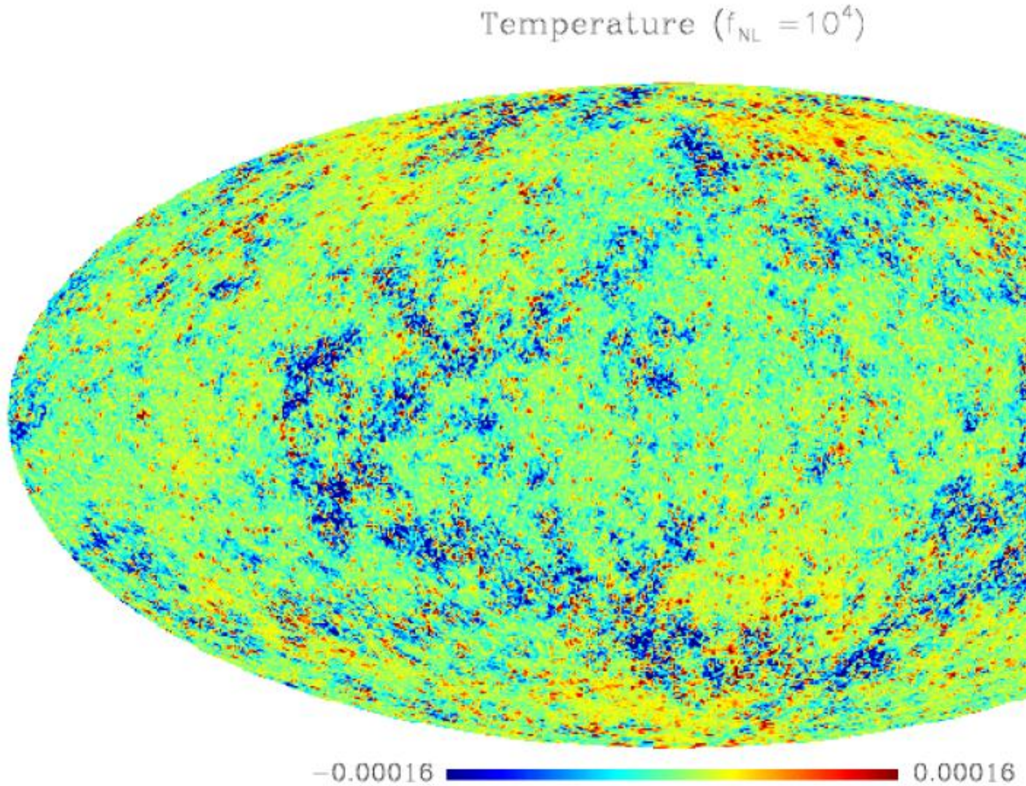


can be explained as correlated noise

Unexpected signals?..

Bispectrum non-Gaussianity

- Local 'squeezed' models: small scale power correlated with large-scale temperature
- Considering large-scale modes to be fixed, expect power anisotropy



Liguori et al 2007

e.g. Local primordial non-Gaussianity

$$\begin{aligned}\Psi &= \Psi_0 + f_{\rm NL}\Psi_0^2 \\ &= \Psi_0(1 + f_{\rm NL}\Psi_0)\end{aligned}$$

Write general quadratic anisotropy estimator:

$$6X_{lm} \equiv \sum_{l_1m_1,l_2m_2} B_{ll_1l_2}(-1)^{m_1}\begin{pmatrix} l & l_1 & l_2 \\ m & -m_1 & m_2 \end{pmatrix}\bar{\Theta}_{l_1m_1}\bar{\Theta}^*_{l_2m_2}$$
$$= \int d\Omega Y^*_{lm} \times \sum_{l_1l_2} b_{ll_1l_2}\left[\sum_{m_1}\bar{\Theta}_{l_1m_1}Y_{l_1m_1}\right]\left[\sum_{m_2}\bar{\Theta}_{l_2m_2}Y_{l_2m_2}\right]$$

Bispectrum estimators are basically the cross-correlation of an anisotropy estimator with the temperature

$$\mathcal{E} = \frac{1}{F_{\mathcal{E}}}\bar{\Theta}^\dagger(\mathbf{X} - 3\langle\mathbf{X}\rangle),$$

In harmonic space

$$\mathcal{E} = \frac{1}{6F_{\mathcal{E}}}\sum_{l_im_i} B_{l_1l_2l_3}\begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \times \left[\bar{\Theta}_{l_1m_1}\bar{\Theta}_{l_2m_2}\bar{\Theta}_{l_3m_3} - 3C^{-1}_{l_1m_1l_2m_2}\bar{\Theta}_{l_3m_3}\right].$$

Creminelli et al 2005, Babich 2005, Smith & Zaldarriaga 2006

CMB lensing bispectrum

Lensed temperature depends on deflection angle:

$$\tilde{T}(\hat{\mathbf{n}}) = T(\hat{\mathbf{n}}') = T(\hat{\mathbf{n}} + \boldsymbol{\alpha})$$

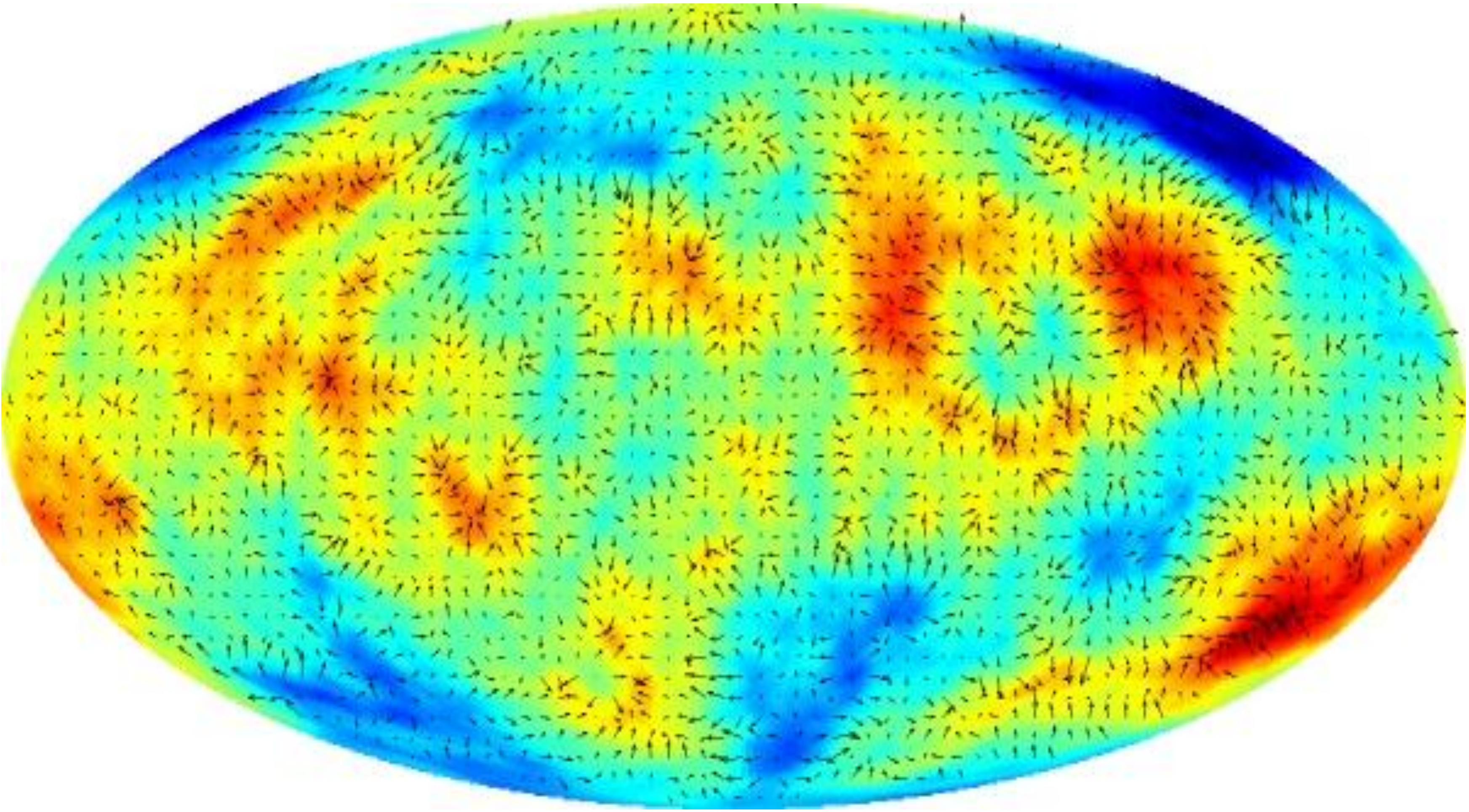
$$\boldsymbol{\alpha} = \delta\theta = -2\int_0^{\chi^*} \mathrm{d}\chi \frac{f_K(\chi^* - \chi)}{f_K(\chi^*)} \nabla_\perp \Psi(\chi\hat{\mathbf{n}}; \eta_0 - \chi)$$

Lensing Potential

Deflection angle on sky given in terms of lensing potential $\boldsymbol{\alpha} = \nabla\psi$

$$\psi(\hat{\mathbf{n}}) = -2\int_0^{\chi_*} \mathrm{d}\chi\, \Psi(\chi\hat{\mathbf{n}}; \eta_0 - \chi) \frac{f_K(\chi^* - \chi)}{f_K(\chi^*) f_K(\chi)}$$

$$\bar{X}(\mathbf{n}) = X(\mathbf{n}') = X(\ \mathbf{n} + \nabla\psi(\mathbf{n})\ )$$



LensPix sky simulation code:
http://cosmologist.info/lenspix
Lewis 2005, Hammimeche & Lewis 2008

Bispectrum as statistical anisotropy correlation

Lensing by fixed ψ field introduced statistical anisotropy

Construct QML estimator for ψ (following Hu and Okomoto 2003)

$$\langle \tilde{T}(\mathbf{l}_2)\tilde{T}(\mathbf{l}_1 - \mathbf{l}_2)\rangle_T \propto \psi(\mathbf{l}_1)$$

Bispectrum measures cross-correlation of quadratic estimator for ψ with the large-scale temperature

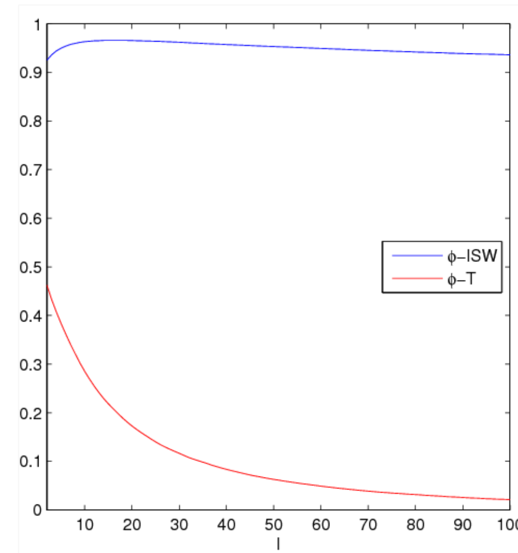
For squeezed triangles, $l_1 \ll l_2, l_3$,

$$\tilde{T}(\mathbf{l}_1) \sim T(\mathbf{l}_1) \text{ and } \langle \tilde{T}(\mathbf{l}_2)\tilde{T}(\mathbf{l}_3)\rangle_T \propto \psi(\mathbf{l}_1)$$

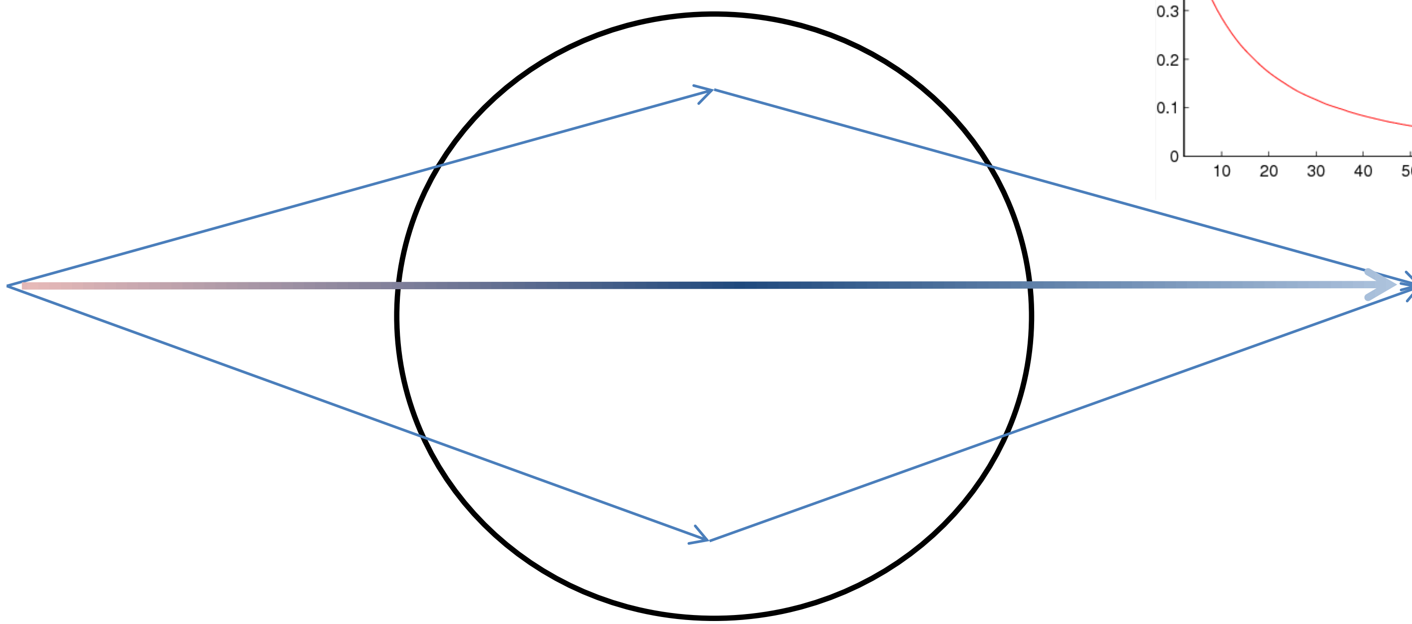
$$\Rightarrow \quad \langle \tilde{T}(\mathbf{l}_1)\tilde{T}(\mathbf{l}_2)\tilde{T}(\mathbf{l}_3)\rangle \sim \langle T(\mathbf{l}_1)\psi(\mathbf{l}_1)\rangle \sim C_{l_1}^{\psi T}$$

Why is there a correlation between large-scale lenses and the temperature?

(small-scales: also SZ , Rees-Sciama..)



$$\Delta T_{\rm ISW}(\hat{\mathbf{n}}) = 2\int_0^{\chi_*} d\chi\,\dot{\Psi}(\chi\hat{\mathbf{n}};\eta_0-\chi)$$



Overdensity: magnification correlated with positive Integrated Sachs-Wolfe (net blueshift)

Underdensity: demagnification correlated with negative Integrated Sachs-Wolfe (net redshift)

Accurate bispectrum calculation

Assume Gaussian fields. Non-perturbative result:

$$\langle T(\mathbf{l}_1)\tilde{T}(\mathbf{l}_2)\tilde{T}(\mathbf{l}_3)\rangle = C_{l_1}^{T\psi}\left\langle \frac{\delta}{\delta\psi(\mathbf{l}_1)^*}\left(\tilde{T}(\mathbf{l}_2)\tilde{T}(\mathbf{l}_3)\right)\right\rangle$$

Use $\tilde{T}(\boldsymbol{x}) = T(\boldsymbol{x} + \boldsymbol{\nabla}\psi)$ $\Rightarrow$ $$\frac{\delta}{\delta\psi(\mathbf{l}_1)^*}\tilde{T}(\mathbf{l}) = -\frac{i}{2\pi}\mathbf{l}_1\cdot\widetilde{\boldsymbol{\nabla} T}(\mathbf{l}+\mathbf{l}_1),$$

$$\begin{aligned}\langle T(\mathbf{l}_1)\tilde{T}(\mathbf{l}_2)\tilde{T}(\mathbf{l}_3)\rangle &= -\frac{i}{2\pi}C_{l_1}^{T\psi}\,\mathbf{l}_1\cdot\left\langle\widetilde{\boldsymbol{\nabla} T}(\mathbf{l}_1+\mathbf{l}_2)\tilde{T}(\mathbf{l}_3)\right\rangle + (\mathbf{l}_2\leftrightarrow\mathbf{l}_3)\\ &= -\frac{1}{2\pi}\delta(\mathbf{l}_1+\mathbf{l}_2+\mathbf{l}_3)C_{l_1}^{T\psi}\left[(\mathbf{l}_1\cdot\mathbf{l}_2)\tilde{C}_{l_2}^{T\nabla T} + (\mathbf{l}_1\cdot\mathbf{l}_3)\tilde{C}_{l_3}^{T\nabla T}\right]\end{aligned}$$

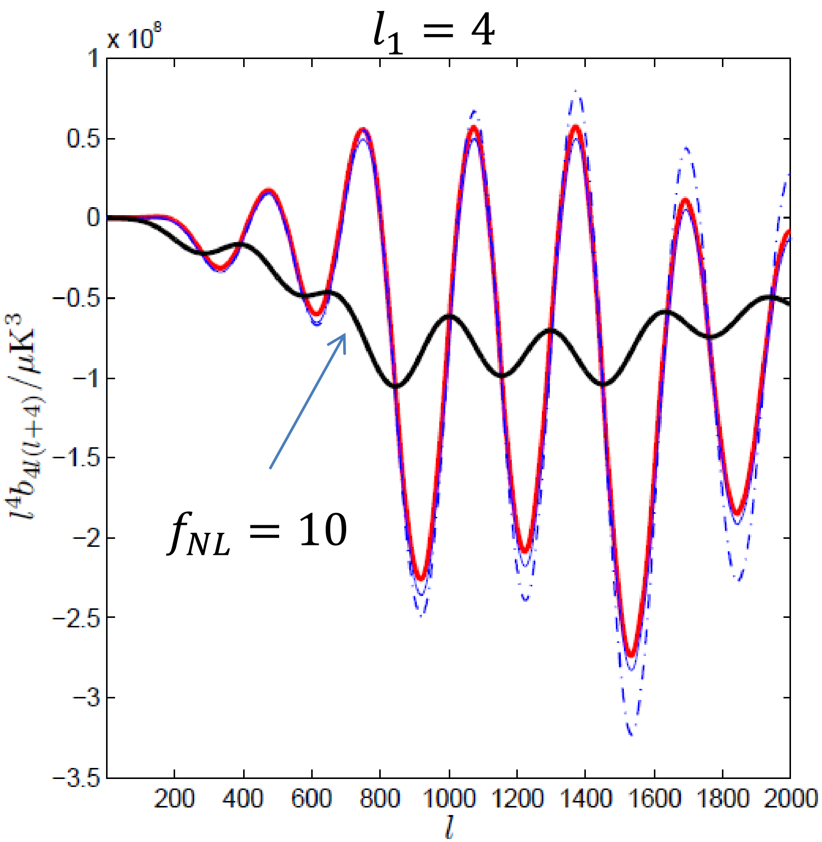
~ Lensed temperature power spectrum

Lensing bispectrum depends on *changes* in the small-scale *lensed* power

$$b_{l_1 l_2 l_3} \approx -C^{T\psi}_{l_1}\left[(\mathbf{l}_1\cdot\mathbf{l}_2)\tilde{C}^{TT}_{l_2} + (\mathbf{l}_1\cdot\mathbf{l}_3)\tilde{C}^{TT}_{l_3}\right]$$

$$\approx l_1^2 C^{T\psi}_{l_1}\left[\frac{(\mathbf{l}_1\cdot\mathbf{l}_2)^2}{l_1^2 l_2^2}\left.\frac{\mathrm{d}\tilde{C}^{TT}_l}{\mathrm{d}\ln l}\right|_{l_2} + \tilde{C}^{TT}_{l_2}\right].$$

$$(\mathbf{l_1} + \mathbf{l_2} + \mathbf{l_3} = 0)$$



- Quite large signal. Expect $\sim 5\sigma$ with Planck. Cosmic variance $\sim 7\sigma$.

- Using lensed power spectra important at 5-20% level: leading-order result (using unlensed spectra) not accurate enough

If lensing is neglected get bias $\Delta f_{NL} \sim 9$ on primordial local models with Planck
(see e.g. Hanson et al 0905.4732, Mangilli 0906.2317)

BUT:

- Lensing bispectrum depends on power difference: has phase shift compared to any adiabatic primordial bispectrum (and different scale dependence)

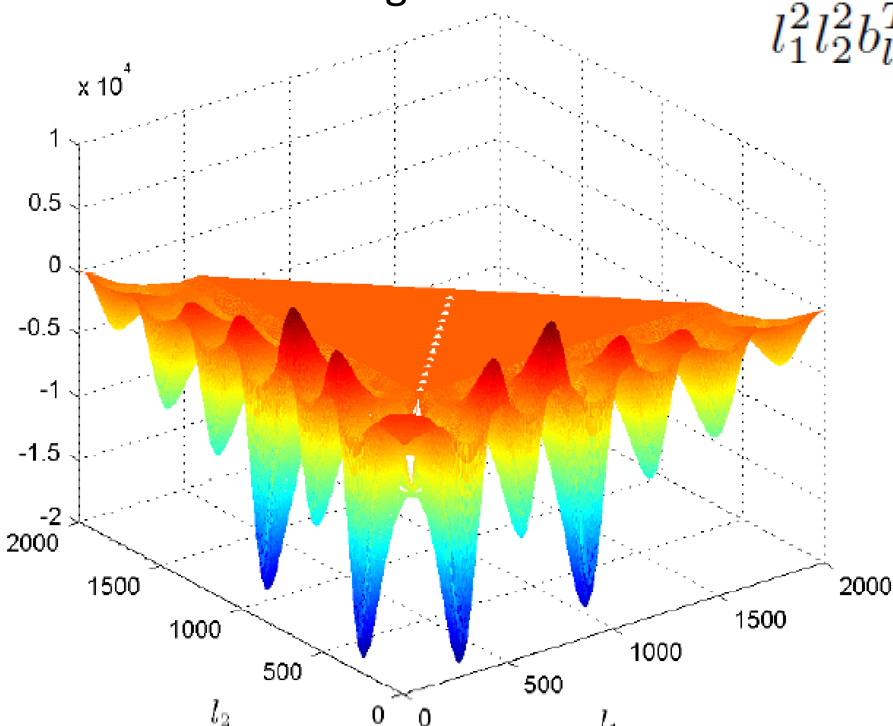
- Lensing bispectrum is strongly scale dependent (small ISW for larger l_1)

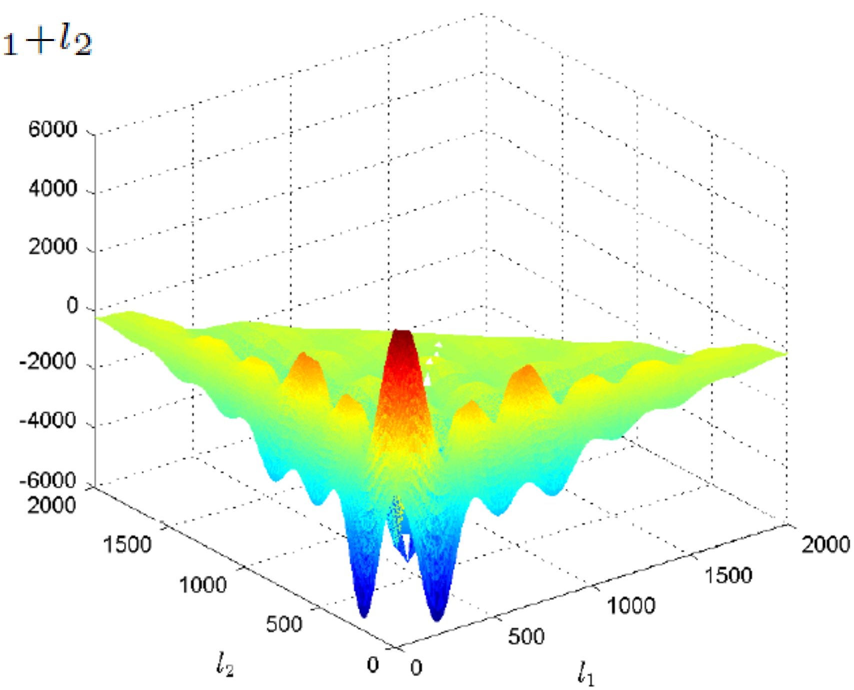
- Lensing bispectrum depends on shape of squeezed triangle ($l_1 \cdot l_2$ factor)

Lensing

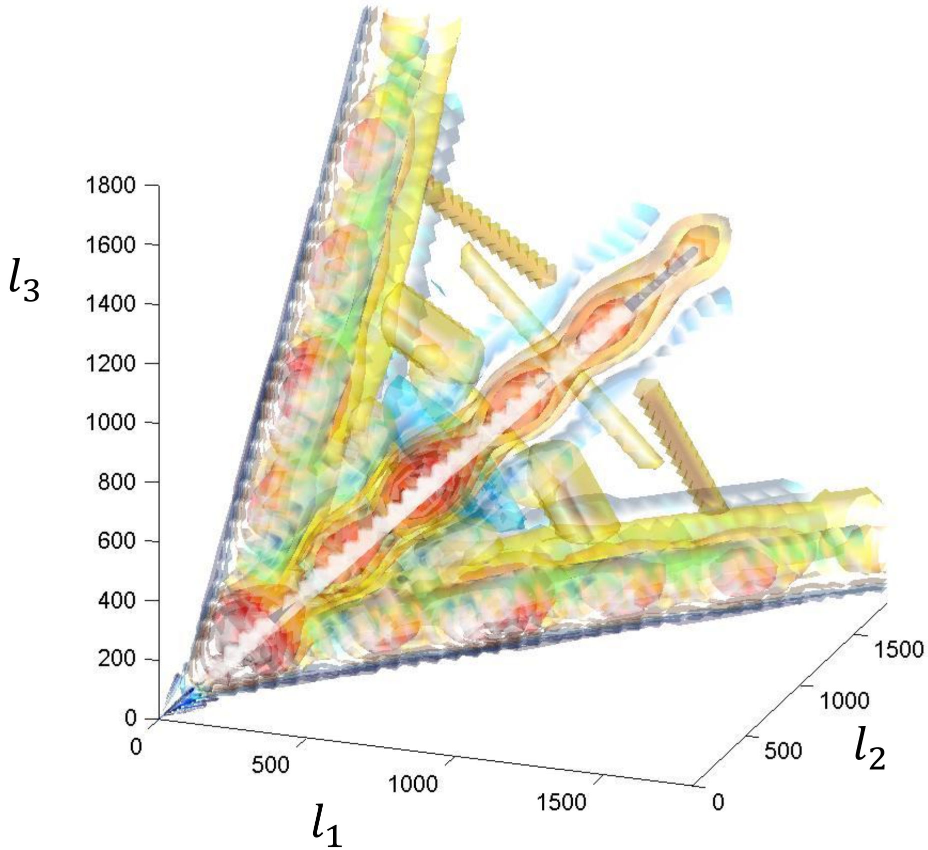
$$l_1^2 l_2^2 b^{TTT}_{l_1 l_2 l_1+l_2}$$

Local f_{NL}

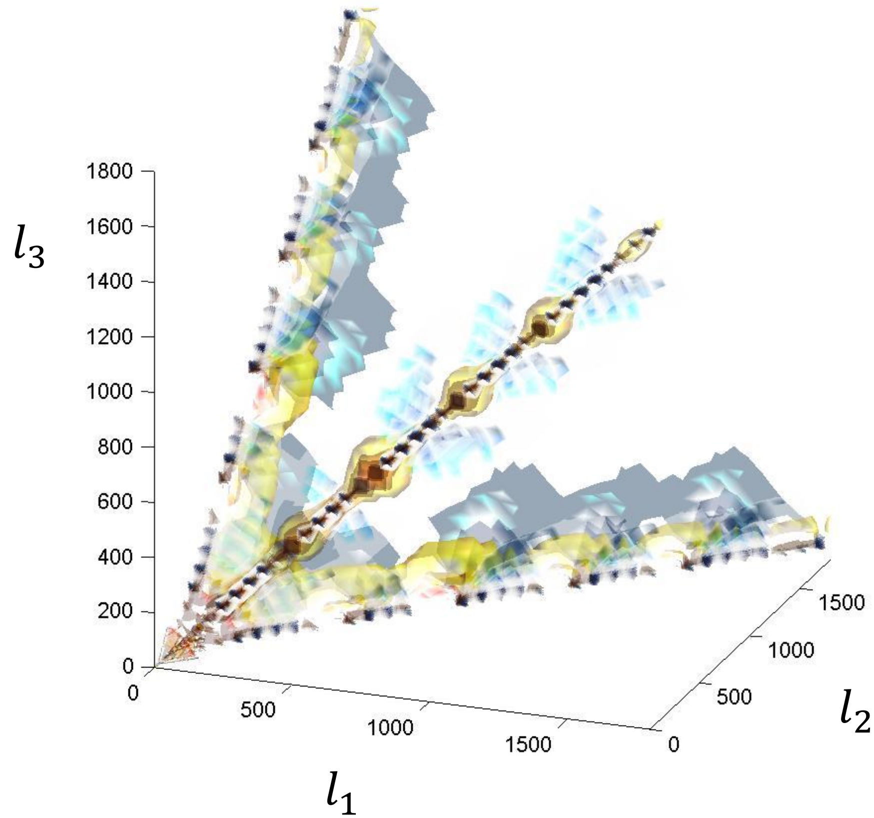




$b_{l_1 l_2 l_3}$



Local f_{NL}



CMB temperature lensing

Lensing bispectrum also squeezed triangles but quite distinctive

Temperature bispectrum correlation with local $f_{NL} \sim 30\%$: in null hypothesis can measure amplitude using optimized estimator and accurately subtract from f_{NL} estimator

CMB polarization

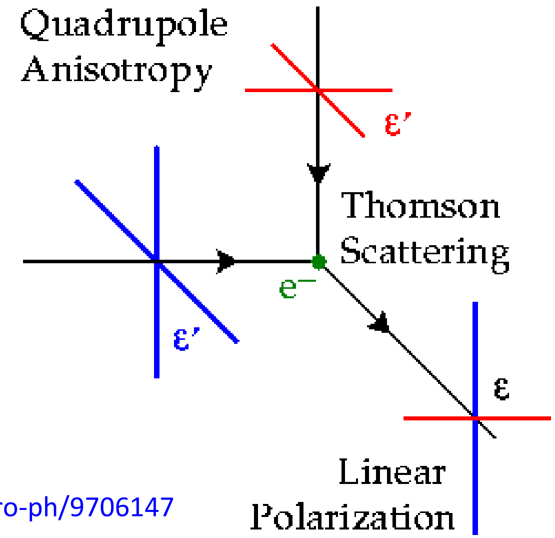
General full-sky bispectrum: $\mathbf{a}_{lm} = (T_{lm}, E_{lm}, B_{lm})^T$

$$B^{ijk}_{l_1 l_2 l_3} = \sum_{m_1 m_2 m_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \langle a^i_{l_1 m_1} a^j_{l_2 m_2} a^k_{l_3 m_3} \rangle$$

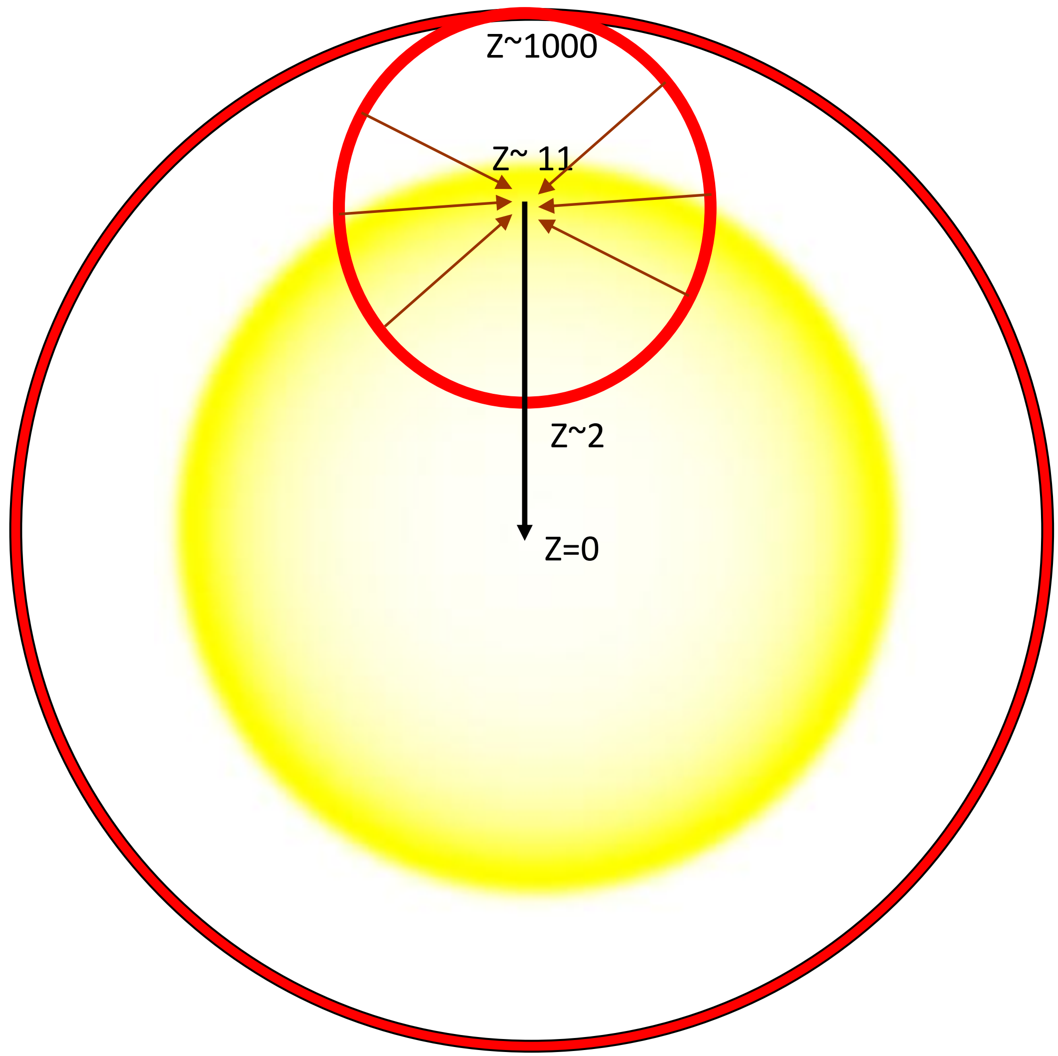
$$\approx F^{s_k}_{l_3 l_1 l_2} C^{a^i \psi}_{l_1} \tilde{C}^{a^j a^k}_{l_2} + i F^{-s_k}_{l_3 l_1 l_2} C^{a^i \psi}_{l_1} \tilde{C}^{a^j \bar{a}^k}_{l_2}$$ + perms

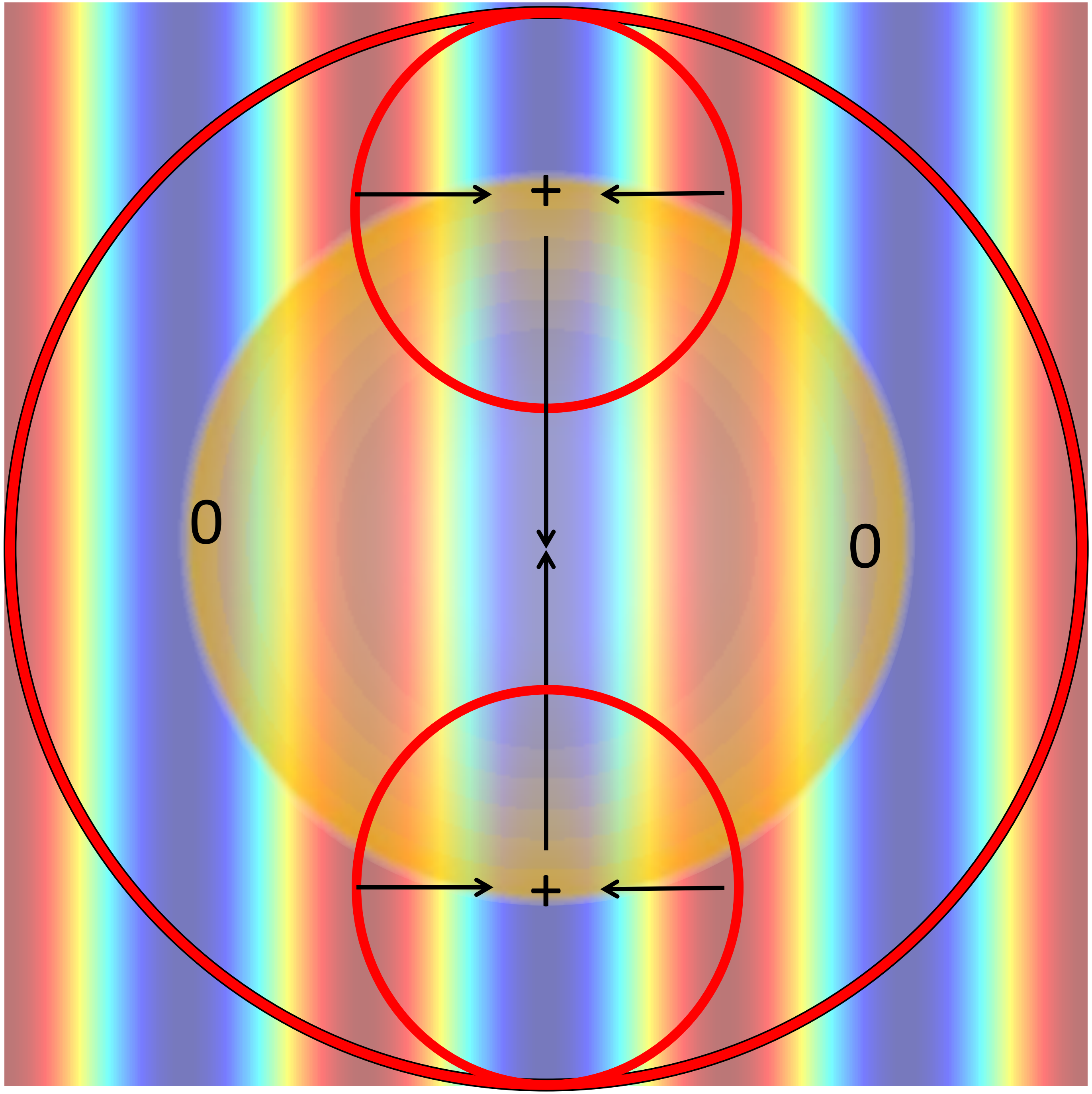
Is the polarization correlated? $C_l^{E\psi} = ?$

Yes! Significant large-scale correlation due to reionization



Hu astro-ph/9706147

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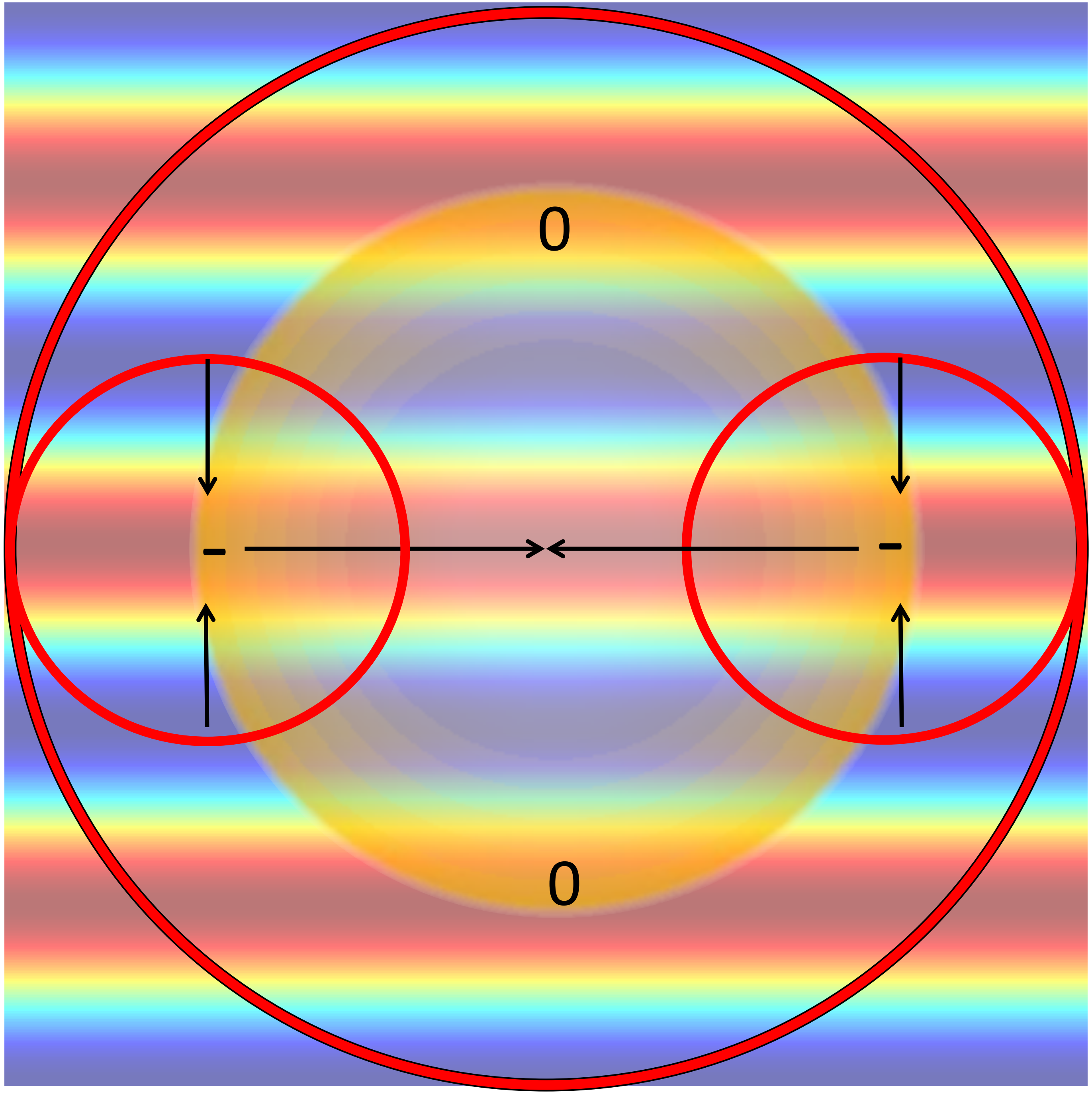
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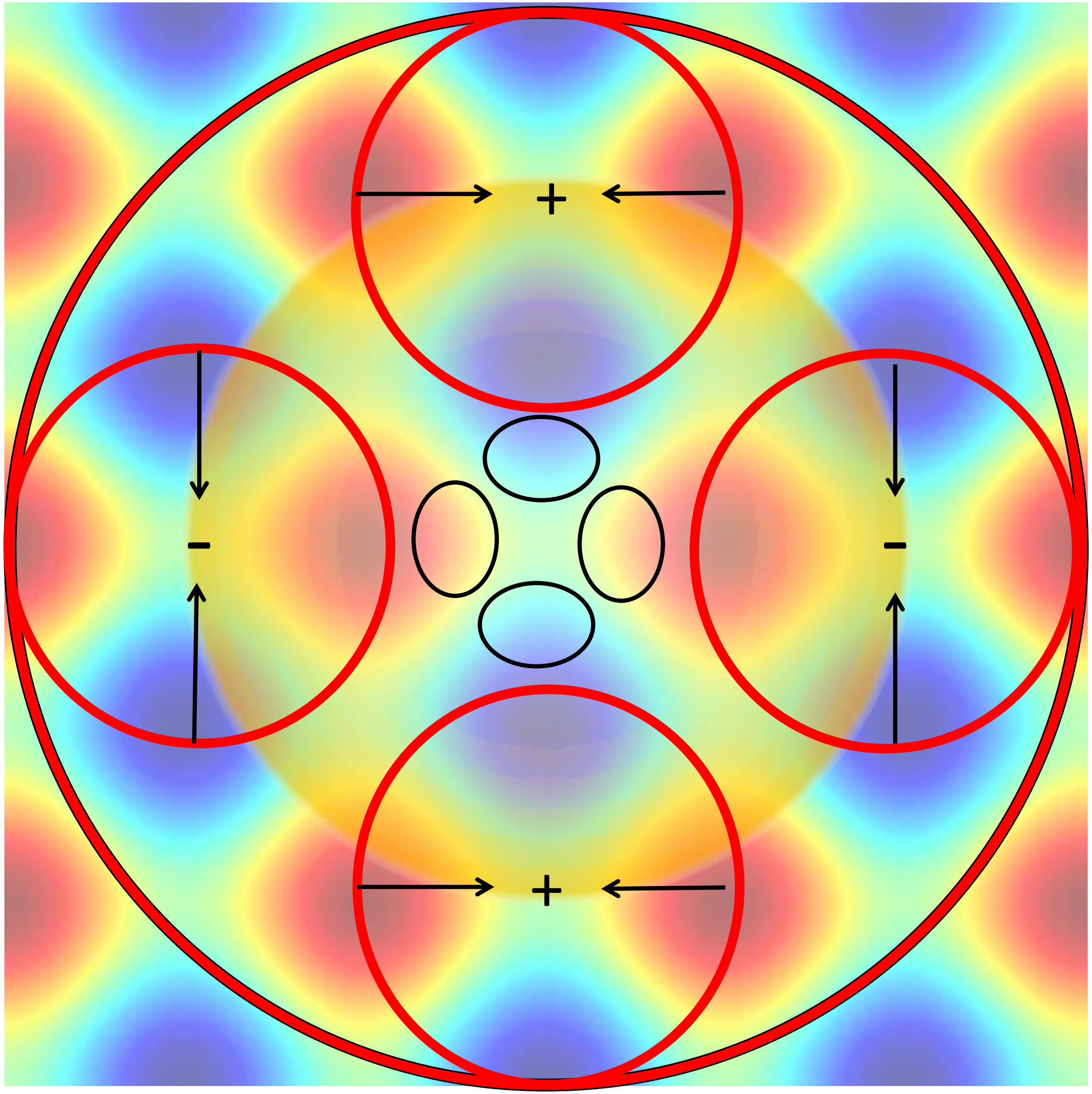
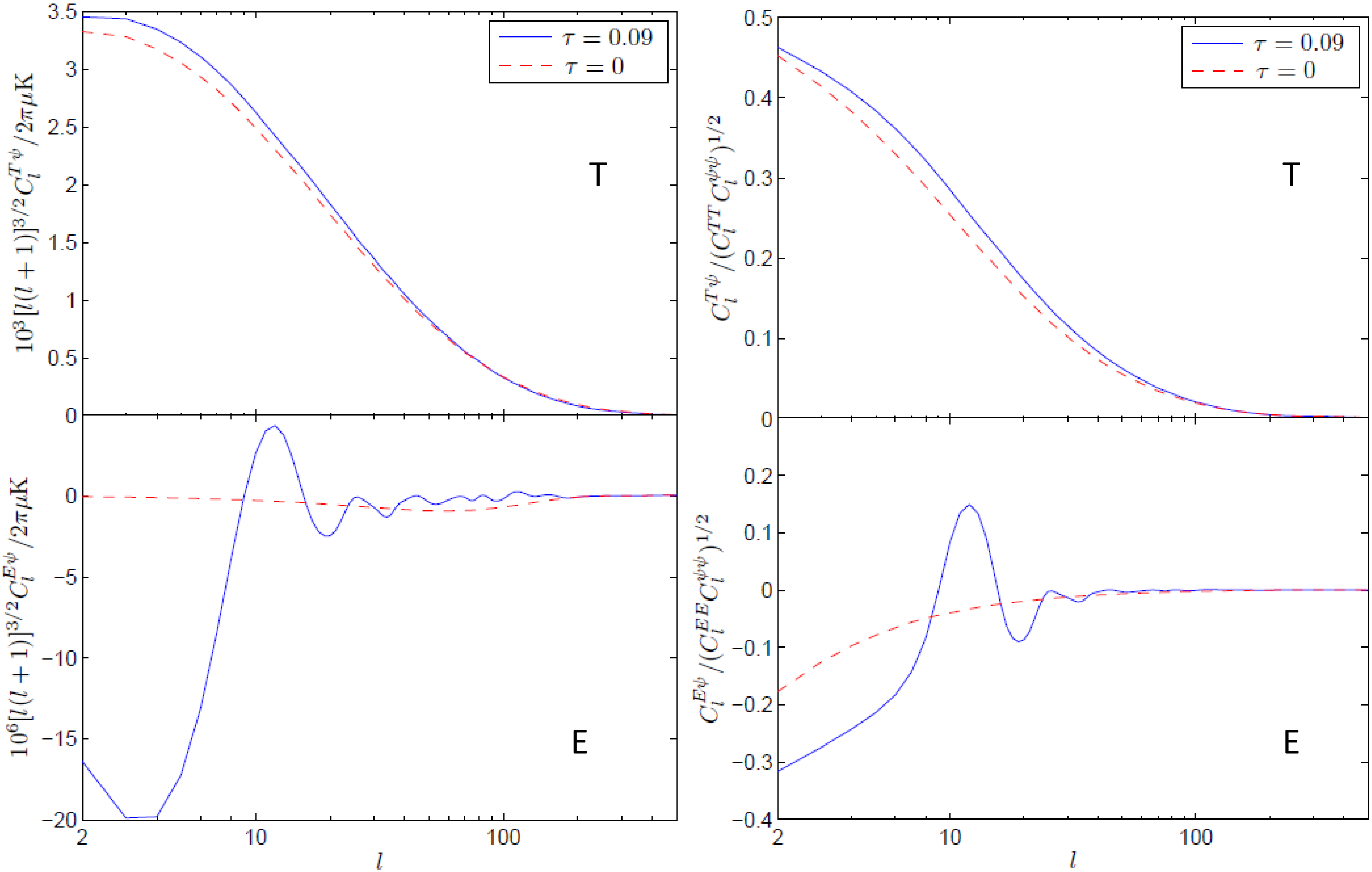
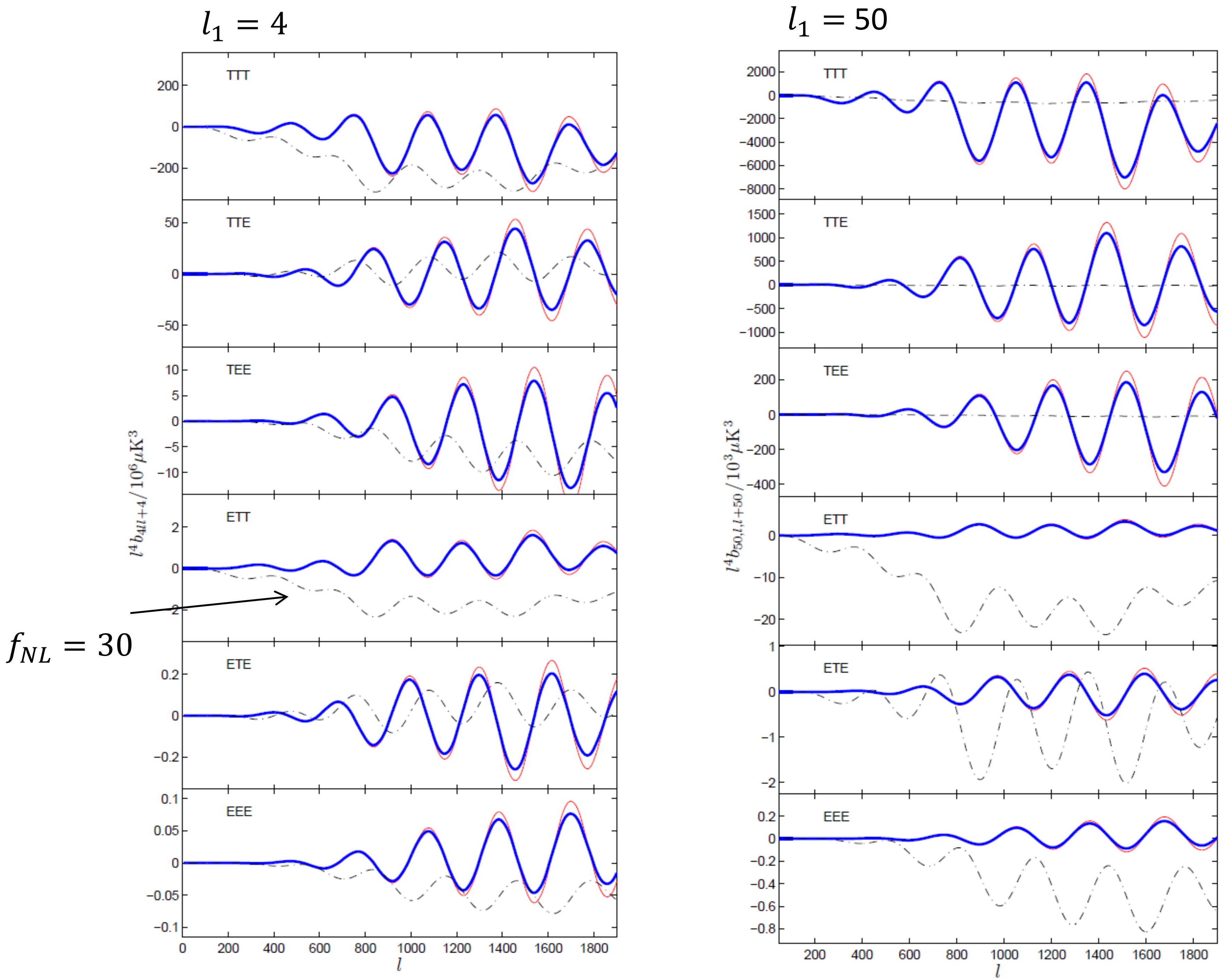


Lensing potential correlation power spectra



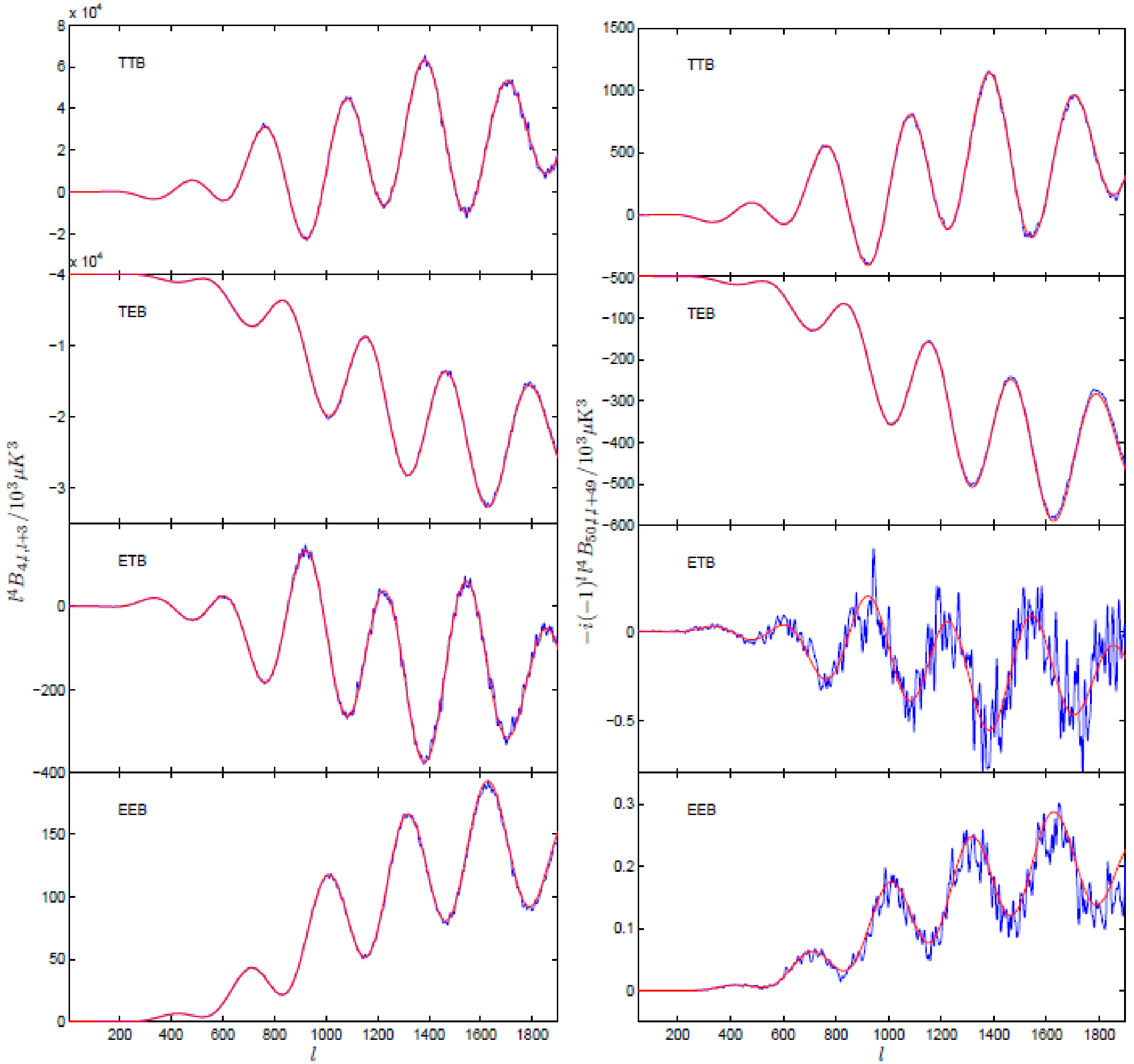
Cosmic variance: $C^{T\psi}$: $\sim 7\sigma$, $C^{E\psi}$: $\sim 2.5\sigma$



$l_1 = 4$
$l_1 = 50$
$f_{NL} = 30$
TTT
TTE
TEE
ETT
ETE
EEE
$l^4 b_{4ll+4}/10^6 \mu K^3$
$l^4 b_{50,l,l+50}/10^3 \mu K^3$
l

Also parity odd bispectra, TEB etc. (compared to sims)

$l_1 = 4$ $l_1 = 50$



Signal to Noise

Signal quite large, so cosmic variance important as well as noise

$$[F^{(l_1)-1}]_{ij} = [\bar{F}^{\text{lens,lens}\,-1}_{l_1}]_{ij} + \frac{[C^{a^i\psi}_{l_1} C^{a^j\psi}_{l_1} + C^{a^i a^j}_{l_1} C^{\psi\psi}_{l_1}]}{(2l_1+1) C^{a^i\psi}_{l_1} C^{a^j\psi}_{l_1}}$$

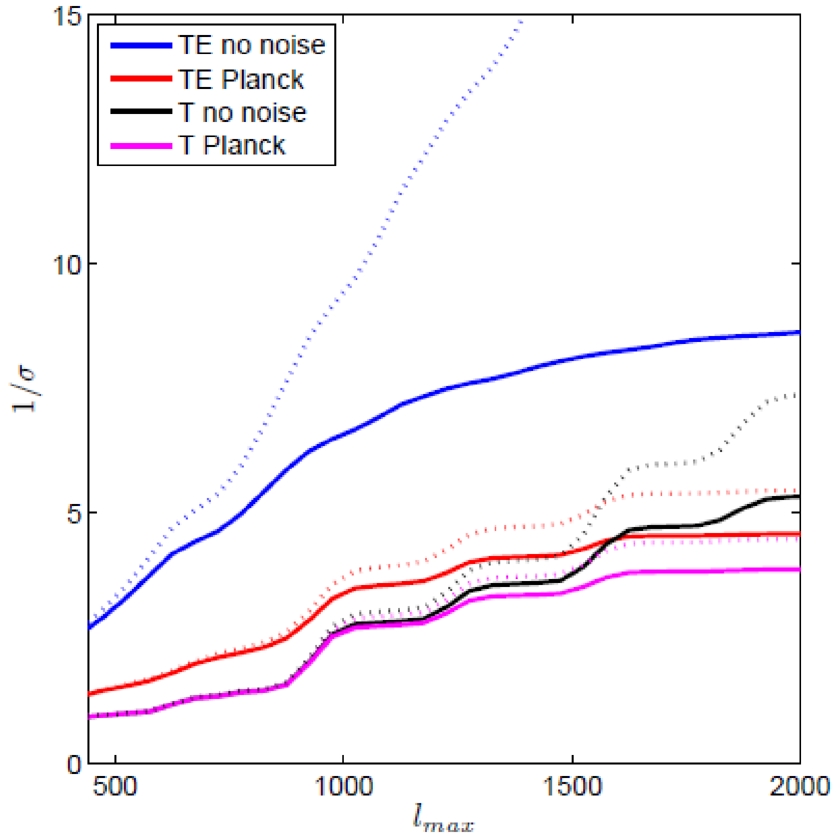
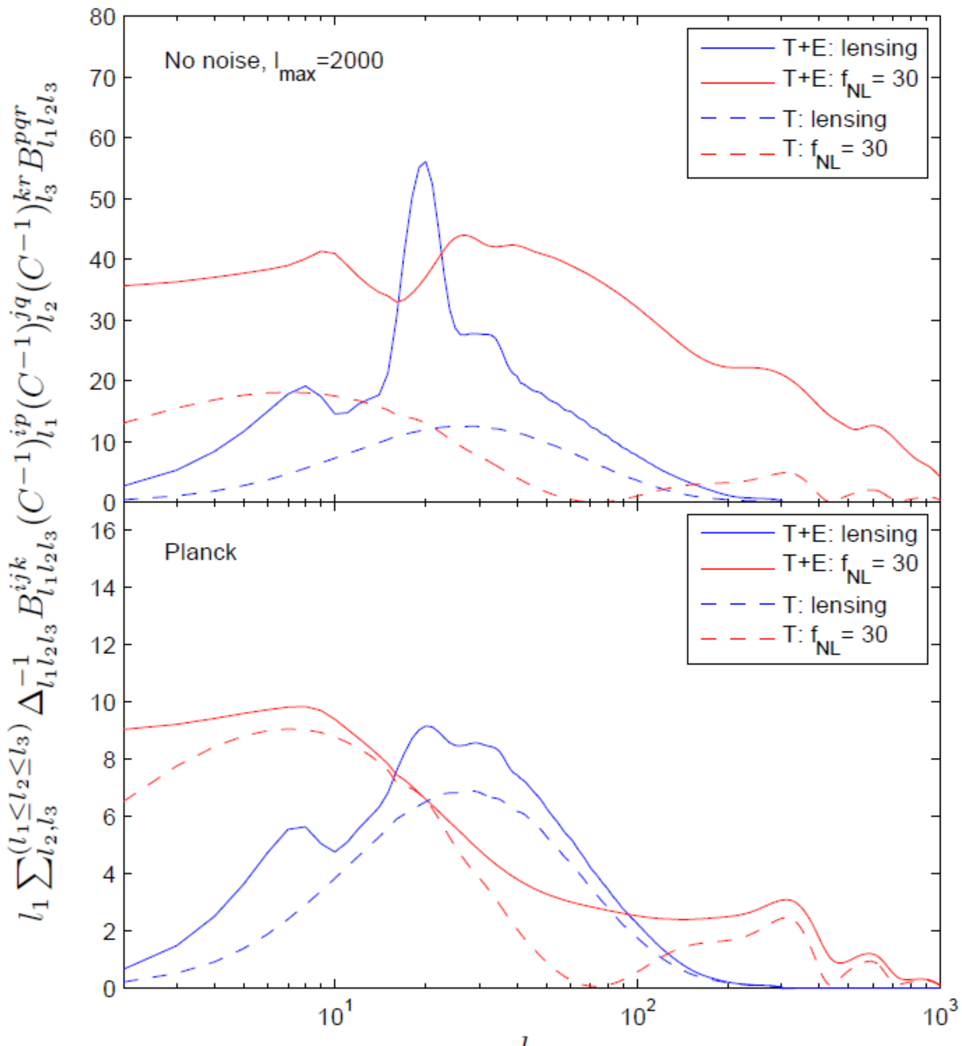


FIG. 10: The Fisher detection significance of the CMB lensing bispectrum as a function of $l_{\max}$ for no noise and Planck, using just the temperature bispectrum or using all the T and E-polarization bispectra. The dotted lines show the (incorrect) results obtained if the signal contribution to the variance is neglected: all results are bounded by the cosmic variance detection limit on a measurement of the low-l cross-correlation spectra $C_l^{T\psi}$ and $C_l^{E\psi}$.

Signal to noise

Contributions to Fisher inverse variance



Lensing signal peaks around $l_1 \sim 30$
- trade-off between size of signal and number of modes

- Cosmic variance limits simply determined by cosmic variance detection limits on $C_l^{T\psi}$ and $C_l^{E\psi}$

Planck $\sim 5\sigma$; Cosmic Variance $\sim 9\sigma$

Conclusions

- Can easily test for and reconstruct many forms of Gaussian statistical anisotropy

  - Fast nearly-optimal Quadratic Maximum Likelihood estimators

  - Strong detection of 'primordial power anisotropy' in WMAP – but actually explained by beam asymmetries

  - Lensing signal expected and can reconstruct lensing potential (learn about cosmological parameters)

- CMB lensing bispectrum is significant

  - Temperature bispectrum from ISW-ψ correlation

  - Also E- ψ correlation ($\sim 2.5\sigma$ cosmic variance limit)

  - Distinctive phase and scale-dependence

  Should be detected by Planck

  - Potential confusion with local f_{NL} but contribution easily distinguished/subtracted

  - Also SZ correlation on smaller scales, but frequency dependent; other terms includes Rees Sciama ($f_{NL} \sim 1$)

- Public codes available:

  $C_l^{E\psi}$, $C_l^{T\psi}$, $C_l^{\psi\psi}$, Local f_{NL} and lensing bispectrum in CAMB update: http://camb.info