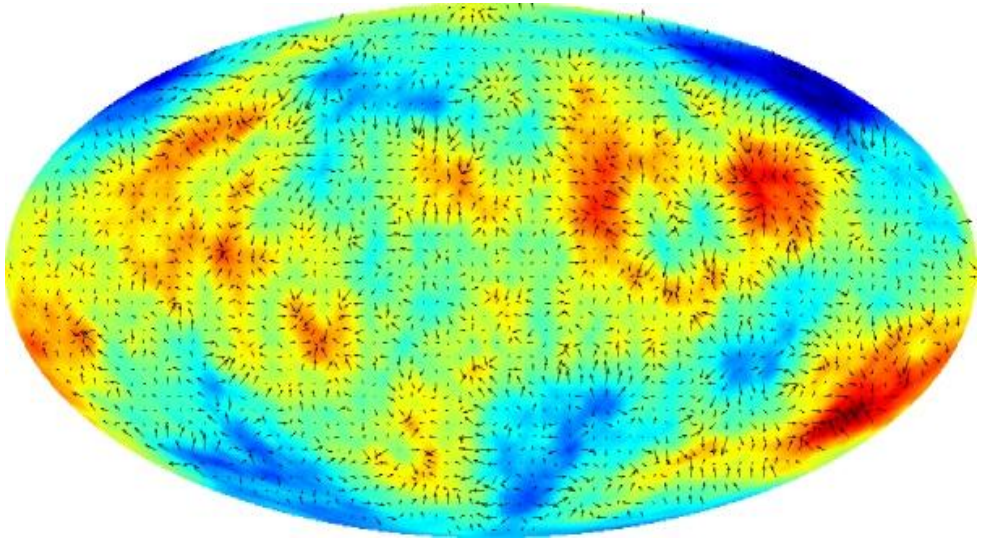
CMB beyond the power spectrum



University of Sussex

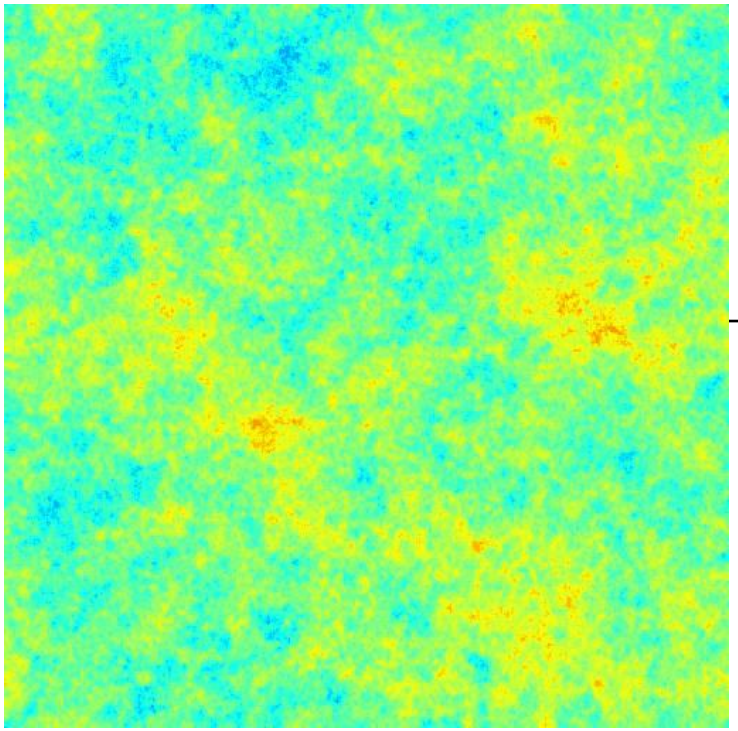
Antony Lewis

http://cosmologist.info/

Lewis *arXiv:1204.5018*
Lewis *arXiv:1107.5431*
Lewis, Challinor & Hanson *arXiv:1101.2234*
Pearson, Lewis & Regan *arXiv:1201.1010*
Planck Collaboration 2013

CMB temperature

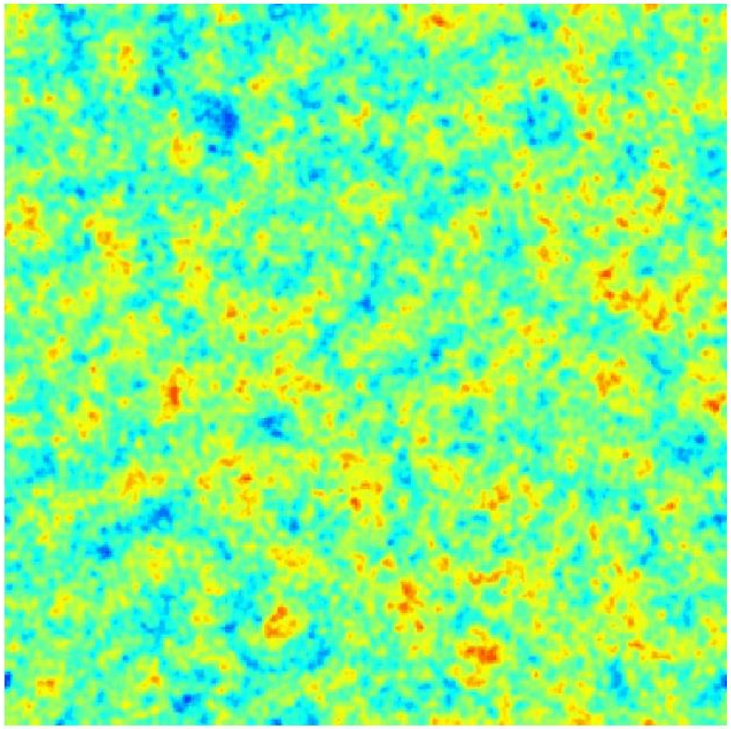
End of inflation



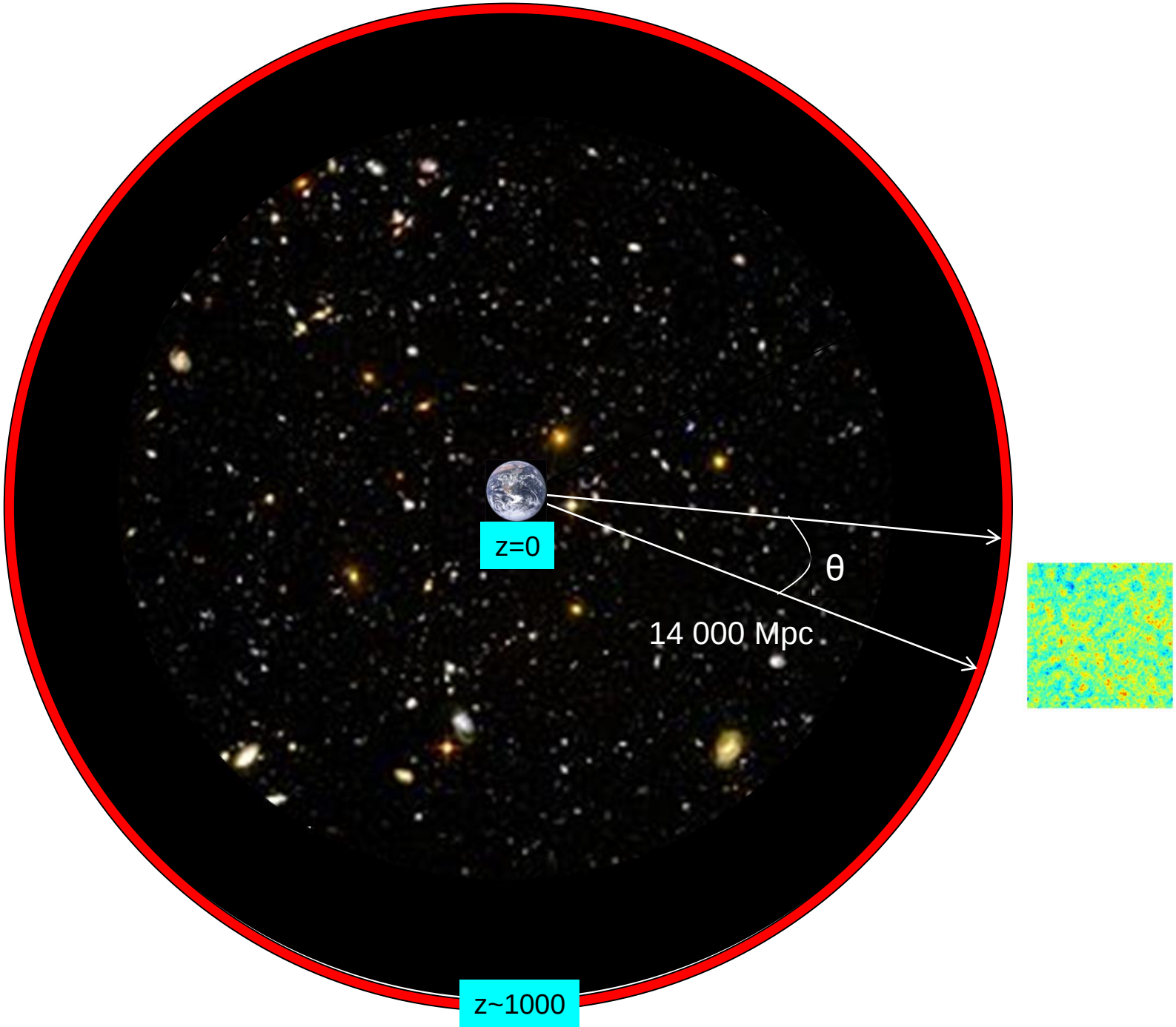
Perturbations super-horizon

gravity+
pressure+
diffusion

Last scattering surface

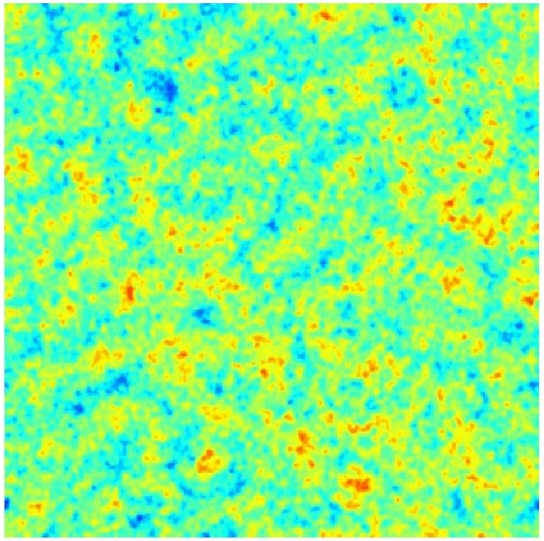


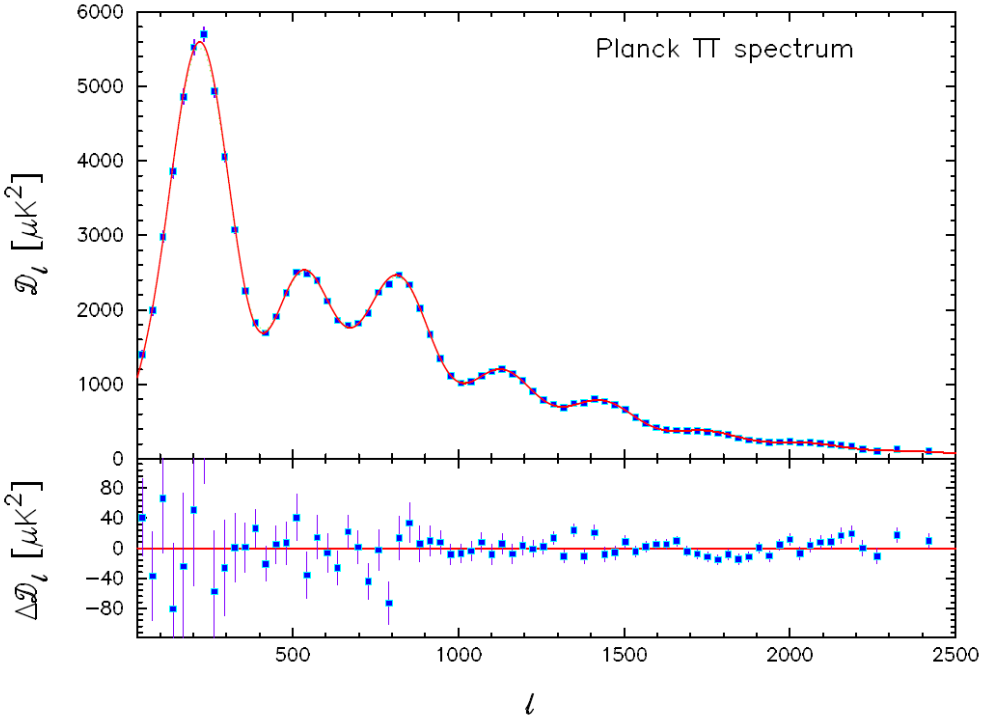
Sub-horizon acoustic oscillations
+ modes that are still super-horizon


z=0
θ
14 000 Mpc
z~1000

Observed CMB temperature power spectrum

Primordial perturbations + known physics with unknown parameters





Observations

Constrain theory of early universe
+ evolution parameters and geometry

Beyond Gaussianity – general possibilities

Flat sky approximation: $\Theta(x) = \frac{1}{2\pi}\int d^2 l\, \Theta(l) e^{ix\cdot l}$ $(\Theta = T)$

Gaussian + statistical isotropy

$$\langle \Theta(l_1)\Theta(l_2)\rangle = \delta(l_1 + l_2) C_l$$

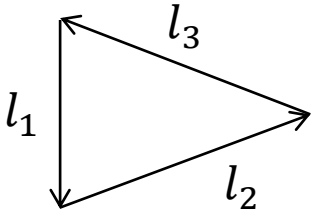
- power spectrum encodes all the information
- modes with different wavenumber are independent

Higher-point correlations

Gaussian: can be written in terms of C_l

Non-Gaussian: non-zero connected n-point functions

Bispectrum



$$\boldsymbol{l_1} + \boldsymbol{l_2} + \boldsymbol{l_3} = \mathbf{0}$$

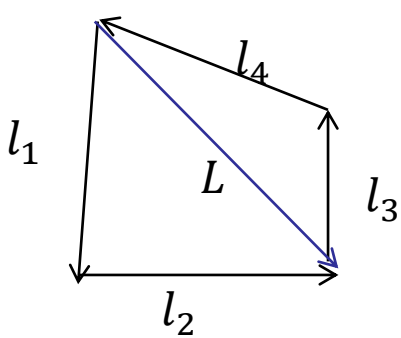
Flat sky approximation: $\langle \Theta(l_1)\Theta(l_2)\Theta(l_3)\rangle = \frac{1}{2\pi}\delta(l_1 + l_2 + l_3) b_{l_1 l_2 l_3}$

If you know $\Theta(l_1), \Theta(l_2)$, sign of $b_{l_1 l_2 l_3}$ tells you which sign of $\Theta(l_3)$ is more likely

Trispectrum

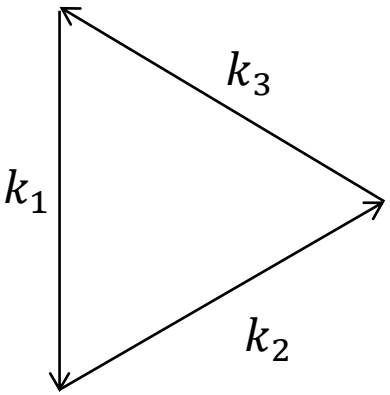
$$\langle \Theta(\mathrm{l}_1)\Theta(\mathrm{l}_2)\Theta(\mathrm{l}_3)\Theta(\mathrm{l}_4)\rangle_C = (2\pi)^{-2}\delta(\mathrm{l}_1 + \mathrm{l}_2 + \mathrm{l}_3 + \mathrm{l}_4) T(\mathrm{l}_1, \mathrm{l}_2, \mathrm{l}_3, \mathrm{l}_4)$$

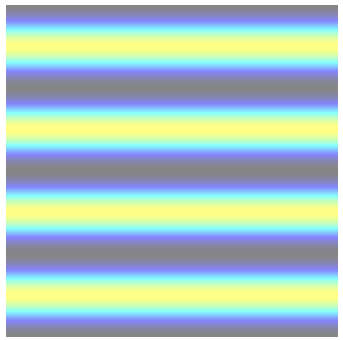
$$\langle \Theta(\mathrm{l}_1)\Theta(\mathrm{l}_2)\Theta(\mathrm{l}_3)\Theta(\mathrm{l}_4)\rangle_C = \frac{1}{2}\int \frac{d^2\mathbf{L}}{(2\pi)^2}\delta(\mathrm{l}_1 + \mathrm{l}_2 + \mathbf{L})\delta(\mathrm{l}_3 + \mathrm{l}_4 - \mathbf{L})\mathbb{T}^{(\ell_1\ell_2)}_{(\ell_3\ell_4)}(L) + \text{perms.}$$



N-spectra…

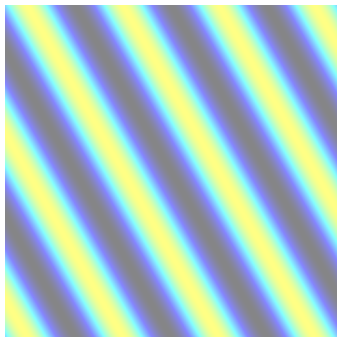
Equilateral $k_1 + k_2 + k_3 = 0, |k_1| = |k_2| = |k_3|$





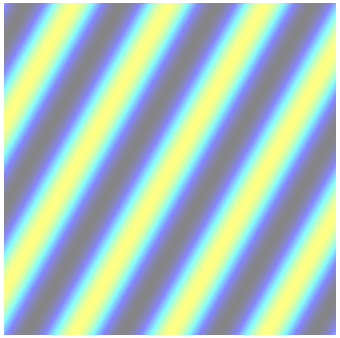
$T(k_1)$

+



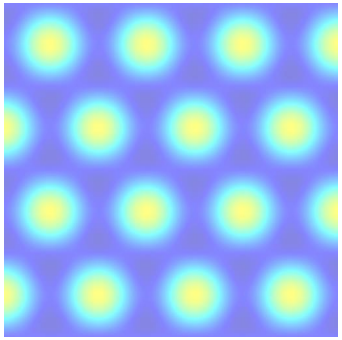
$T(k_2)$

+



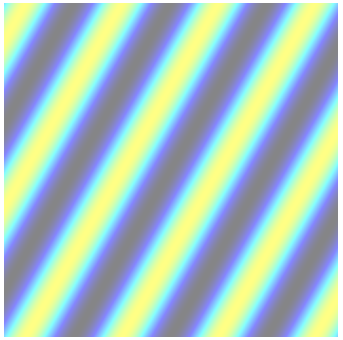
$T(k_3)$

=

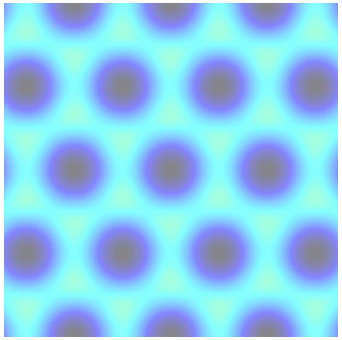


b>0

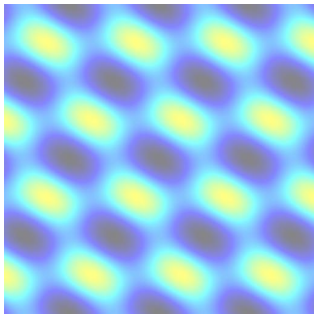
+

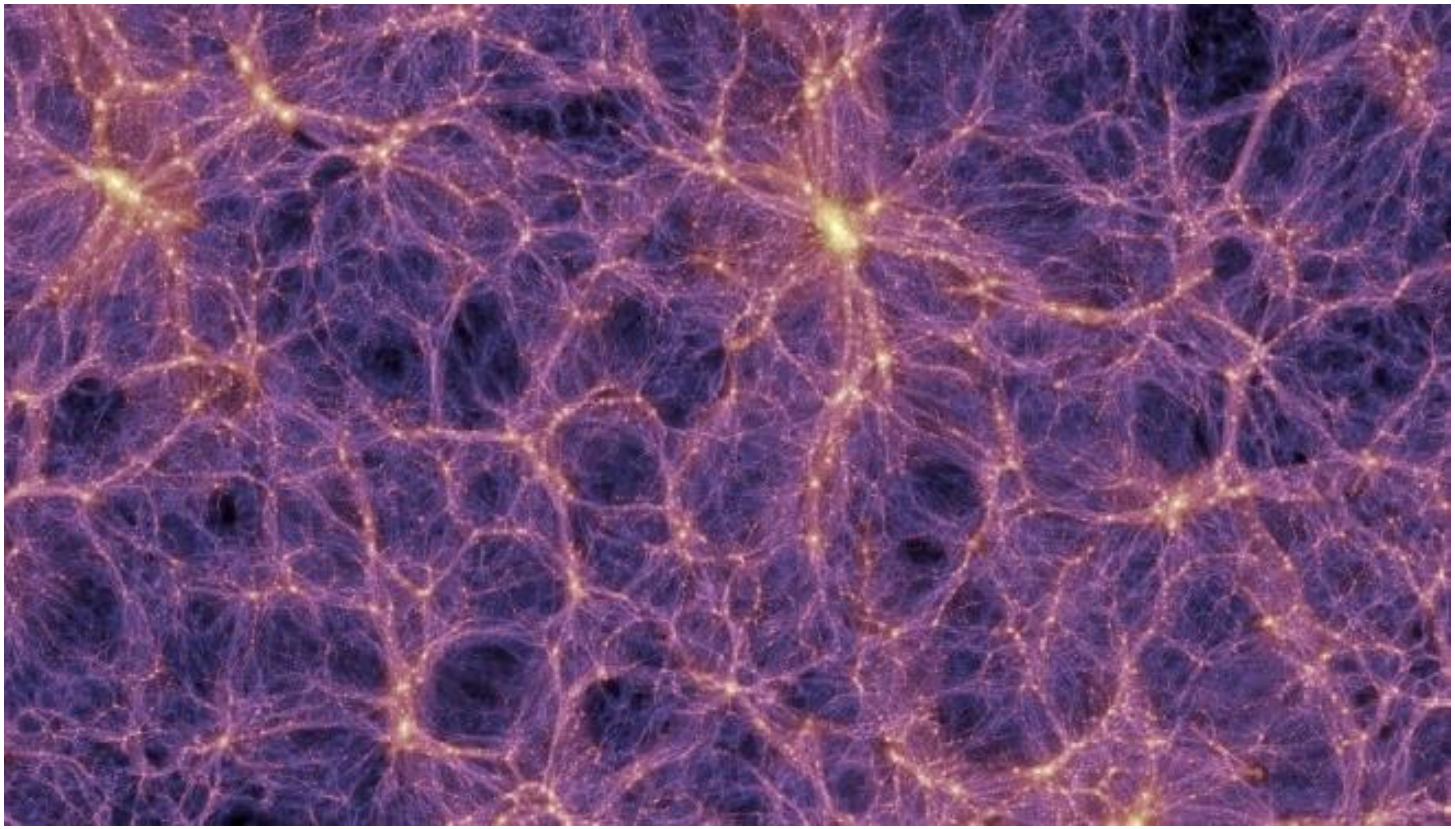


$-T(k_3)$



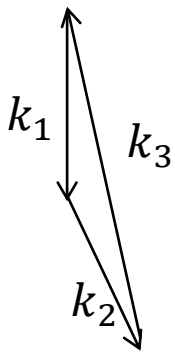
b<0





Millennium simulation

Near-equilateral to flattened:



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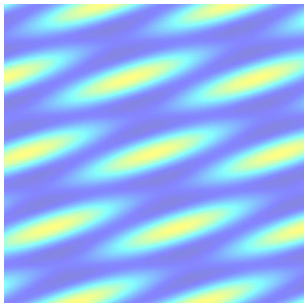
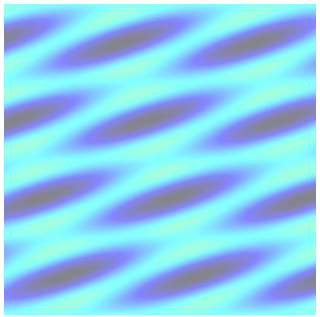
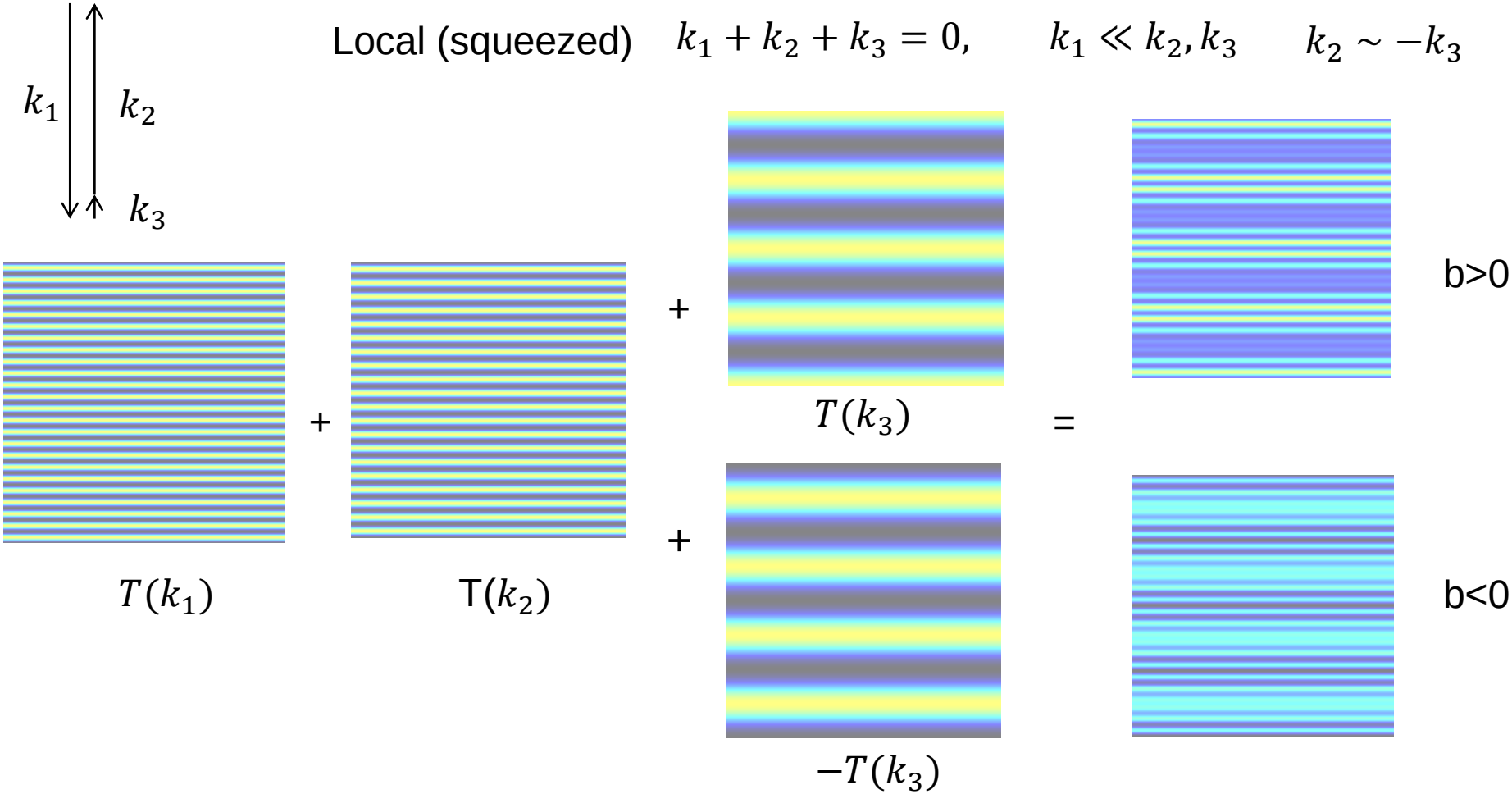


b<0



Local (squeezed) $k_1 + k_2 + k_3 = 0, \quad k_1 \ll k_2, k_3 \quad k_2 \sim -k_3$



Squeezed bispectrum is a *correlation* of small-scale power with large-scale modes

For more pretty pictures and trispectrum see: The Real Shape of Non-Gaussianities, arXiv:1107.5431

Calculating the squeezed bispectrum

'Linear-short leg' approximation very accurate for large scales where cosmic variance is large

$l_1 \ll l_2 \leq l_3$ with modulation field(s) X_i

$$\langle \tilde{T}_{l_1 m_1} \tilde{T}_{l_2 m_2} \tilde{T}_{l_3 m_3} \rangle \approx C_{l_1}^{T X_i} \left\langle \frac{\delta}{\delta X^*_{i,l_1 m_1}} \left(\tilde{T}_{l_2 m_2} \tilde{T}_{l_3 m_3} \right) \right\rangle$$

Correlation of the modulation with the large-scale field

Response of the small-scale power to changes in the modulation field (non perturbative)

Note: uses *only* the linear short leg approximation, otherwise non-perturbatively exact

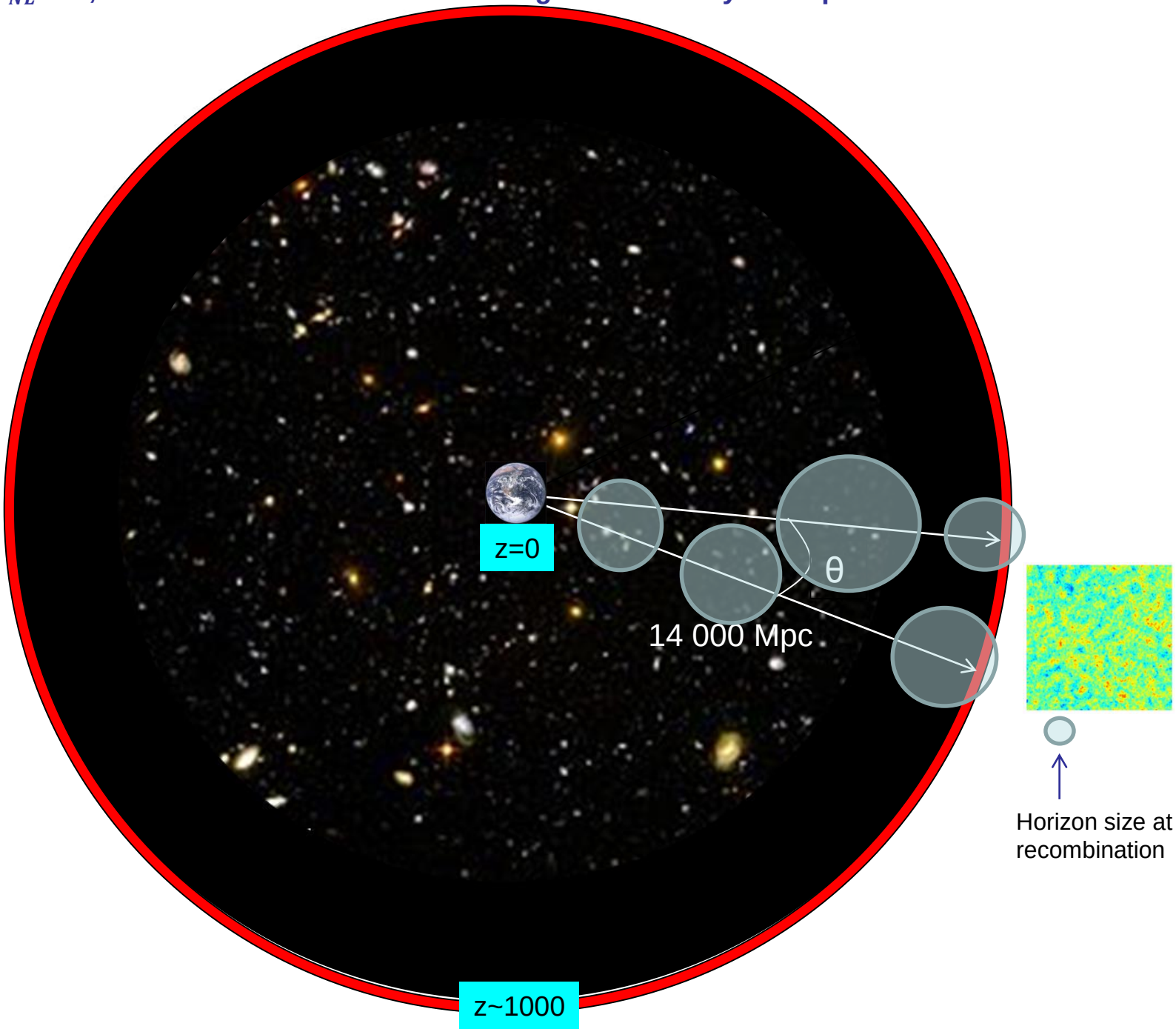
Example: local primordial non-Gaussianity

Primordial curvature perturbation is modulated as $\zeta = [1 + (3 f_{\rm NL}/5)\zeta_g]\zeta_g$

$l_1 \ll 100$: modulation super-horizon and constant through last-scattering, $\zeta_g = \zeta_0^*$

$$b_{l_1 l_2 l_3} \approx \frac{6}{5} f_{\rm NL} C_{l_1}^{T \zeta_0^*} (\tilde{C}_{l_2} + \tilde{C}_{l_3})$$

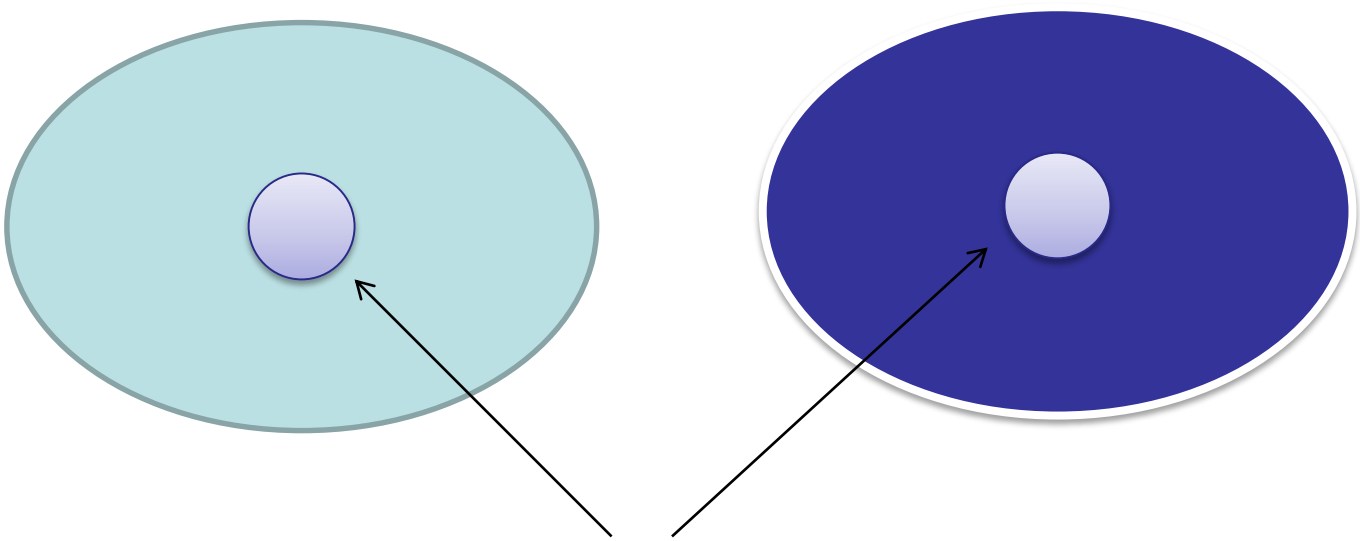
But even with $f_{NL} = 0$, we observe CMB at last scattering modulated by other perturbations



What is the modulating effect of large-scale super-horizon perturbations?

Single-field inflation: only one degree of freedom, e.g. everything determined by local temperature (density) on super-horizon scales

Cannot locally observe super-horizon perturbations (to $O(\frac{k^2}{H^2})$)



Observers in different places on LSS will see statistically exactly the same thing (at given fixed temperature/time from hot big bang)

- local physics is identical in Hubble patches that differ only by super-horizon modes

BUT: a distant observer *will* see modulations due to the large modes <~ horizon size today
- can see and compare multiple different Hubble patches

- Super-horizon modes induce linear perturbations on all scales

  linear CMB anisotropies on large scales ($l < 100$)

- Sub-horizon perturbations are observed in perturbed universe:
  -small-scale perturbations are modulated by the effect large-scale modes

  squeezed-shape non-Gaussianities

Linear CMB anisotropies

Linear perturbation theory with $ds^2 = a(\eta)^2 \left[(1+2\Psi)d\eta^2 - (1-2\Phi)d\mathbf{x}^2\right]$

Using the geodesic equation in the Conformal Newtonian Gauge:

$$E(\eta_0) = a(\eta)E(\eta)\left[1 + \Psi(\eta) - \Psi_0 + \int_\eta^{\eta_0} d\eta(\Psi' + \Phi')\right]$$

All photons redshift the same way, so $kT \sim E$.

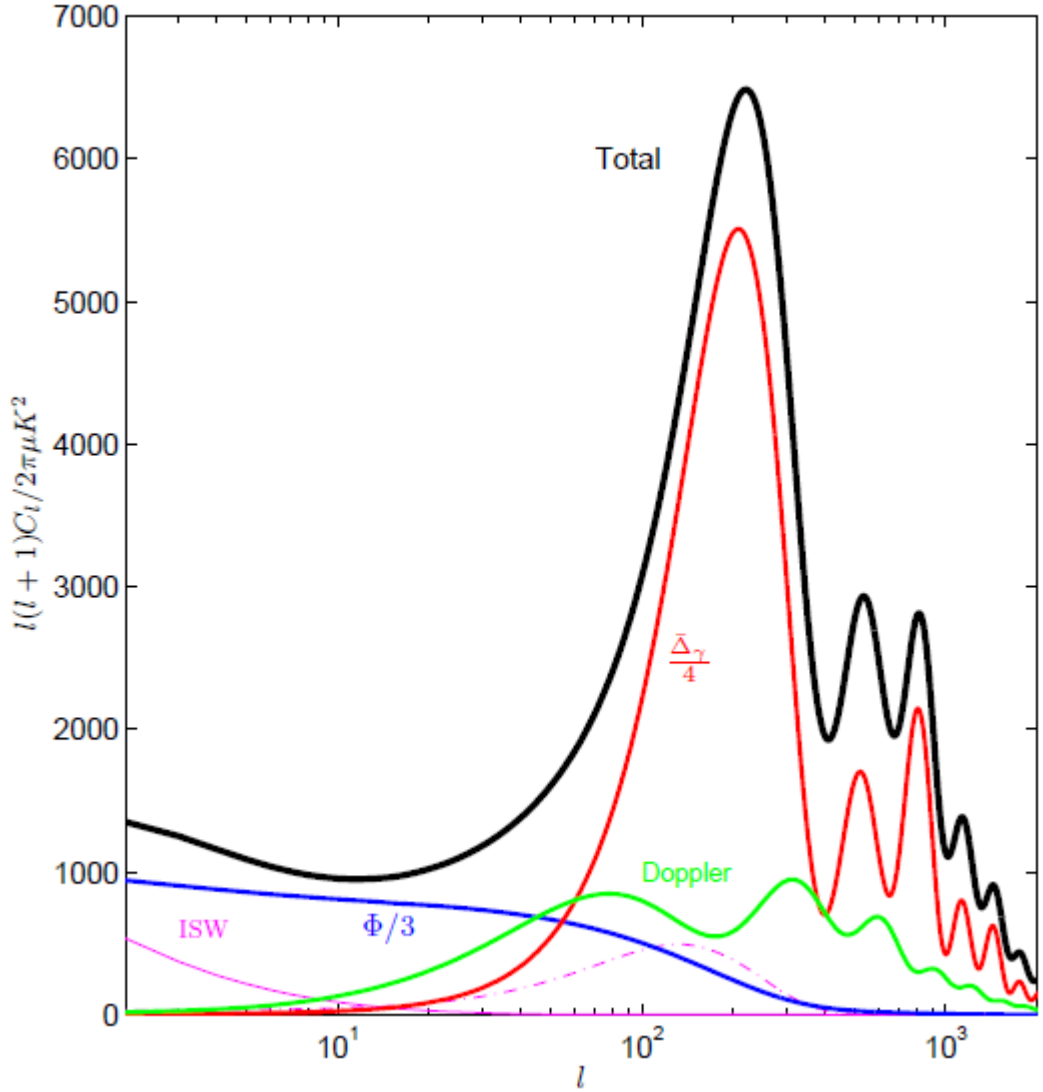
Recombination fairly sharp at background time η_*: ~ *constant temperature* surface.
Also add Doppler effect:

$$\begin{aligned} T(\hat{\mathbf{n}}, \eta_0) &= (a_* + \delta a)T_* \left[1 + \Psi(\eta_*) - \Psi_0 + \hat{\mathbf{n}} \cdot (\mathbf{v}_o - \mathbf{v}) + \int_{\eta_*}^{\eta_0} d\eta(\Psi' + \Phi')\right] \\ &= T_0 \left[1 + \frac{\delta a}{a_*} + \Psi(\eta_*) - \Psi_0 + \hat{\mathbf{n}} \cdot (\mathbf{v}_o - \mathbf{v}) + \int_{\eta_*}^{\eta_0} d\eta(\Psi' + \Phi')\right] \end{aligned}$$

$$\rho_\gamma \propto T^4 \propto a^4$$

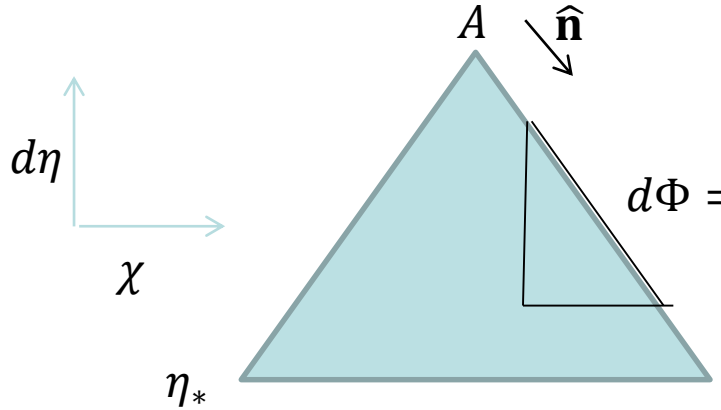
$$\Rightarrow \quad \frac{\Delta T_0}{T}(\hat{\mathbf{n}}) = \frac{\Delta_\gamma(\eta_*)}{4} + \underbrace{\Psi(\eta_*) - \Psi_0}_{\text{Sachs-Wolfe}} + \underbrace{\hat{\mathbf{n}} \cdot (\mathbf{v}_o - \mathbf{v})}_{\text{Doppler}} + \underbrace{\int_{\eta_*}^{\eta_0} d\eta(\Psi' + \Phi')}_{\text{ISW}}$$

Temperature perturbation at recombination



Note: no scale on which Sachs-Wolfe $\Phi/3$ result is accurate
Doppler dominates at $l \sim 60$ because other terms cancel

Alternative



$$d\Phi = \frac{\partial \Phi}{\partial \eta}\, d\eta + \frac{\partial \Phi}{\partial \chi}\, d\chi$$

$$a_A E_A = a(\eta)E(\eta)\left[1 + \Psi(\eta) - \Psi_A + \int_\eta^{\eta_A} \mathrm{d}\eta \partial_\eta(\Psi + \Phi)\right]$$

$$= a(\eta)E(\eta)\left[1 - \Phi(\eta) + \Phi_A + \int_\eta^{\eta_A} \mathrm{d}\eta \partial\chi(\Psi + \Phi)\right]$$

$$\Delta T(\hat{\mathbf{n}}) = \frac{\Delta_\gamma}{4} - \Phi + \hat{\mathbf{n}} \cdot (\mathbf{v}_A - \mathbf{v}) + \int_\eta^{\eta_A} \mathrm{d}\eta\, \hat{\mathbf{n}} \cdot \boldsymbol{\nabla}(\Psi + \Phi)$$

$$= \zeta_\gamma + \hat{\mathbf{n}} \cdot (\mathbf{v}_A - \mathbf{v}) + \int_\eta^{\eta_A} \mathrm{d}\eta\, \hat{\mathbf{n}} \cdot \boldsymbol{\nabla}(\Psi + \Phi)$$

Gauge-invariant 3-curvature on constant temperature hypersurfaces;
Redshifting from δN expansion of the beam makes the δN expansion from inflation observable
(but line of sight integral is larger on large-scales: overdensity looks colder)

Non-linear effect due to redshifting by large-scale modes?

Large-scale *linear* anisotropies are due to the *linear* anisotropic redshifting of the otherwise uniform (zero-order) temperature last scattering surface

$$T \to (1 + \Delta T)T$$

Also *non-linear* effect due to the *linear* anisotropic redshifting of the *linear* last scattering surface

$$\Delta T_{\text{small}} \to \left(1 + \Delta T_{\text{large}}\right)\Delta T_{\text{small}}$$

Reduced bispectrum $b_{l_1 l_2 l_3} \approx C_{l_1}\left(\tilde{C}_{l_2} + \tilde{C}_{l_3}\right)$

Large-scale power spectrum

Small-scale (non-perturbative) power spectrum

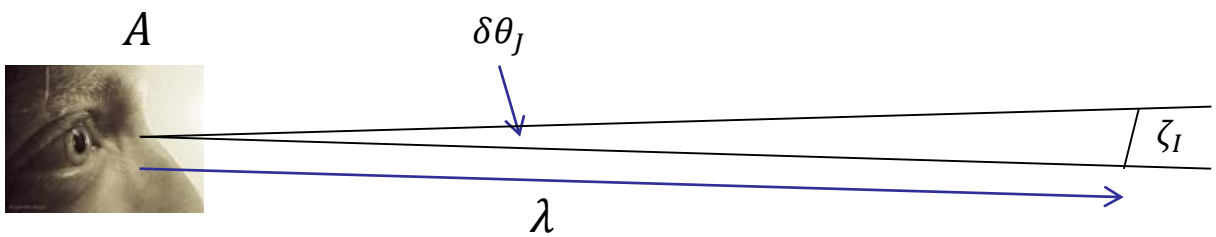
(Actually very small, so not very important)

Linear effects of large-scale modes

- Redshifting as photons travel through perturbed universe

- Transverse directions also affected:
  perturbations at last scattering are distorted as well as anisotropically redshifted

Jabobi map relates observed angle to physical separation of pair of rays



$$\xi_I(\lambda) = \mathcal{D}_{IJ}(\lambda)\delta\theta_J$$

Physical separation vector orthogonal to ray

Jacobi map

Angular separation seen by observer at A

Evolution of Jacobi map: $\dfrac{\mathrm{d}^2\mathcal{D}_{IJ}}{\mathrm{d}\lambda^2} = \mathcal{T}_{IK}\mathcal{D}_{KJ}$

Optical tidal matrix depends on the Riemann tensor: $\mathcal{T}_{IJ} \equiv -E_I^b E_J^c k^a k^d R_{abcd}$

(k^a is wave vector along ray, E_I^a projects into ray-orthogonal basis)

‘Riemann = Weyl + Ricci’

Non-local part (does not depend on local density):
- e.g. determined by Weyl (Newtonian) potential $\frac{1}{2}(\Phi + \Psi)$

differential deflection of light rays ⇒
convergence and shear of beam

- (Weyl) **lensing**

(can be modelled as transverse deflection angle)

Einstein equations relate Ricci to stress-energy tensor: depends on local density

⇒ ray area changes due to expansion of spacetime as the light propagates

- **Ricci focussing**

(cannot be modelled by deflection angle)

FRW background universe has Weyl=0, Ricci gives standard angular diameter distance

At radial distance χ_*, trace of Jacobi map determines physical areas: $\mathcal{D}/2 = \chi_* a_*$

Beam propagation in a perturbed universe, e.g. Conformal Newtonian Gauge

$$ds^2 = a^2(\eta)[(1+2\Psi)d\eta^2 - (1-2\Phi)\delta_{ij}dx^i dx^j]$$

$$\xi_I(\lambda) = \mathcal{D}_{IJ}(\lambda)\delta\theta_J$$

Trace-free part of Jacobi map depends on the shear: $\mathcal{D}_{\langle IJ\rangle} = a_* \chi_* \gamma_{IJ}$

$$\gamma_{IJ} = \nabla_{\langle I}\nabla_{J\rangle}\psi \qquad \psi \equiv -2\int_0^{\chi_*} d\chi \frac{\chi_* - \chi}{\chi\chi_*}\Psi_W(\chi\hat{\mathbf{n}}, \eta_A - \chi)$$

Area of beam determined by trace of Jacobi map:

$$\mathcal{D}(\hat{\mathbf{n}},\eta)/2 = \chi(\hat{\mathbf{n}},\eta)a(\eta)[1 + \Phi_A - \Phi - \kappa + \hat{\mathbf{n}}\cdot\mathbf{v}_A]$$

CMB is constant temperature surface:

$$\eta = \eta_* + \delta\eta$$

$$\rho_\gamma \propto T^4 \propto a^4$$

$$\mathcal{D}/2 = \chi_* a_* \left[1 + \frac{\delta\chi}{\chi_*} + \underbrace{\frac{\Delta_\gamma}{4} - \Phi}_{\zeta_\gamma \equiv \Delta_\gamma/4 - \Phi} - \kappa + \hat{\mathbf{n}}\cdot\mathbf{v}_A\right]$$

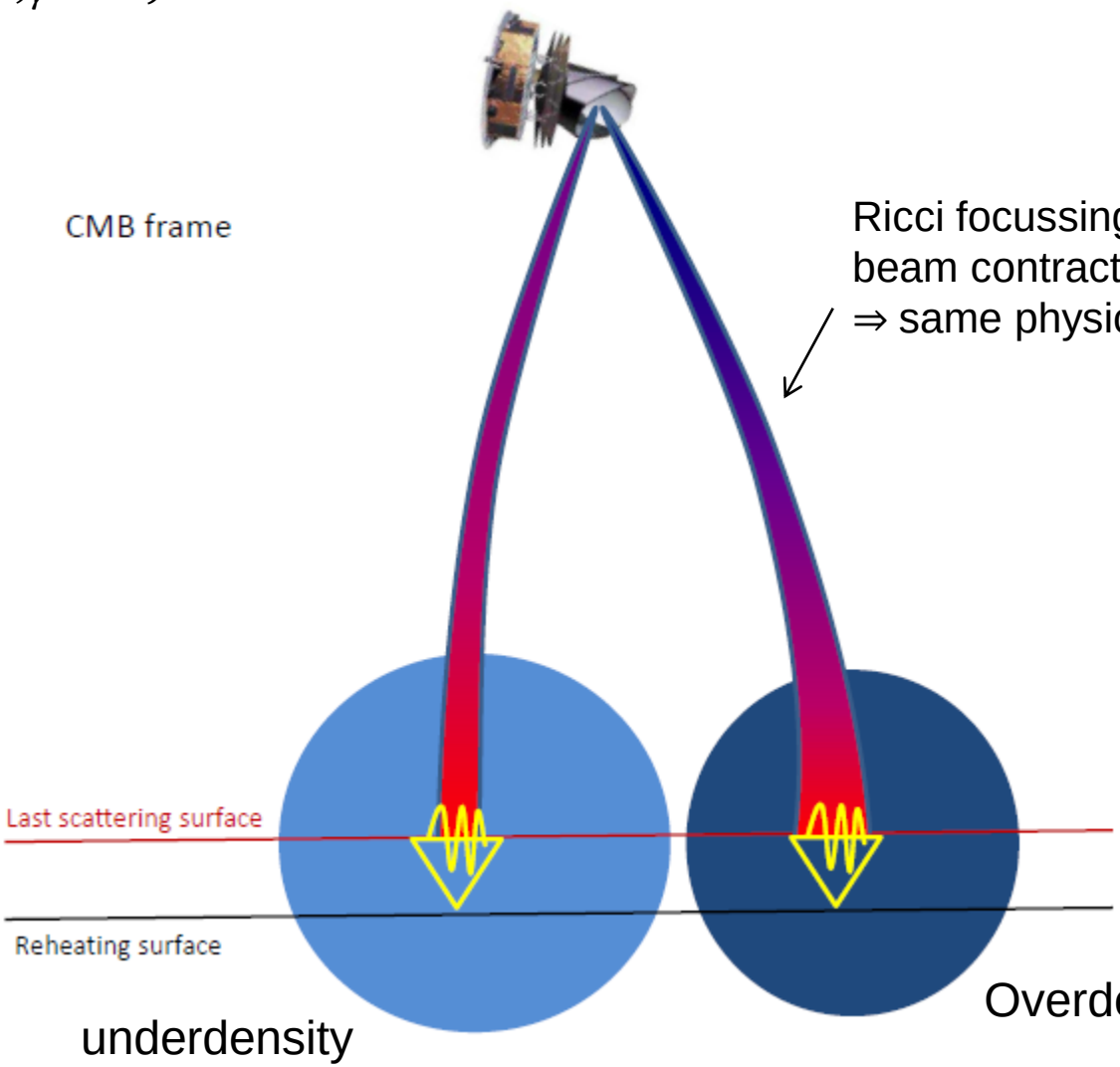
Radial displacement (small, $\delta\chi \ll \chi_*$)

Ricci focussing

(Weyl) convergence

Local aberration

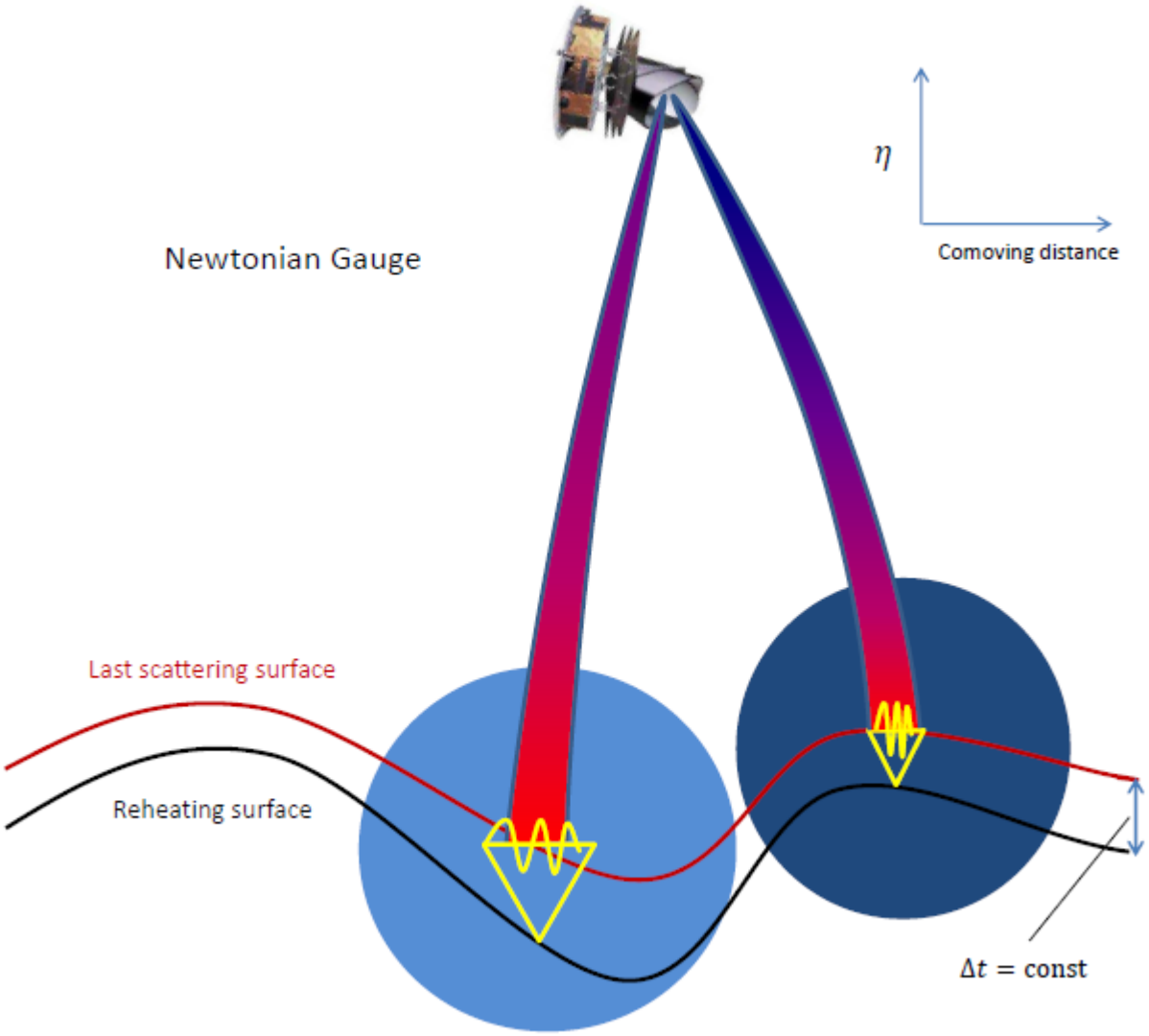
$$\frac{D}{2} \approx \chi_* a_* (1 + \zeta_\gamma - \kappa)$$



Ricci focussing:
beam contracts more leaving LSS
⇒ same physical size looks smaller

(Weyl lensing effect not shown and partly cancels area effect)

Gauge-invariant Ricci focussing $\zeta_\gamma \equiv \Delta_\gamma/4 - \Phi$



Observable CMB bispectrum from single-field inflation

Linear-short leg approximation for nearly-squeezed shapes:

$$\langle \tilde{T}_{l_1 m_1} \tilde{T}_{l_2 m_2} \tilde{T}_{l_3 m_3} \rangle \approx C_{l_1}^{T X_i} \left\langle \frac{\delta}{\delta X^*_{i, l_1 m_1}} \left(\tilde{T}_{l_2 m_2} \tilde{T}_{l_3 m_3} \right) \right\rangle$$

Where X_i here is δT, κ and ζ_γ, with $\frac{D}{2} \approx \chi_* a_* (1 + \zeta_\gamma - \kappa)$. For super-horizon adiabatic modes $\zeta_\gamma = \zeta_0$.

Weyl lensing bispectrum

$$b_{l_1 l_2 l_3} = \frac{1}{2} \left[(l_1(l_1+1) + l_2(l_2+1) - l_3(l_3+1) \right] C_{l_1}^{T\psi} \tilde{C}_{l_2} + \text{perms}$$

Squeezed limit ($l_1 \ll l$) $\quad b_{l_1 l_2 l_3} \approx C_{l_1}^{T\kappa} \left[\frac{1}{l^2} \frac{\mathrm{d}(l^2 \tilde{C}_l)}{d \ln l} + \cos 2\phi_{l_1 l} \frac{\mathrm{d}\tilde{C}_l}{\mathrm{d} \ln l} \right]$ $\qquad \mathrm{l} \equiv (\mathrm{l}_2 - \mathrm{l}_3)/2$

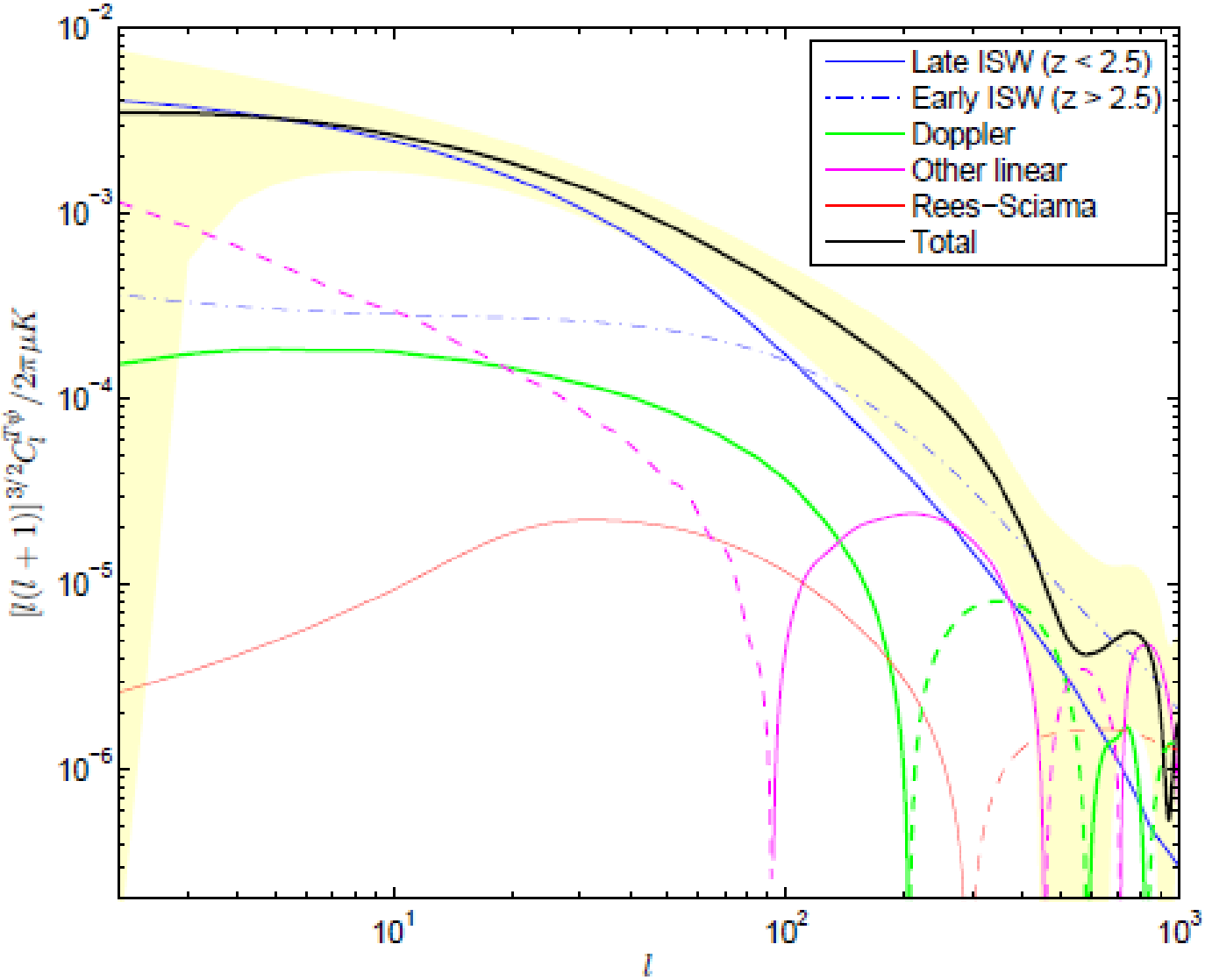
Ricci focussing bispectrum

$$b_{l_1 l_2 l_3} \approx C_{l_1}^{T\zeta_0^*} \frac{1}{2} \frac{\mathrm{d}}{\mathrm{d} \ln \chi_*} \left[\tilde{C}_{l_2} + \tilde{C}_{l_3} \right]$$

Squeezed limit ($l_1 \ll l$) $\quad b_{l_1 l_2 l_3} \approx -C_{l_1}^{T\zeta_0^*} \frac{1}{l^2} \frac{\mathrm{d}}{\mathrm{d} \ln l} (l^2 \tilde{C}_l)$

+ anisotropic redshifting bispectrum (from before) $\quad b_{l_1 l_2 l_3} \approx C_{l_1} \left(\tilde{C}_{l_2} + \tilde{C}_{l_3} \right)$

Weyl lensing depends on cross-correlation, $C_l^{T\psi}$



(note Rees-Sciama contribution is small, numerical problem with much larger result of Verde et al, Mangilli et al.; see also Junk et al. 2012 who agree with me)

Does this look like squeezed non-Gaussianity f_{NL} from multi-field inflation (local modulation of small scale perturbation amplitudes in each Hubble patch)?

forecast

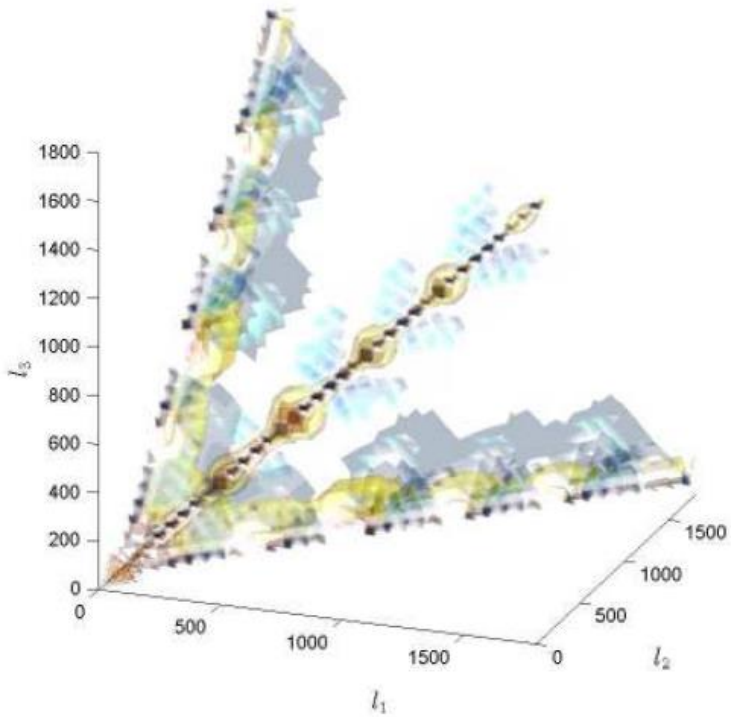
		bias on $f_{\rm NL}$			
Data used	$\sigma_{f_{\rm NL}}$	Weyl	Ricci	Redshift	Total
T	4.3	9.5	1.5	-0.22	10.7
Planck T	5.9	6.4	1.0	-0.22	7.1
T ($l_1 < 60$)	4.6	10.6	1.7	-0.25	12.0
Planck T ($l_1 < 60$)	6.2	7.0	1.1	-0.25	7.9
T+E	2.1	2.6	1.1	-0.05	3.7
Planck T+E	5.2	4.3	1.0	-0.15	5.2

TABLE I: Individual and total biases on primordial local-model non-Gaussianity parameterized by $f_{\rm NL}$ for CMB temperature and E-polarization data with Planck-like noise (assuming isotropic coverage over the full sky with sensitivity $\Delta T = \Delta Q/2 = \Delta U/2 = 50\,\mu$K arcmin $[N_l^T = N_l^E/4 = 2\times10^{-4}\mu\mathrm{K}^2]$ and a beam FWHM of 7 arcmin) or cosmic-variance limited data with $l_{\rm max} = 2000$. Results are assuming that non-$f_{\rm NL}$ contributions are only significant at $l_1 \leq 300$ and negligible dynamical effects; the $l_1 < 60$ results are filtered to only use large scale modulations and are therefore immune to small-scale modulation effects. The bias is the systematic error on $f_{\rm NL}$ if the given contribution is neglected, which can be compared to $\sigma_{f_{\rm NL}}$ which is the Fisher error estimate (including lensing signal variance).

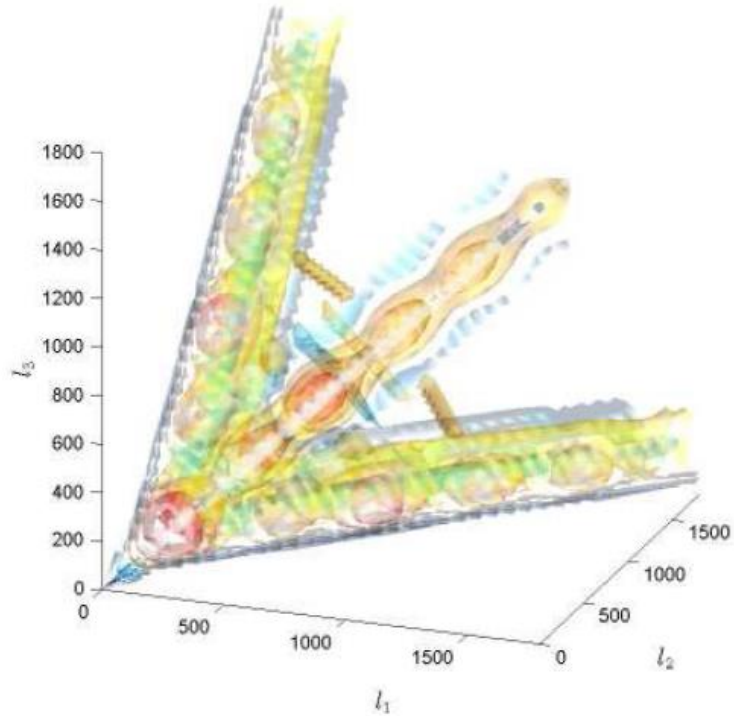
Dominated by lensing $f_{NL} \sim 6 - 10$
Ricci is an $O(1)$ correction

Calculation reliable for $l_1 < 60$ where dynamical effects suppressed by small $\frac{k^2}{H^2}$: do not need fully non-linear dynamical calculation of bispectrum a la Pitrou et al to make reliable f_{NL} constraint

Signal easily modelled
Squeezed shape but different phase, angle and scale dependence



Lensing



f_{NL}

Lewis, Challinor, Hanson 1101.2234

Note: ‘Maldacena’ bispectrum

$$\langle\tilde{\zeta}(\mathbf{k}_1)\tilde{\zeta}(\mathbf{k}_2)\tilde{\zeta}(\mathbf{k}_3)\rangle \approx -\frac{1}{(2\pi)^{3/2}}\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)P_{\zeta\zeta}(k_1)\frac{1}{2}\left[\frac{1}{k_3^3}\frac{\mathrm{d}}{\mathrm{d}\ln k_3}(k_3^3 P_{\zeta\zeta}(k_3)) + \frac{1}{k_2^3}\frac{\mathrm{d}}{\mathrm{d}\ln k_2}(k_2^3 P_{\zeta\zeta}(k_2))\right]$$

Consistency relation: $f_{NL} \sim O(n_s - 1)$

is *not* an observable

- cannot measure comoving curvature perturbations on scales larger than the horizon directly
- $d/d(\ln k)$ and CMB transfer functions do not commute: cannot get correct result from primordial $f_{NL} \sim (n_s - 1)$

Observable CMB analogue is Ricci focussing bispectrum

$$b_{l_1 l_2 l_3} \approx -C_{l_1}^{T\zeta_0^*}\frac{1}{l^2}\frac{\mathrm{d}}{\mathrm{d}\ln l}(l^2\tilde{C}_l)$$

- larger because of acoustic oscillations, non-zero for $n_s = 1$
- different shape to f_{NL} in CMB, but projects as $f_{NL} = O(1)$

Planck result

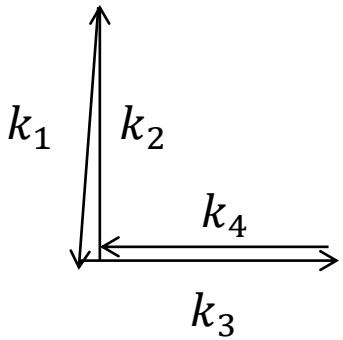
Table 2. Results for the amplitude of the ISW-lensing bispectrum from the SMICA, NILC, SEVEM, and C–R foreground-cleaned maps, for the KSW, binned, and modal (polynomial) estimators; error bars are 68% CL.

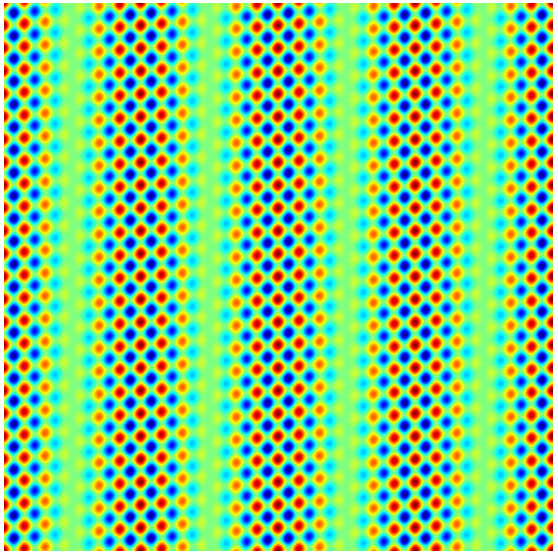
	SMICA	NILC	SEVEM	C–R
KSW	0.81 ± 0.31	0.85 ± 0.32	0.68 ± 0.32	0.75 ± 0.32
Binned	0.91 ± 0.37	1.03 ± 0.37	0.83 ± 0.39	0.80 ± 0.40
Modal	0.77 ± 0.37	0.93 ± 0.37	0.60 ± 0.37	0.68 ± 0.39

Table 8. Results for the f_{NL} parameters of the primordial local, equilateral, and orthogonal shapes, determined by the KSW estimator from the SMICA foreground-cleaned map. Both independent single-shape results and results marginalized over the point source bispectrum and with the ISW-lensing bias subtracted are reported; error bars are 68% CL.

	Independent KSW	**ISW-lensing subtracted** KSW
SMICA		
Local	9.8 ± 5.8	**2.7 ± 5.8**
Equilateral	−37 ± 75	**−42 ± 75**
Orthogonal	−46 ± 39	**−25 ± 39**

Diagonal squeezed trispectra $|k_1| \sim |k_2|, |k_3| \sim |k_4|, |k_1 + k_2| = |k_3 + k_4| \ll |k_2|, |k_3|$





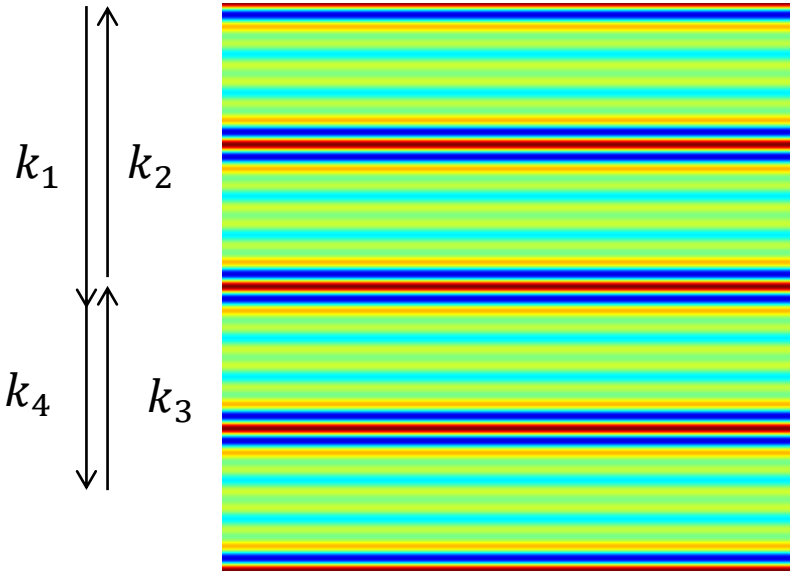
Trispectrum = power spectrum of modulation

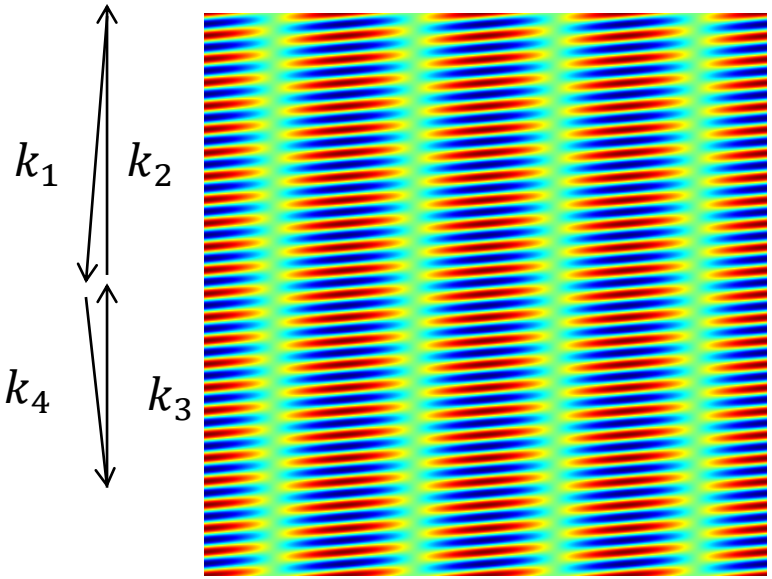
e.g. $\chi = \chi_0 \left(1 + f_{NL} \chi_0\right)$

$\tau_{NL} \sim f_{NL}^2$

or $\chi = \chi_0 (1 + \phi)$

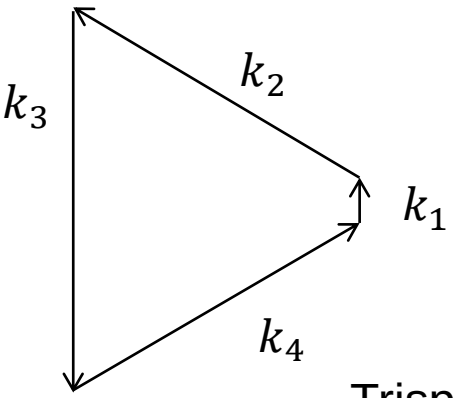
(any correlation, $\tau_{NL} > f_{NL}^2$)





One-leg squeezed trispectra

$|k_1| \ll |k_2| \sim |k_3| \sim |k_4|$

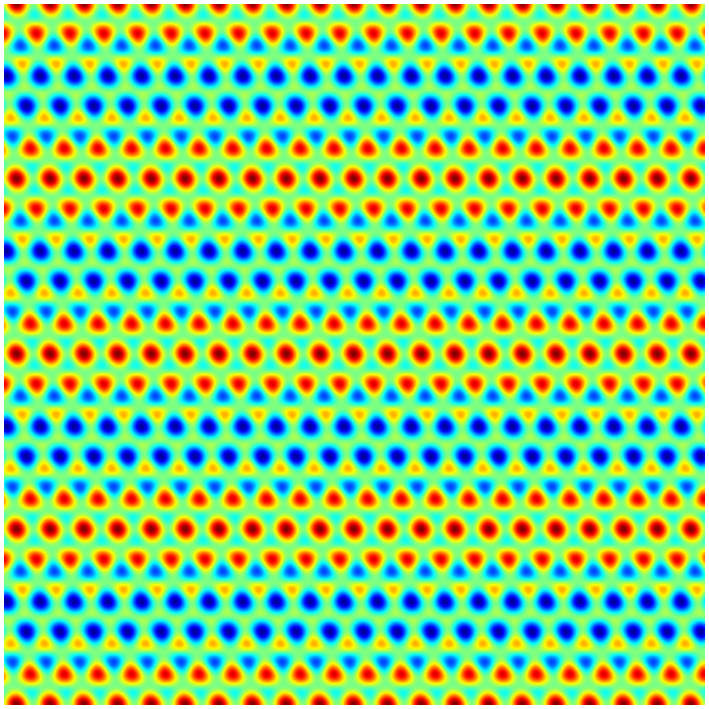


Correlated modulation of equilateral bispectrum

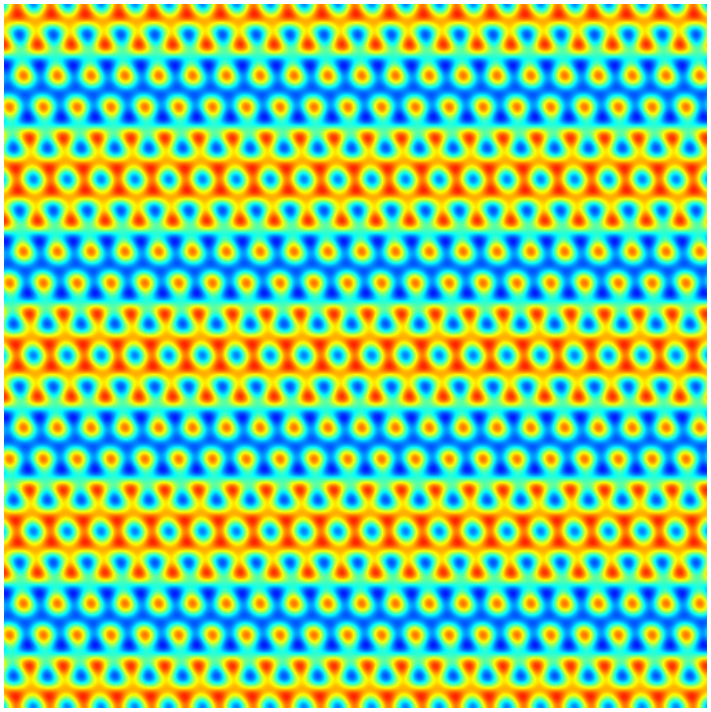
e.g. from $\chi = \chi_0(1 + g_{NL}\chi_0^2)$ [***no*** diagonal squeezed]

Trispectrum>0

also from $\chi = \chi_0(1 + f_{NL}\chi_0)$ [***plus*** diagonal squeezed]



Trispectrum<0



Reconstructing the modulation field

Marginalized over modulation field:

$$T \sim \int P(T,\psi)d\psi$$

- Non-Gaussian statistically isotropic temperature distribution

- Bispectrum + significant connected 4-point function

For a given modulation field：

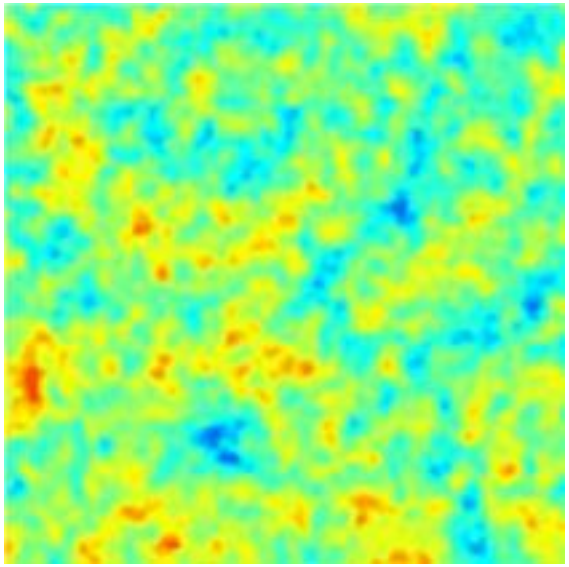
$$T \sim P(T|\psi)$$

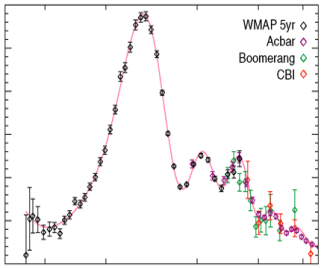
- Anisotropic Gaussian temperature distribution

- Re-construct the modulation field

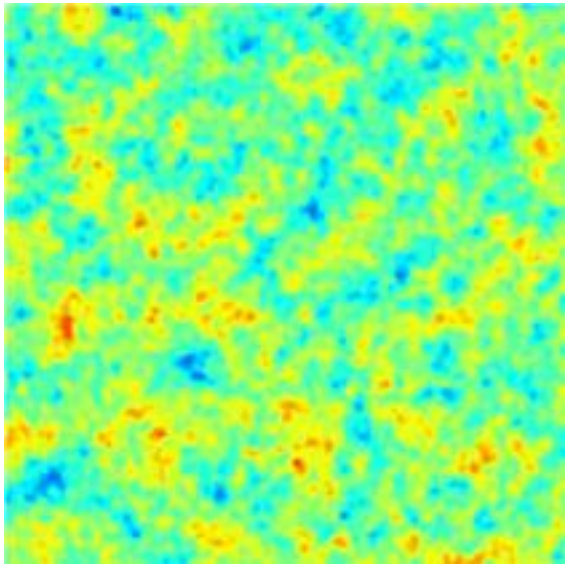
Anisotropy estimators – e.g. reconstruct the modulating lensing field

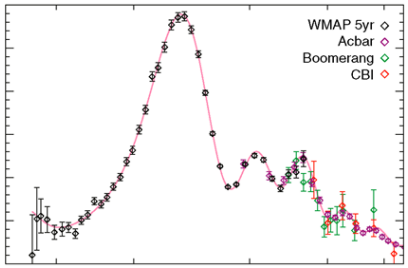
Magnified



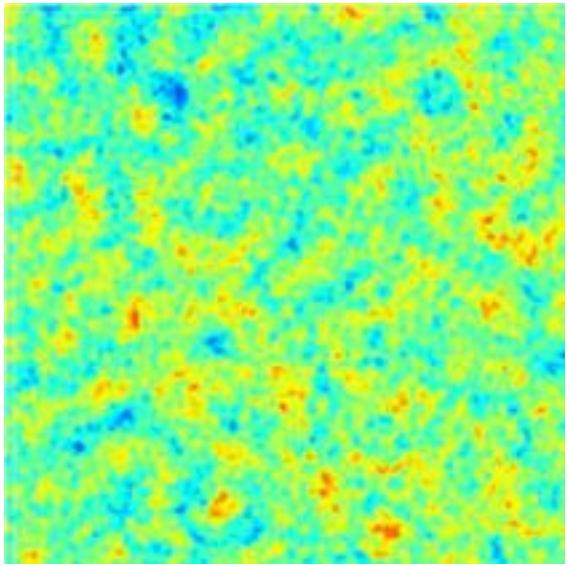


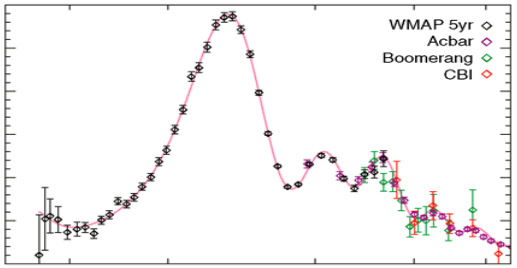
Unlensed





Demagnified





Reconstructing the modulation field

For a *given* (fixed) modulation field:

$$T \sim P(T|X)$$

- Anisotropic Gaussian temperature distribution

- Modes correlated for $\mathbf{k}_2 \neq \mathbf{k}_3$

$$\langle \tilde{T}(\mathbf{k}_2)\tilde{T}(\mathbf{k}_3)\rangle_{P(\tilde{T}|X)} \approx \int \mathrm{d}\mathbf{K} X(\mathbf{K})^* \underbrace{\left\langle \frac{\delta}{\delta X(\mathbf{K})^*}\left(\tilde{T}(\mathbf{k}_2)\tilde{T}(\mathbf{k}_3)\right)\right\rangle}_{\mathcal{A}(K,k_2,k_3)\delta(K+k_2+k_3)} \approx \mathcal{A}(K,k_2,k_3)\; X(\mathbf{K})^*|_{\mathbf{K}=-\mathbf{k}_2-\mathbf{k}_3}$$

Model-dependent function you can calculate for $X(\mathbf{K}) = 0$

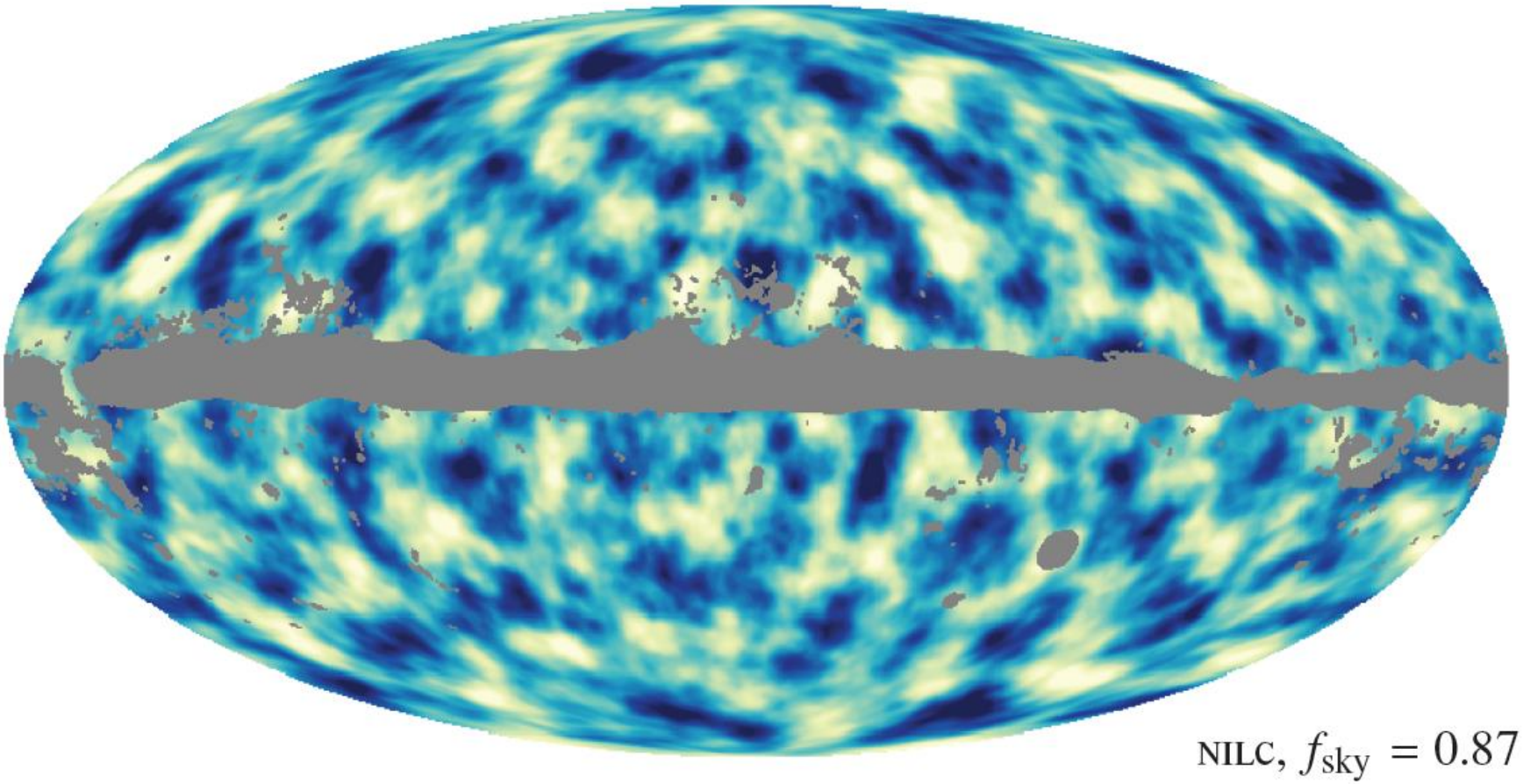
Can reconstruct the modulation field X!

- For small modulations can construct "optimal" QML estimator $\hat{X}(K)$ by summing filtered fields appropriately over k_2, k_3

$$\hat{X}(K) \sim N\left(\sum_{\mathbf{k}_2,\mathbf{k}_3} A(K,k_2,k_3)\, \bar{T}(\mathbf{k}_2)\bar{T}(\mathbf{k}_3) - \text{mean field}\right)$$

Zaldarriaga, Hu, Hanson, etc..

Planck lensing potential reconstruction (north and south galactic)



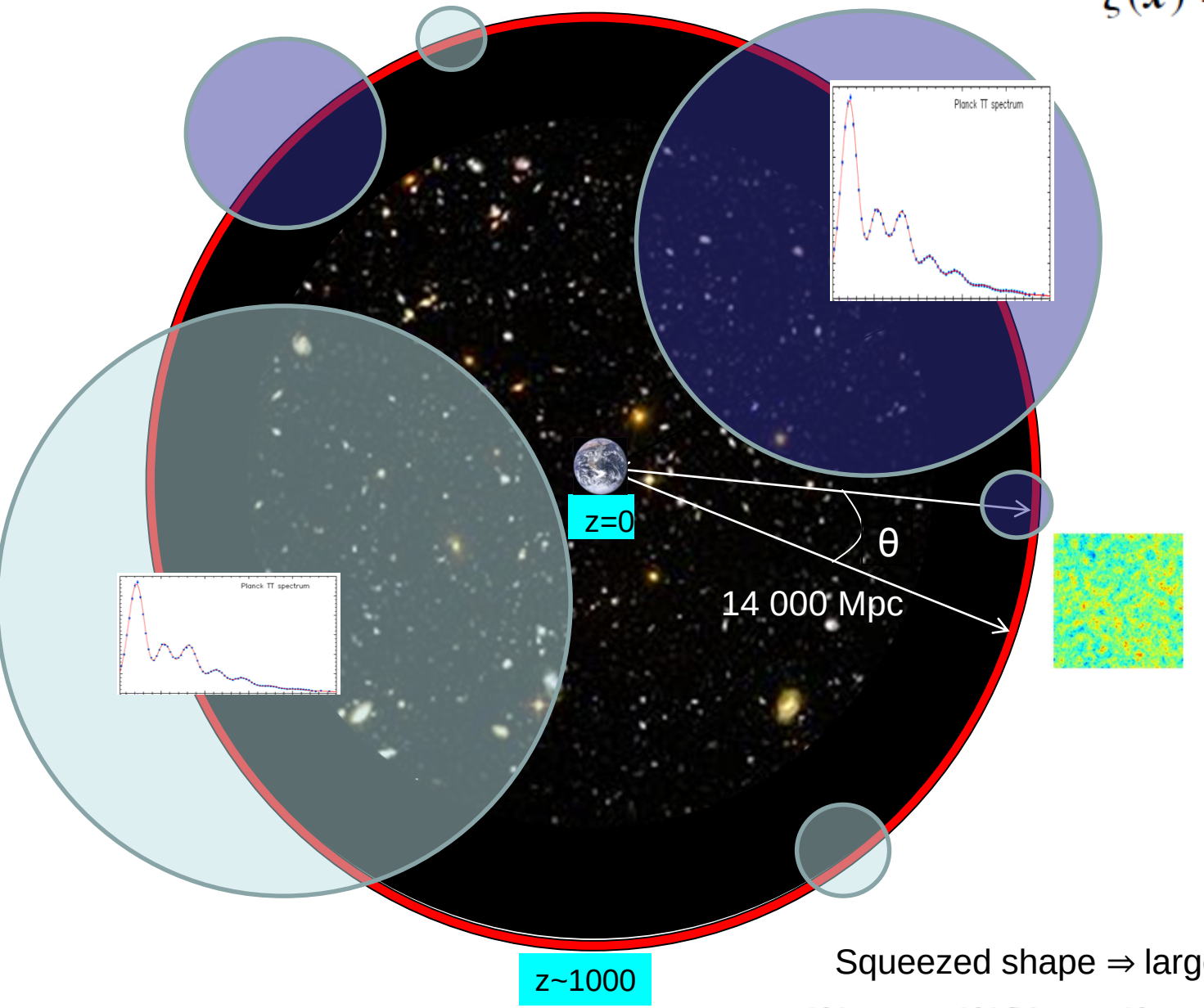
Note – about half signal, half noise, not all structures are real
map is effectively Wiener filtered

Correlation with T ⇒ Bispectrum; Power spectrum ⇒ Trispectrum

Primordial modulations?

Primordial curvature modulation:

$$\zeta(\boldsymbol{x}) = \zeta_0(\boldsymbol{x})[1 + \phi(\boldsymbol{x})]$$



Squeezed shape ⇒ large-scale modulation

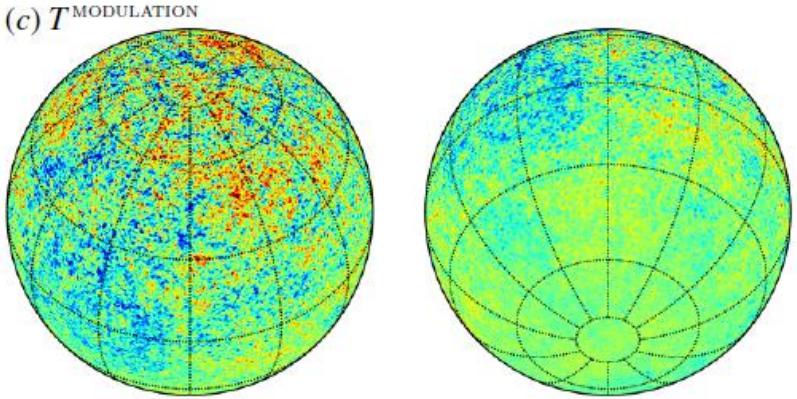
$$T(\hat{\boldsymbol{n}}) \approx T_{\rm g}(\hat{\boldsymbol{n}})[1 + \phi(\hat{\boldsymbol{n}}, r_*)] \equiv T_{\rm g}(\hat{\boldsymbol{n}})[1 + f(\hat{\boldsymbol{n}})]$$

Complication: Kinematic dipole modulation signal

Modulation

$$\Delta\Theta(\hat{n}) \rightarrow \left[1 + \hat{n}\cdot\boldsymbol{v} + T\frac{\mathrm{d}^2 I_\nu/\mathrm{d}T^2}{\mathrm{d}I_\nu/\mathrm{d}T}\hat{n}\cdot\boldsymbol{v}\right]\Delta\Theta(\hat{n})$$
$$= (1 + [x\coth(x/2) - 1]\,\hat{n}\cdot\boldsymbol{v})\,\Delta\Theta(\hat{n}),$$

$$x \equiv h\nu/k_b T$$

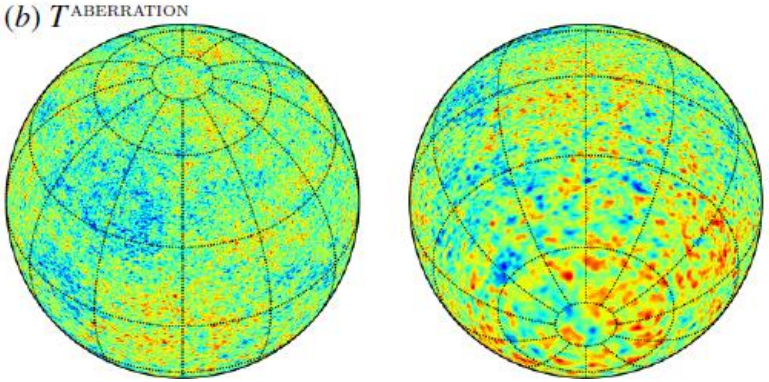


Illustrated for $\frac{v}{c} = 0.85$

Aberration

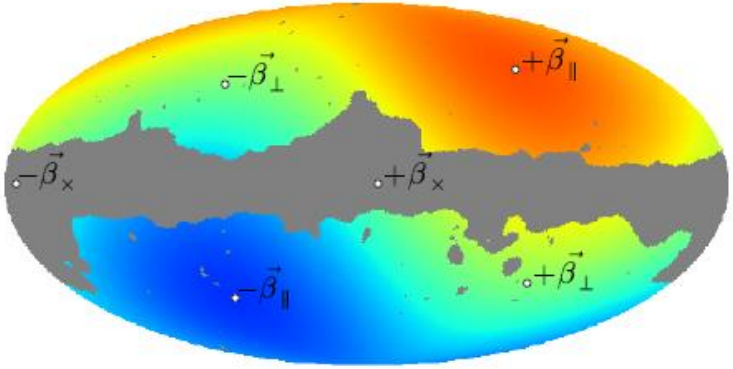
$$\hat{\boldsymbol{n}} \rightarrow \hat{\boldsymbol{n}} \quad + \nabla(\hat{\boldsymbol{n}}\cdot\boldsymbol{v})$$

- just like a dipole lensing convergence



Challinor & van Leeuwen 2002

known dipole amplitude and direction



$$\frac{v}{c} \approx 1.23 \times 10^{-3}$$

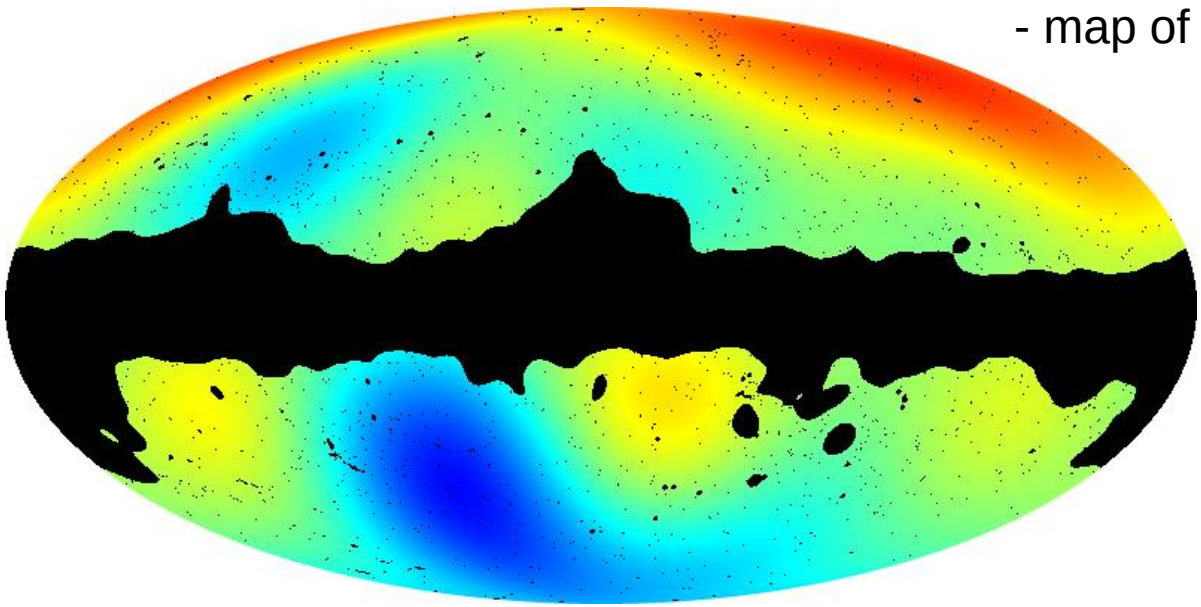
Modulation $f = \left(x \coth \frac{\left(\frac{x}{2}\right)}{2} - 1\right) \hat{\boldsymbol{n}} \cdot \boldsymbol{v} \equiv b_\nu \, \hat{\boldsymbol{n}} \cdot \boldsymbol{v}$

		Approx boost factor $b_\nu = x \coth\left(\frac{x}{2}\right)/2 - 1$	Map modulation amplitude
Planck maps:	100 GHz	1.5	0.18%
	143 GHz	2	0.24%
	217 GHz	3	0.37%
	353 GHz	5	0.64%
	545 GHz	9	1.1%
	857 GHz	14	1.7%

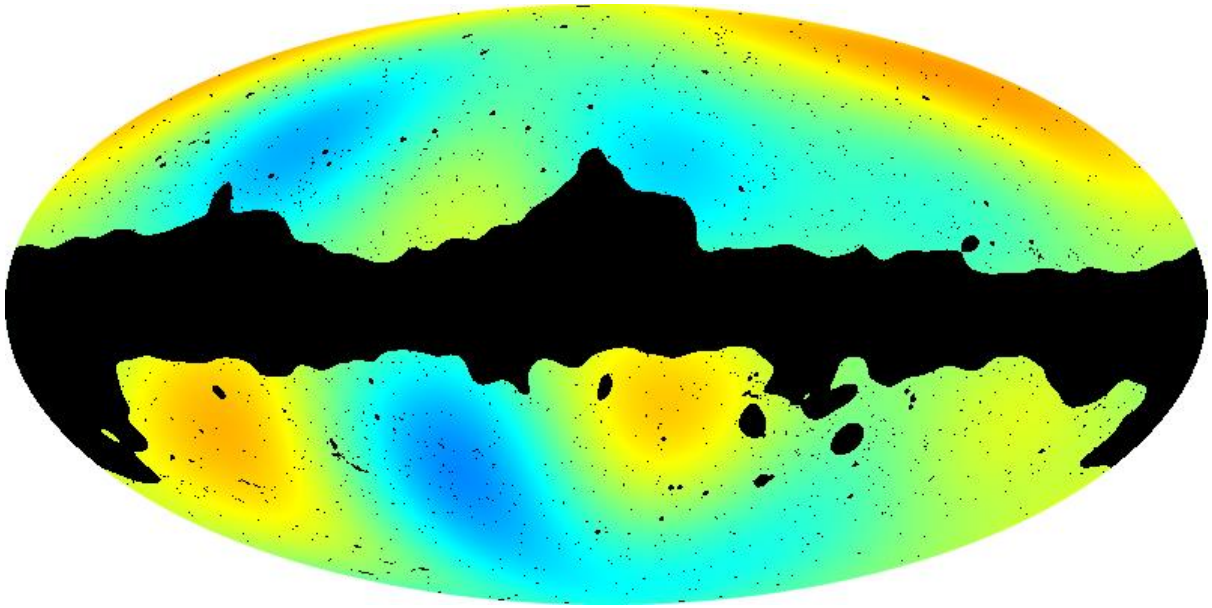
Use 143, 217 only (with dust subtraction from 857)

Note: SMICA maps are a complicated mixture; modulation effect not currently included in FFP6 sims

143x217 modulation reconstruction ($L \leq 5$)
- map of estimated modulation field f

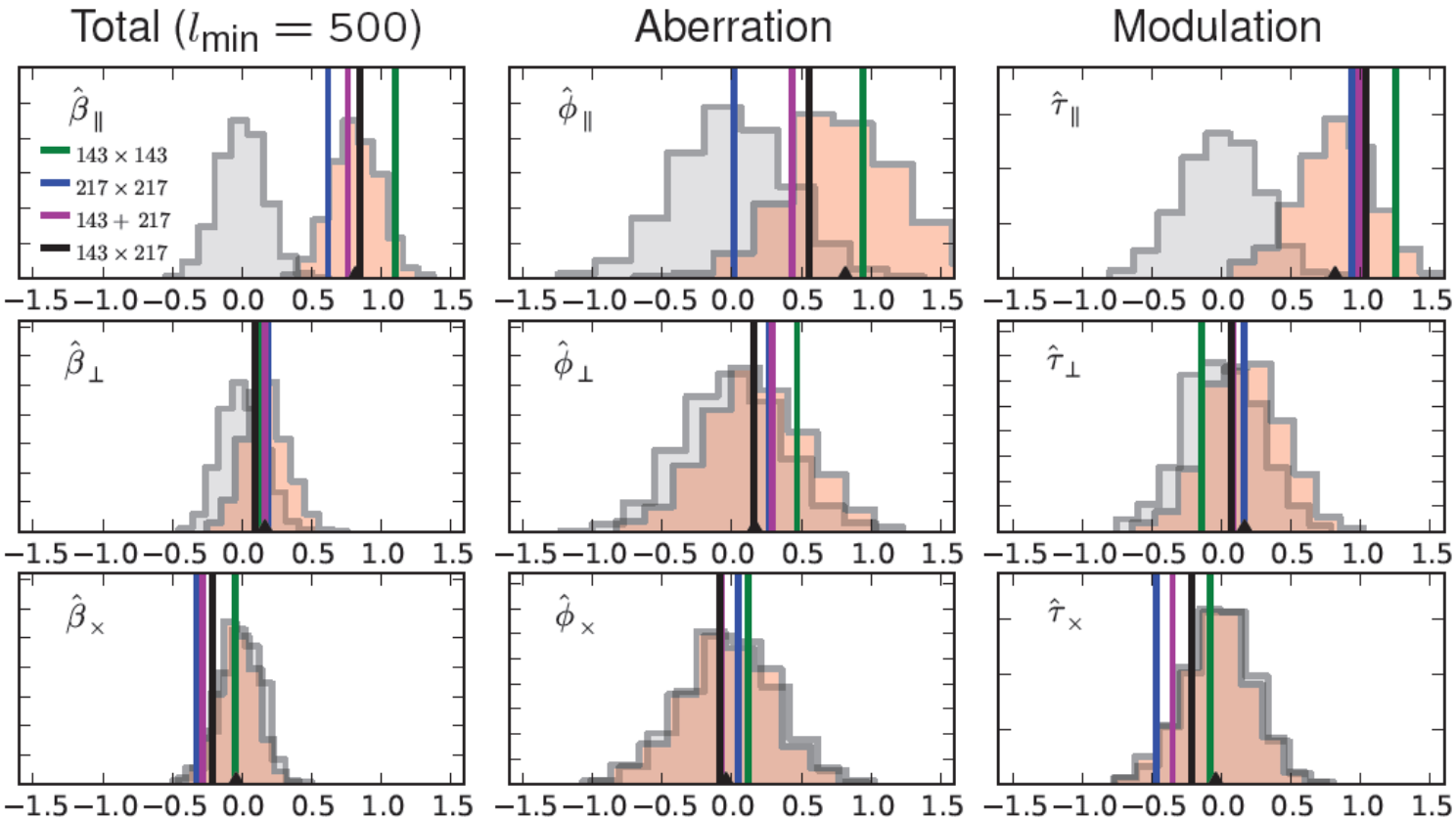


Kinematics not subtracted



Kinematics subtracted
in mean field from sims

Dipole kinematic effect using appropriate quadratic estimators



Simulations without velocity effects (143 × 217)
Simulations with velocity effects

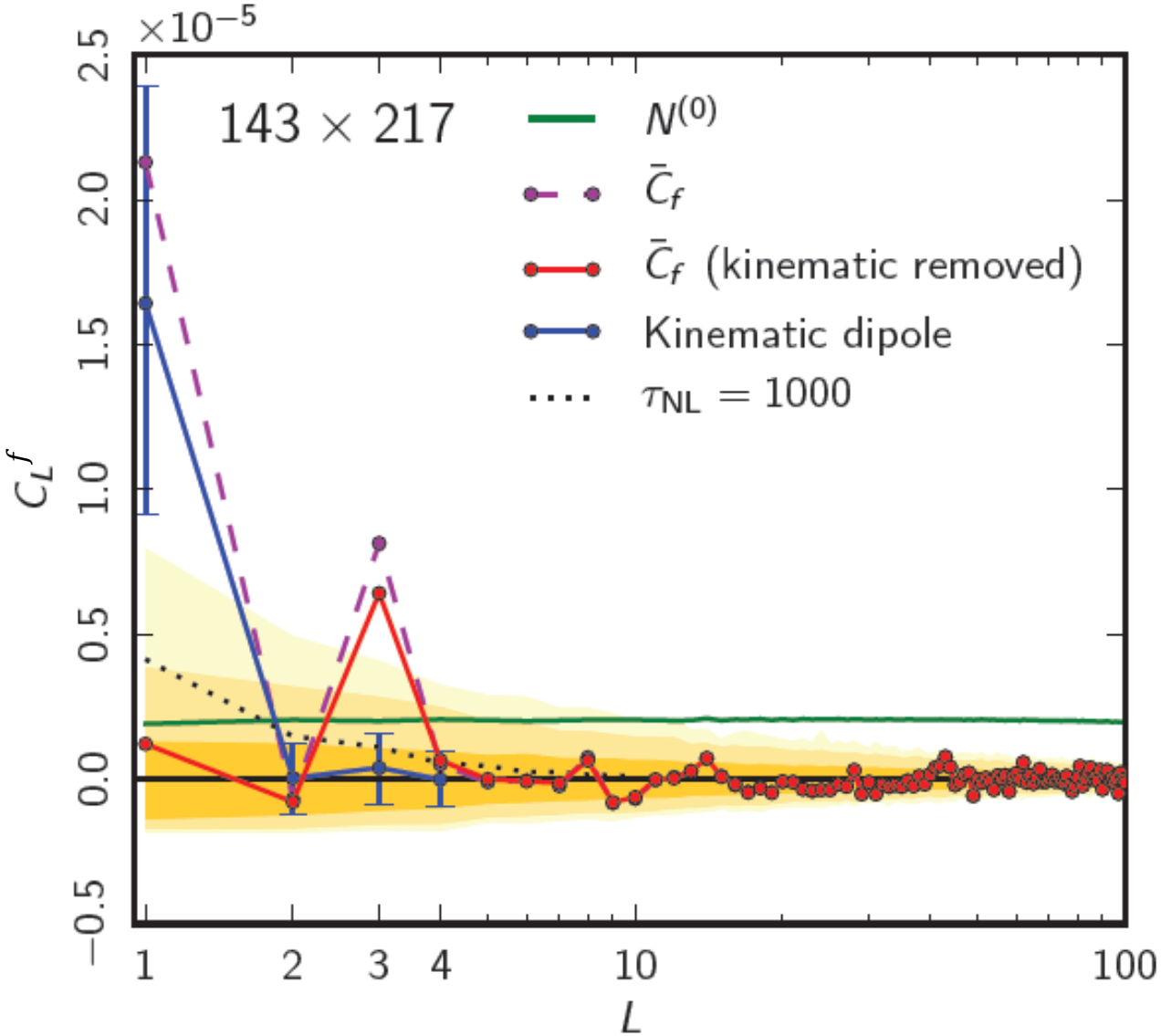
- 5σ detection in 143×217: $v_\parallel = (384 \pm 78)\,\mathrm{km\,s^{-1}}$
- Foreground issue at 217×217 in $\widehat{\beta}_\times$ (driven by $\widehat{\tau}_\times$)?

Note: not included in parameter analysis $\theta_* = (1.04148 \pm 0.00066) \times 10^{-2} = 0.596724^\circ \pm 0.00038^\circ$

- bias due to aberration average over mask $\sim 0.25\sigma$

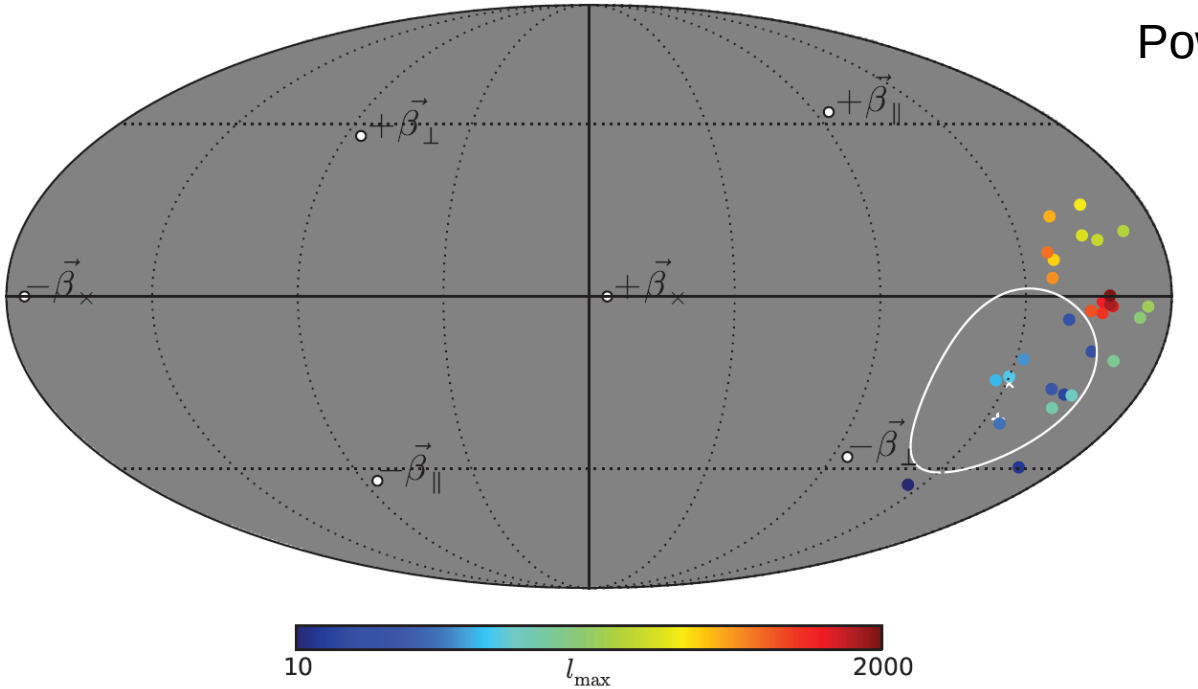
Modulation pseudo-power spectrum

$$\tau_{\rm NL}(L) \equiv \frac{C_L^f}{C_L^{\zeta_\star}}$$

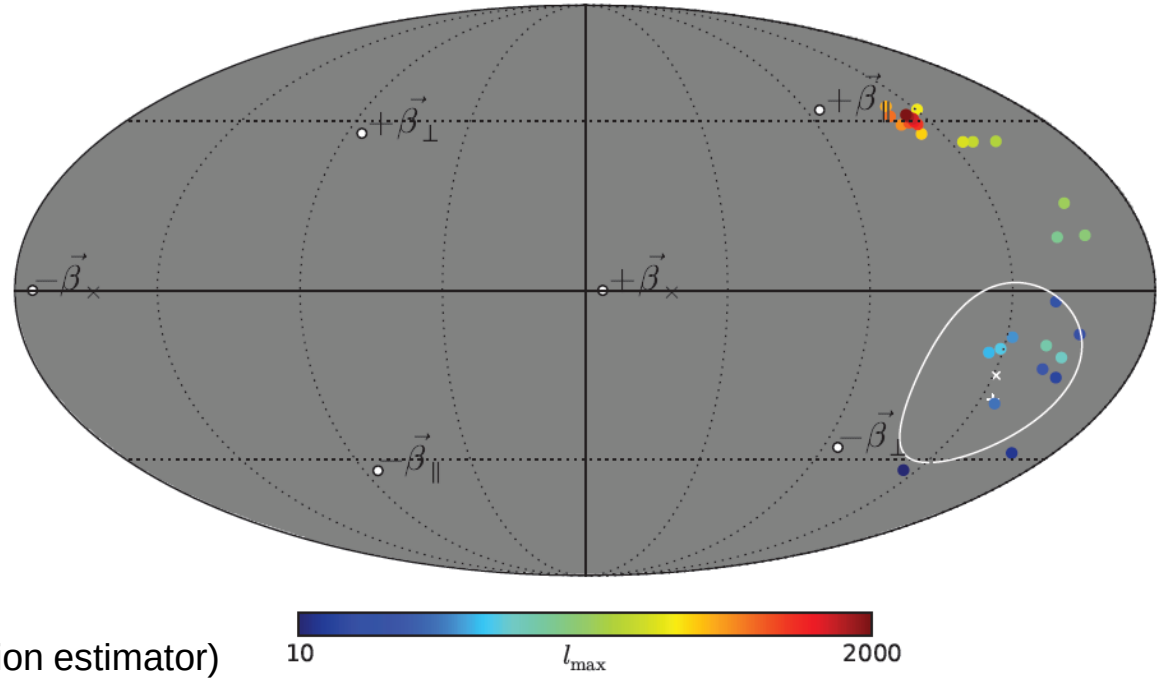


Consistent with zero except for anomalous octopole

Power dipole directions ($l \leq l_{\max}$)



Kinematic subtracted

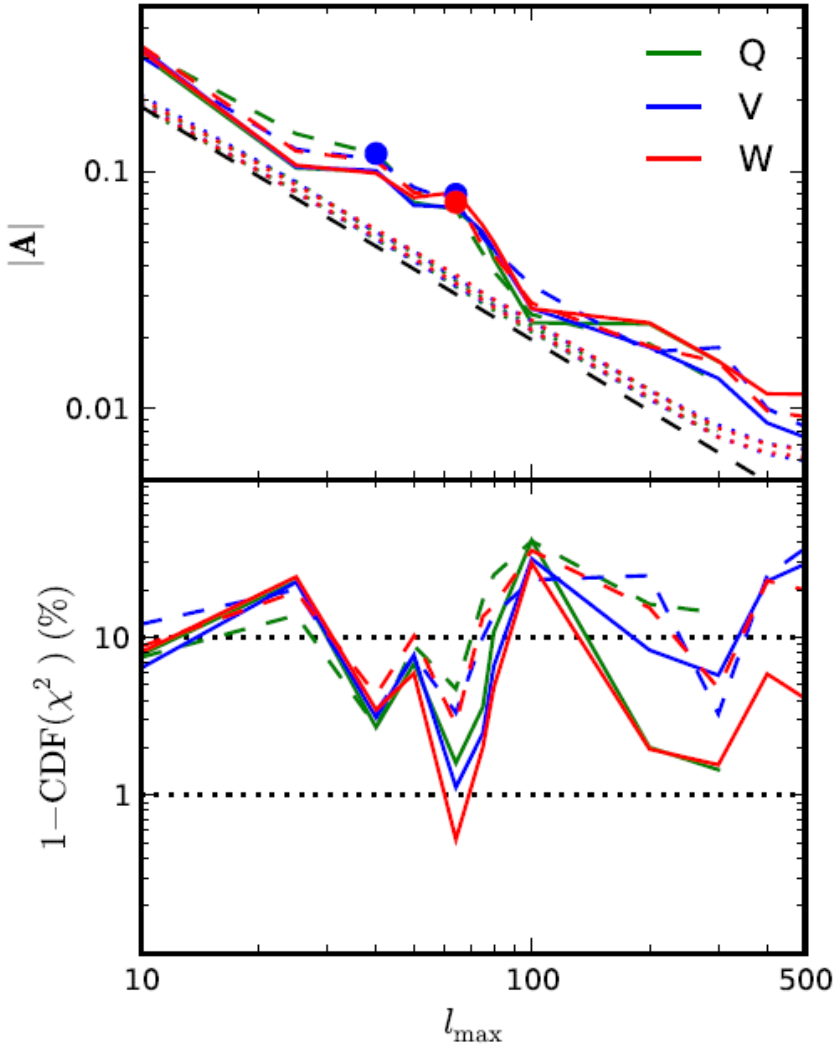


Kinematics not subtracted

(as in Doppler paper but here pure modulation estimator)

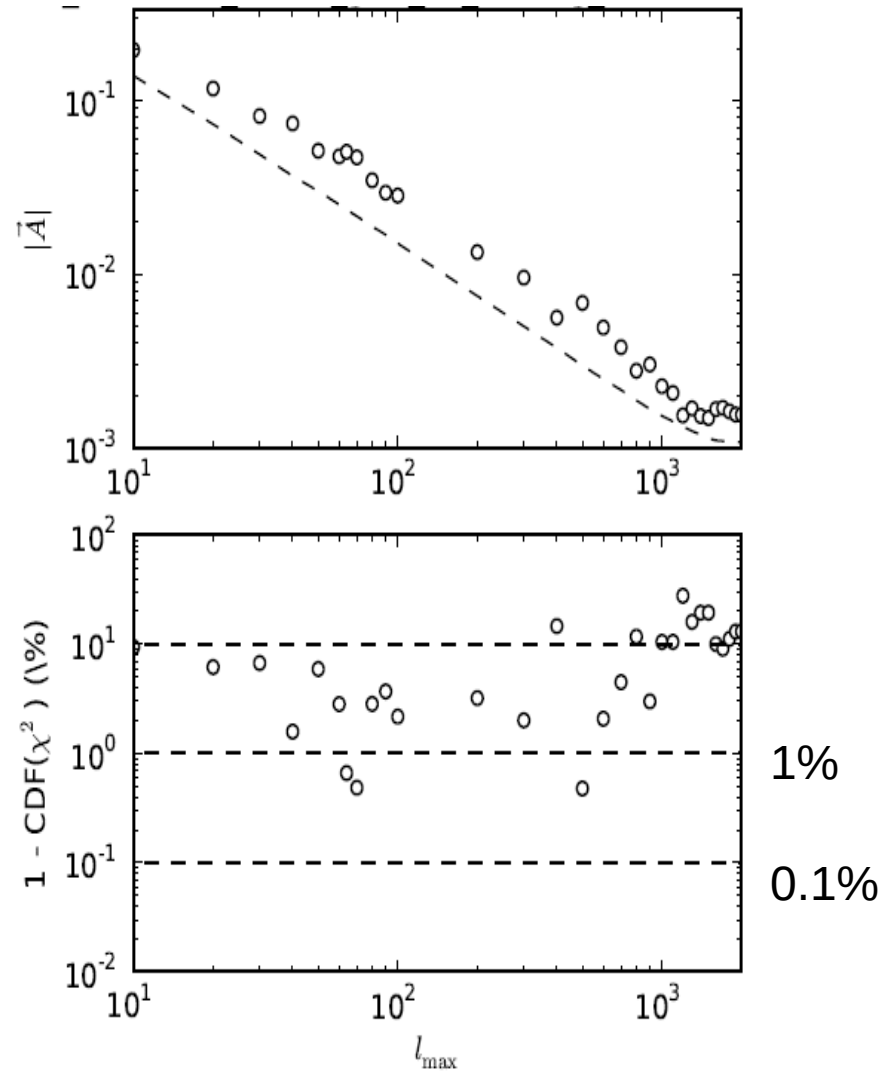
Power modulation dipole? Result for amplitude at $l \leq l_{\max}$

WMAP 5 (Hanson & Lewis 2009)



Modulation < 1% for $l \leq l_{\max} = 500$

Planck 217x143 (kinematic subtracted)



Modulation < 0.2% for $l_{\max} = 1500 - 2000$

Using full resolution to estimate local trispectrum

Local trispectrum often measured by

modulation power

$$\hat{\tau}_{\rm NL} \approx N^{-1} \sum_{L=L_{\rm min}}^{L_{\rm max}} \frac{2L+1}{L^2(L+1)^2} \frac{\hat{C}_L^f}{C_L^{\zeta_\star}}$$

(optimal to percent level)

Conventional normalization to primordial power

Planck τ_{NL} trispectrum constraint

Estimator result $\hat{\tau}_{\rm NL} = 442$

Gaussian simulations:

$$-452 < \hat{\tau}_{\rm NL} < 835 \text{ at } 95\% \text{ CL } (\sigma_{\tau_{\rm NL}} \approx 335)$$

Consistent with Gaussian null hypothesis (octopole has small weight)

Conservative upper limit, allowing octopole to be physical using Bayesian posterior

$$\tau_{\rm NL} < 2800 \text{ at } 95\% \text{ CL}$$

Conclusions

- Single field inflation predicts significant non-Gaussianity in the observed CMB

  - mostly due to (Weyl) lensing
  - total projects onto $f_{NL} \sim 7$ for Planck temperature
  - Ricci focussing expansion of beam recovers the δN from inflation,
    : gives equivalent of consistency relation, but larger
    : small and not quite observable, projects on to $f_{NL} \sim 1$

- Lensing bispectrum signal important but distinctive shape
  - Marginally detected by Planck

- Local Doppler signal predicted in trispectrum (dipole modulation)

- No sign yet of primordial NG once lensing and Doppler signals subtracted..