Dark energy notes

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Try to understand how dark energy perturbations evolve. Updated Jan 09 (fixing bug, thanks Anze).

Fluid equations

Following [1] we use the fluid equations. For varying w these are

$$\delta_i' + 3\mathcal{H}(\hat{c}_{s,i}^2 - w_i)(\delta_i + 3\mathcal{H}(1+w_i)v_i/k) + (1+w_i)kv_i + 3\mathcal{H}w'v/k = -3(1+w_i)h' \tag{1}$$

$$v_i' + \mathcal{H}(1-3\hat{c}_{s,i}^2)v_i + kA = k\hat{c}_{s,i}^2\delta_i/(1+w_i)\,, \tag{2}$$

where hatted quantities are evaluated in the frame co-moving with the dark energy.

For dark energy we are usually interested in ISW, which is sourced by the potential which depends on the perturbations in the rest frame of the total energy. At late times this is close to the rest from of the CDM, so we use the gauge where $v_c = A = 0$ (synchronous gauge). Then the velocity equation integrates to (we assume constant $\hat{c}_s^2$)

$$v_i = ka^{3\hat{c}_{s,i}^2-1}\int d\tau \frac{\hat{c}_{s,i}^2\delta_i}{1+w_i}a^{1-3\hat{c}_{s,i}^2} \tag{3}$$

and the CDM perturbation is related to the evolution of the local scale factor by $h' = -\delta_c'/3$. We also assume $w' = 0$ which is not a good approximation for many models. The CDM perturbations obey the same equation as h:

$$\delta_c'' + \mathcal{H}\delta_c' = \frac{1}{2}\kappa a^2(\delta\rho + 3\delta p) = \frac{1}{2}\kappa a^2\sum_i \rho_i\delta_i(1+3c_{s,i}^2). \qquad \text{(CDM frame)} \tag{4}$$

where in general

$$\delta_i c_{s,i}^2 = \delta_i\hat{c}_{s,i}^2 + \frac{3\mathcal{H}v_i}{k}(1+w_i)\left(\hat{c}_{s,i}^2 - \frac{p_i'}{\rho_i'}\right).$$

One can derive this using the frame invariance of $\delta p - p'/\rho'\delta\rho$ which implies $c_s^2\delta = \hat{\delta}\hat{c}_s^2 + (p'/\rho')\rho' v/k\rho$, which together with $\hat{\delta\rho} = \delta\rho - \rho' v/k$ and the background result for ρ' gives the above.

Radiation domination

The dark energy is a negligible fraction of the density, and $a = \Omega_R^{\frac{1}{2}}H_0\tau$. The CDM perturbation is $\delta_c = C\tau^2$, which implies $h' = -C\tau^2/3$ where C is determined by the primordial perturbation amplitude. So we can solve the equation in the $k << \mathcal{H}$ limit to give, for $\hat{c}_s^2 = 1$,

$$\delta = \frac{C(1+w)}{7-6w}\tau^2 + A\tau^{3w/2-1}\cos\left[\sqrt{20-12w-9w^2}\ln(\tau)/2 + \alpha\right] \tag{5}$$

where A and α specify the initial conditions. For $\hat{c}_s^2 \neq 1$ the solution is very similar but messier with the homogenous solution also decaying similarly and the inhomogeneous solution

$$\delta = \frac{C(w+1)(4-3\hat{c}_s^2)}{3\hat{c}_s^2+4-6w}\tau^2 \tag{6}$$

Thus for $w > -1$ and $\hat{c}_s^2 \leq 1$ the perturbations are the same sign.

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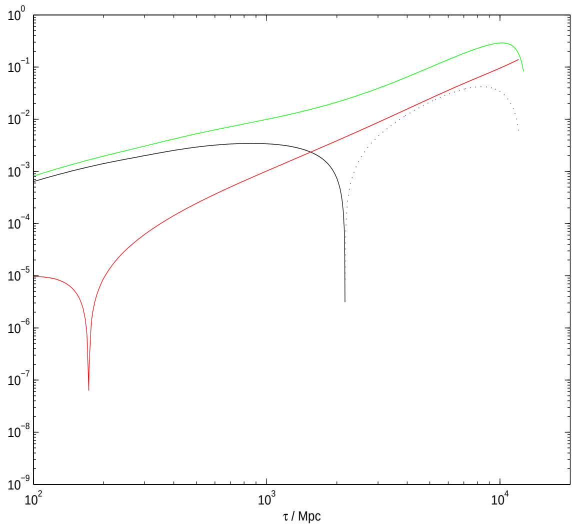

FIG. 1: Evolution of $k = 10^{-3}$Mpc mode in the CDM frame (red, w=-0.8), compared with the frame comoving with the total energy (black: w=-0.8; green: w=-2). All have $\hat{c}_s^2 = 1$.

Matter domination

Now $a = \Omega_m H_0^2 \tau^2/4$ (for $a(t=0)=0$), and $\delta_c = D\tau^2$. Similar to before for $k << \mathcal{H}$ and $\hat{c}_s^2 = 1$,

$$\delta = -\frac{D(1+w)}{14-15w}\tau^2 + A\tau^{3w-3/2}\cos\left[\sqrt{7-4w-4w^2}3\ln(\tau)/2+\alpha\right] \tag{7}$$

but this time the forced perturbations are of opposite sign. In general the forced solution is

$$\delta = -\frac{D(w+1)(6\hat{c}_s^2-5)}{5+9\hat{c}_s^2-15w}\tau^2. \tag{8}$$

For sensible w the criterion for the perturbations to be the same sign is $\hat{c}_s^2 < 5/6$ and quintessence perturbations ($\hat{c}_s^2 = 1$) are therefore anticorrelated with the matter during the matter era. The velocity term of opposite sign is winning over the source term. The damping of the homogeneous solution is independent of $\hat{c}_s^2$.

[1] J. Weller and A. M. Lewis, Mon. Not. Roy. Astron. Soc. **346**, 987 (2003), astro-ph/0307104.