CMB Lensing Overview

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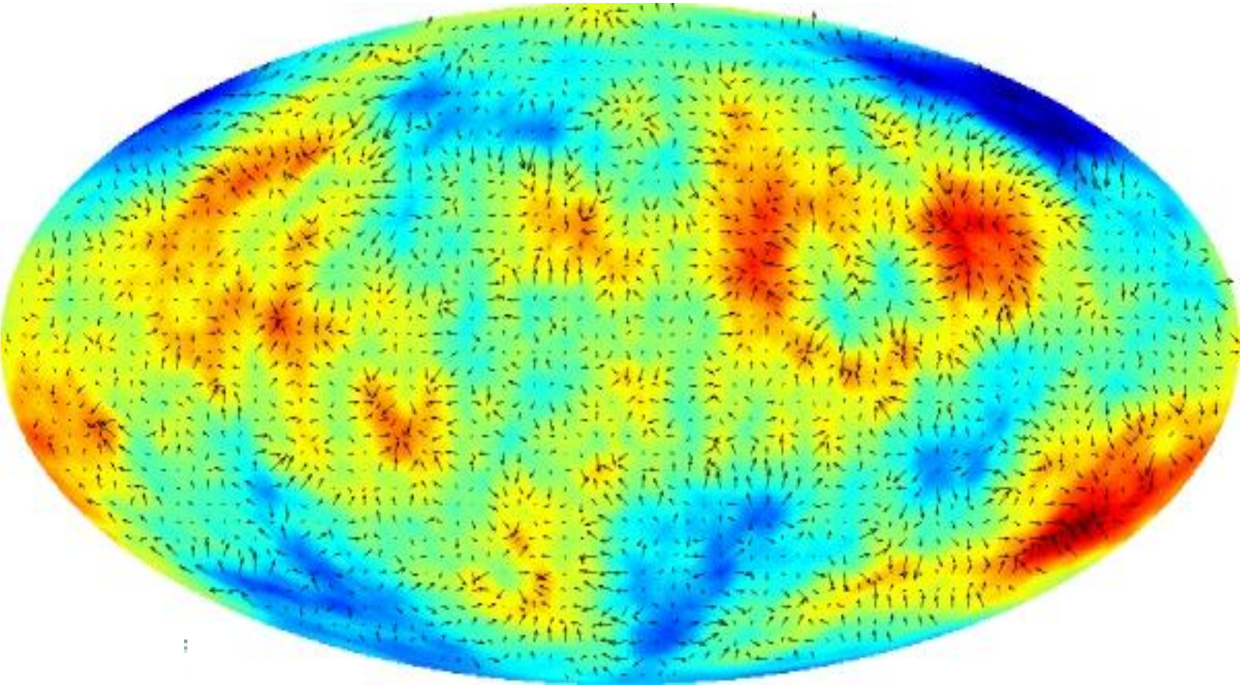
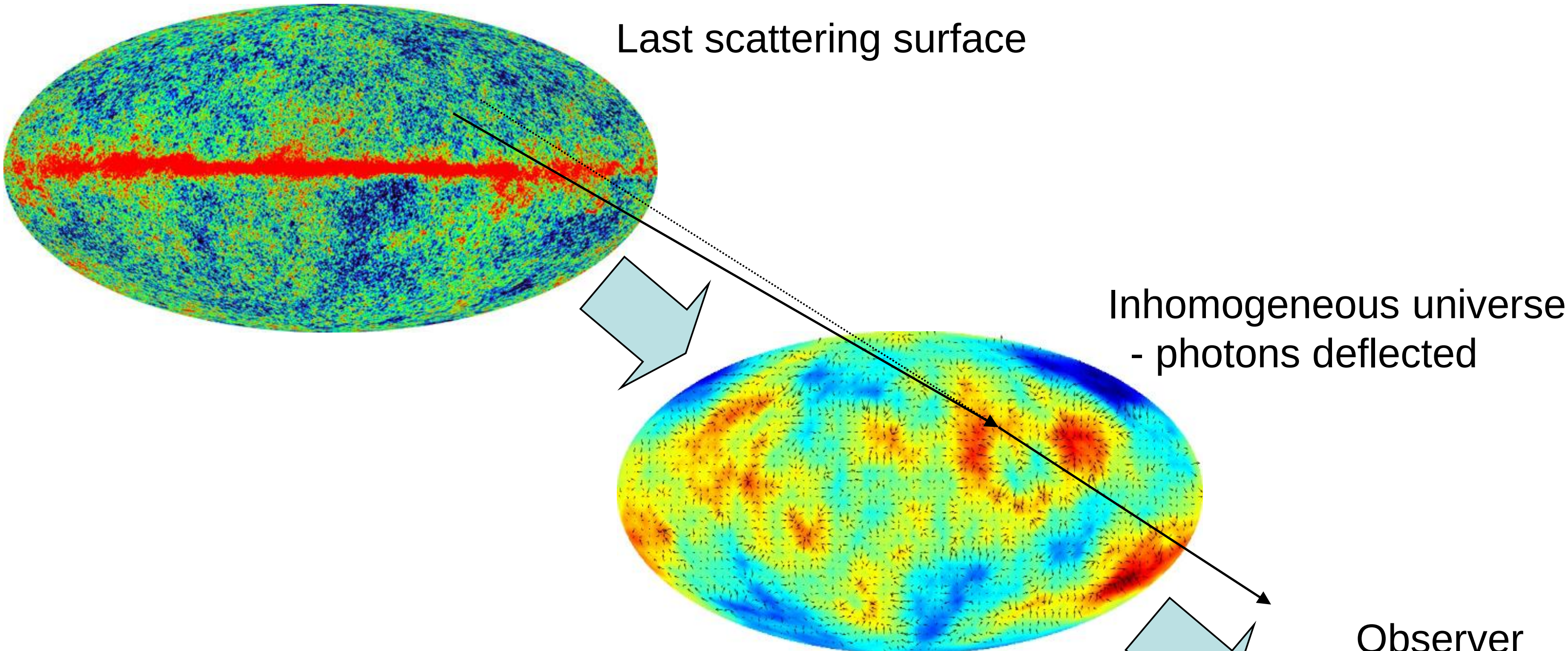
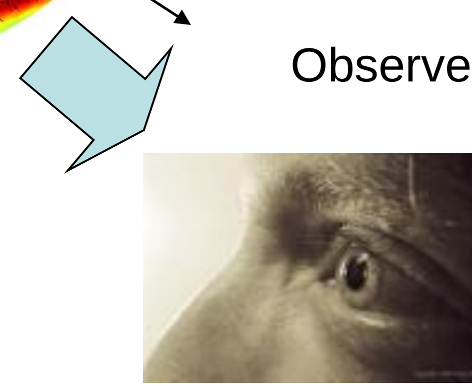


University of Sussex

Weak lensing of the CMB



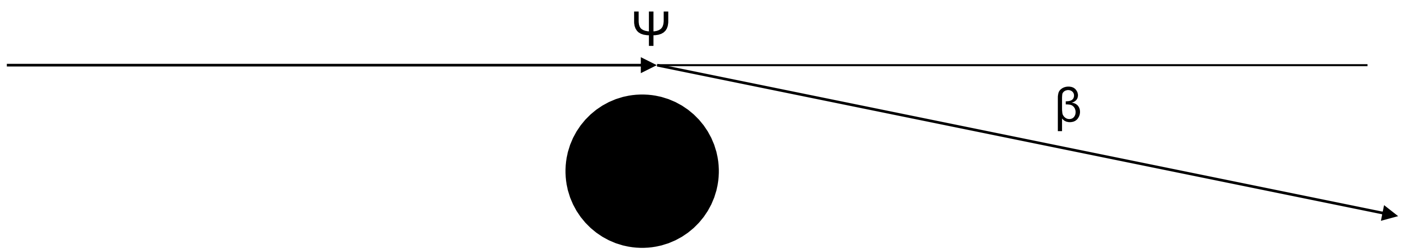


$$\tilde{T}(\hat{\mathbf{n}}) = T(\hat{\mathbf{n}}') = T(\hat{\mathbf{n}} + \boldsymbol{\alpha})$$

$$\boldsymbol{\alpha} = \nabla\psi$$

$$\psi(\hat{\mathbf{n}}) = -2\int_0^{\chi_*} \mathrm{d}\chi\, \Psi(\chi\hat{\mathbf{n}}; \eta_0 - \chi)\, \frac{f_K(\chi^* - \chi)}{f_K(\chi^*) f_K(\chi)}$$

Lensing order of magnitudes



General Relativity: $\beta = 4\,\Psi$ $(\beta << 1)$

Potentials linear and approx Gaussian: $\Psi \sim 2 \times 10^{-5} \Rightarrow \beta \sim 10^{-4}$

Potentials scale-invariant on large scales, decay on scales smaller than matter-power spectrum turnover:

⇒ most abundant efficient lenses have size ~ peak of matter power spectrum ~ 300Mpc

Comoving distance to last scattering surface ~ 14000 MPc

⇨ pass through ~50 lenses ⇨ total deflection ~ $50^{1/2} \times 10^{-4}$

assume uncorrelated

~ 2 arcminutes

(neglects angular factors, correlation, etc.)

Why lensing is important

- 2arcmin deflections: $l \sim 3000$

  - On small scales CMB is very smooth so lensing dominates the linear signal at high l

- Deflection angles coherent over 300/(14000/2) ~ 2°

  - comparable to CMB scales
  - expect 2arcmin/60arcmin ~ 3% effect on main CMB acoustic peaks

- Non-linear: observed CMB is non-Gaussian

  - more information
  - potential confusion with primordial non-Gaussian signals

- Does not preserve E/B decomposition of polarization: e.g. $E \rightarrow B$

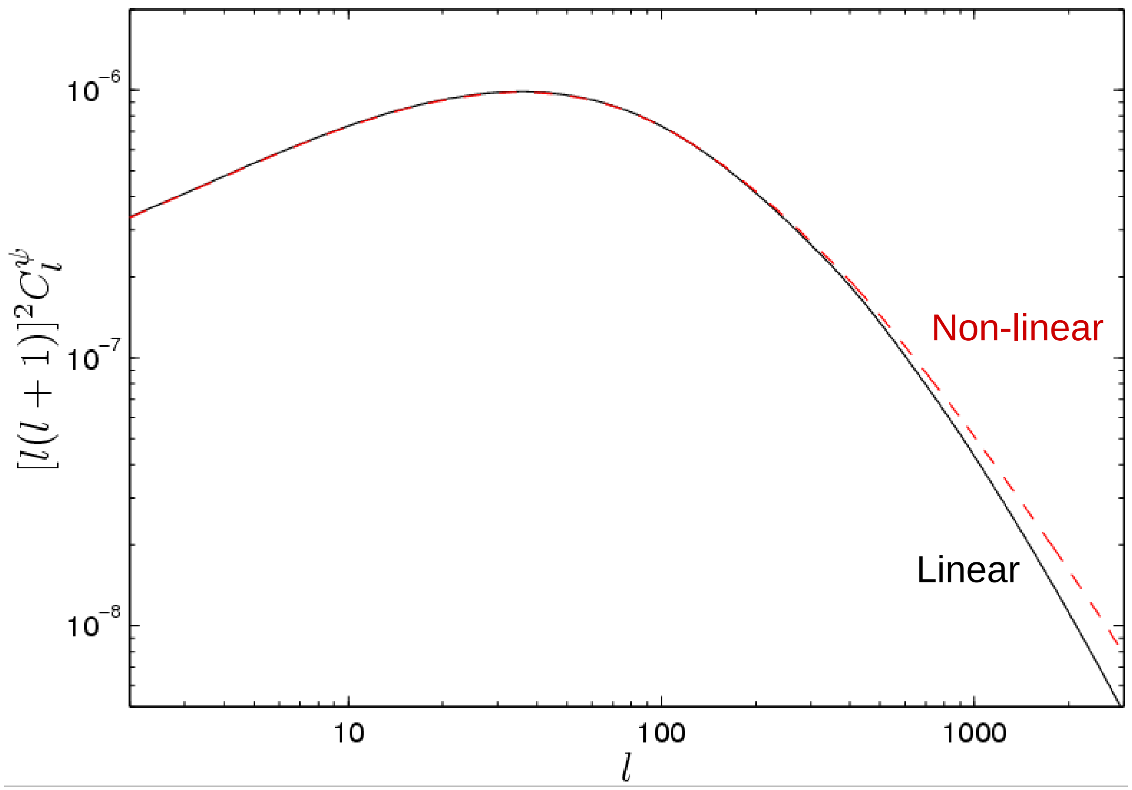
  - Confusion for primordial B modes ("r-modes")
  - No primordial B $\Rightarrow$ B modes clean probe of lensing

Deflection angle power spectrum

$\boldsymbol{\alpha} = \nabla\psi$

Clean physics: potentials nearly linear ⇒ lensing potential nearly Gaussian

(also central limit theorem on small less-linear scales – lots of small lenses)



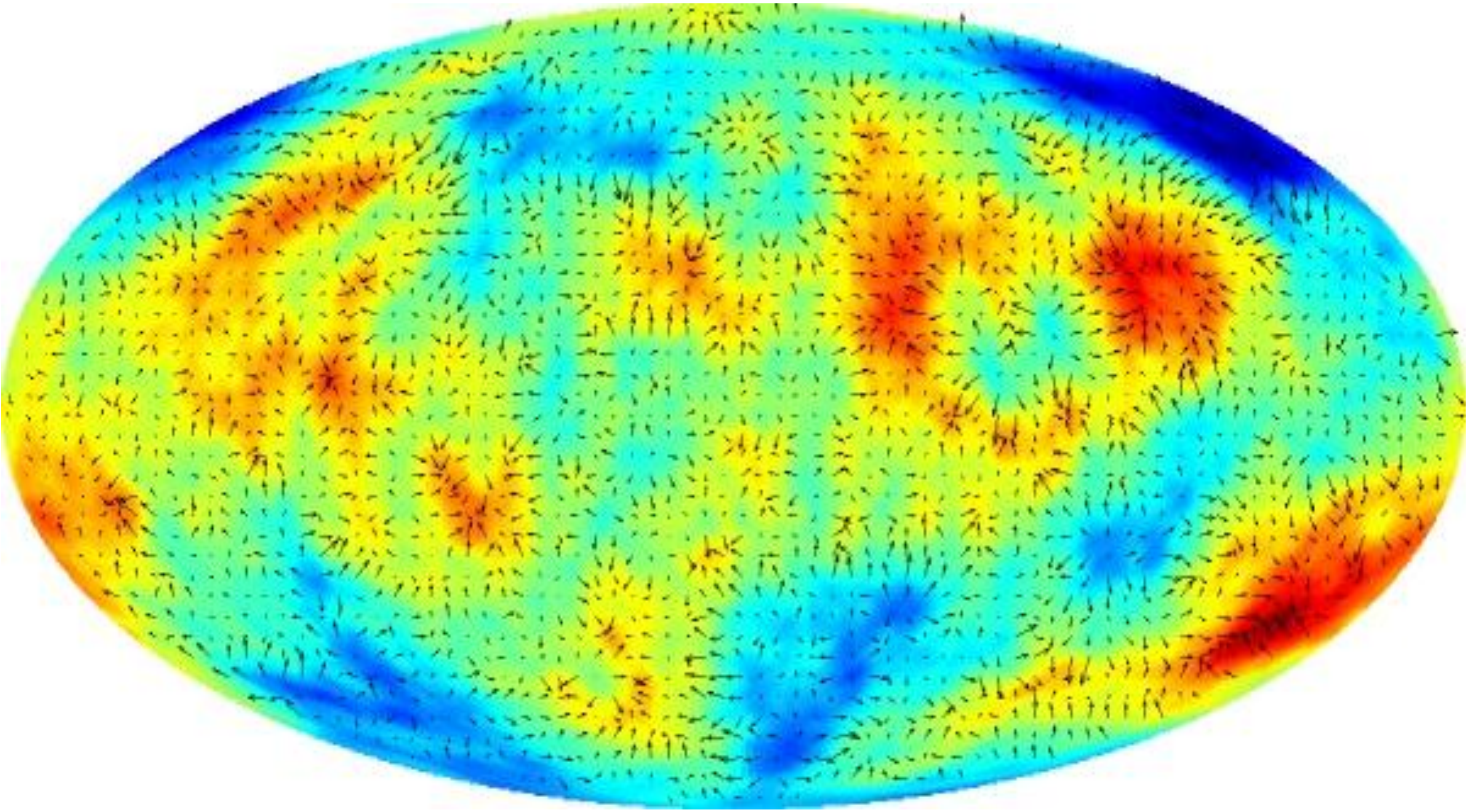
Deflections O(10^{-3}) , but coherent on degree scales

Probes matter distribution at roughly $0.5 < z < 6$ depending on l

Non-linear structure growth effects not a major headache

Note: lensing is *not* a larger effect at low z because of growth of structure: deflections depend on Newtonian potential which is *constant* in matter domination, and actually decaying at low redshift.

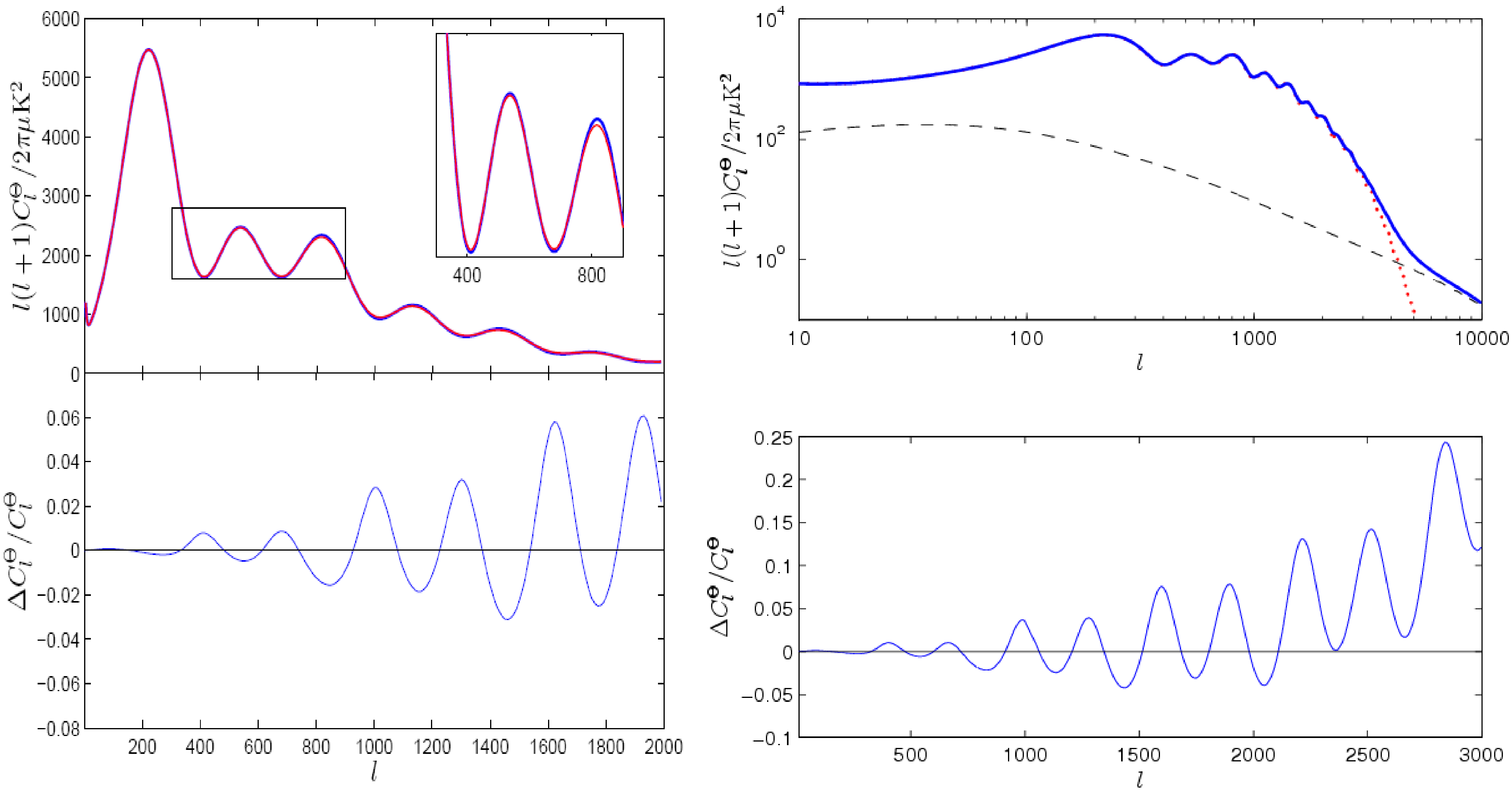
Simulated full sky lensing potential and (englarged) deflection angle fields



Easily simulated assuming Gaussian fields

- just re-map points using Gaussian realisations of CMB and potential

Lensing effect on CMB temperature power spectrum

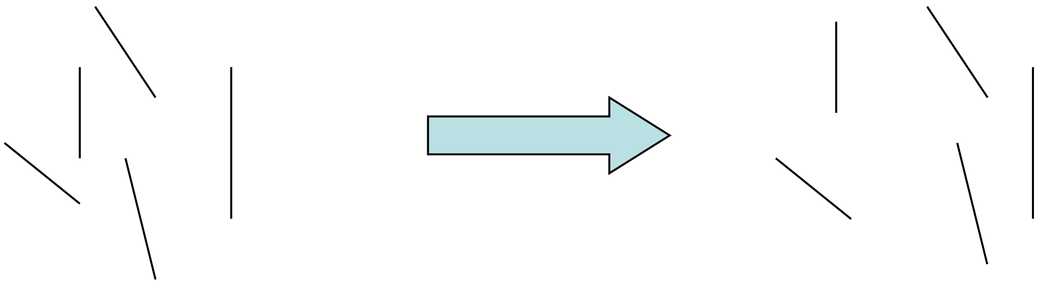


Important, but accurately modelled (e.g. CAMB); only limited additional information

Lensing of polarization

- Polarization not rotated w.r.t. parallel transport (vacuum is not birefringent)
- Q and U Stokes parameters simply re-mapped by the lensing deflection field

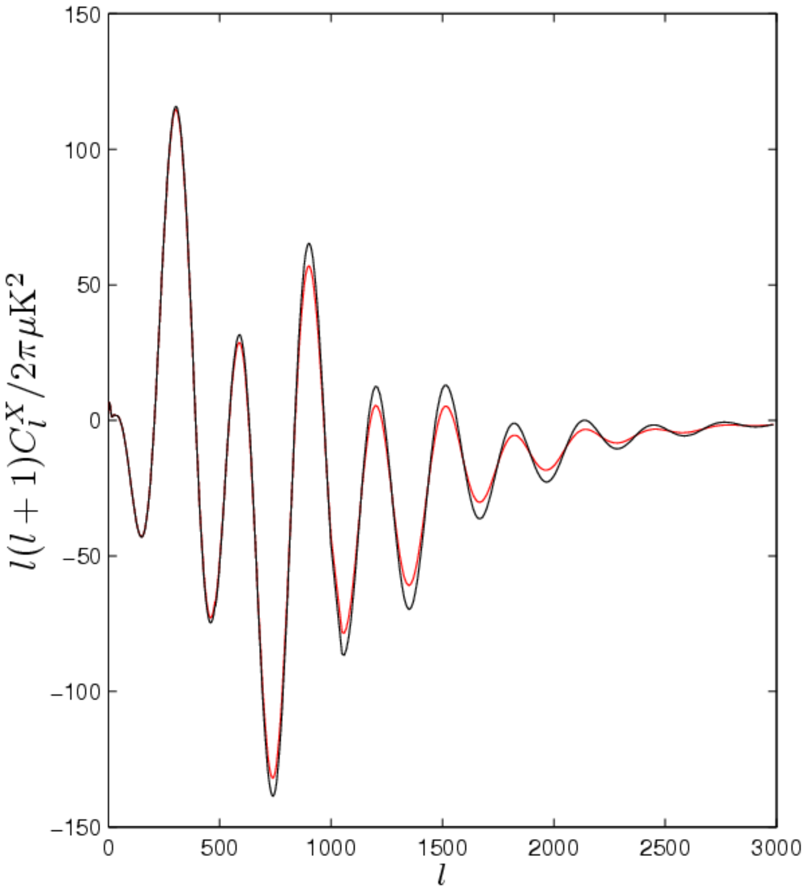
e.g.

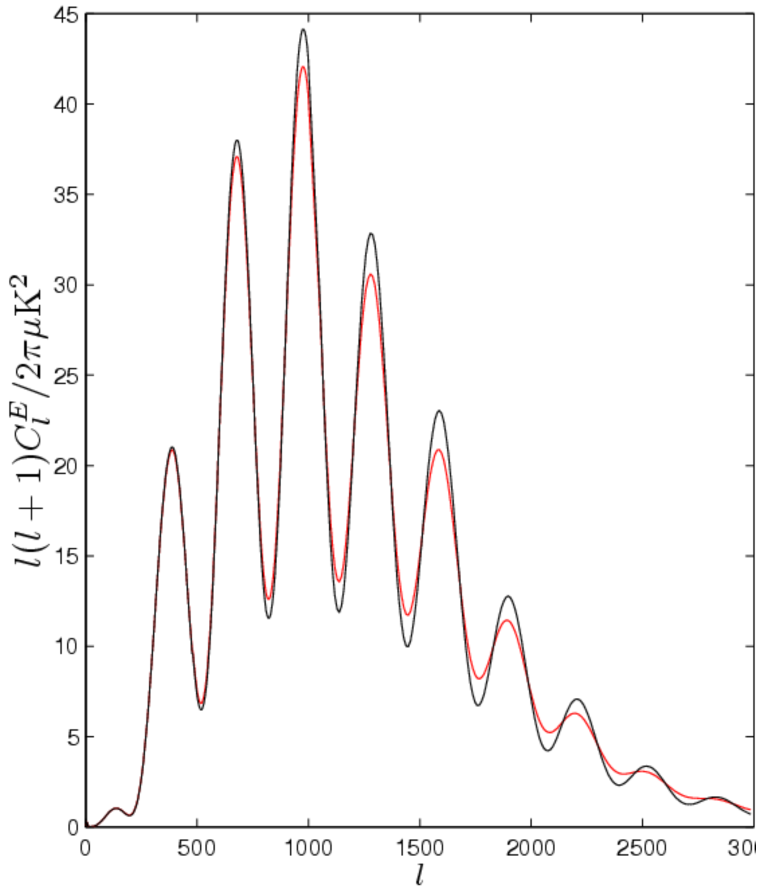


Last scattering

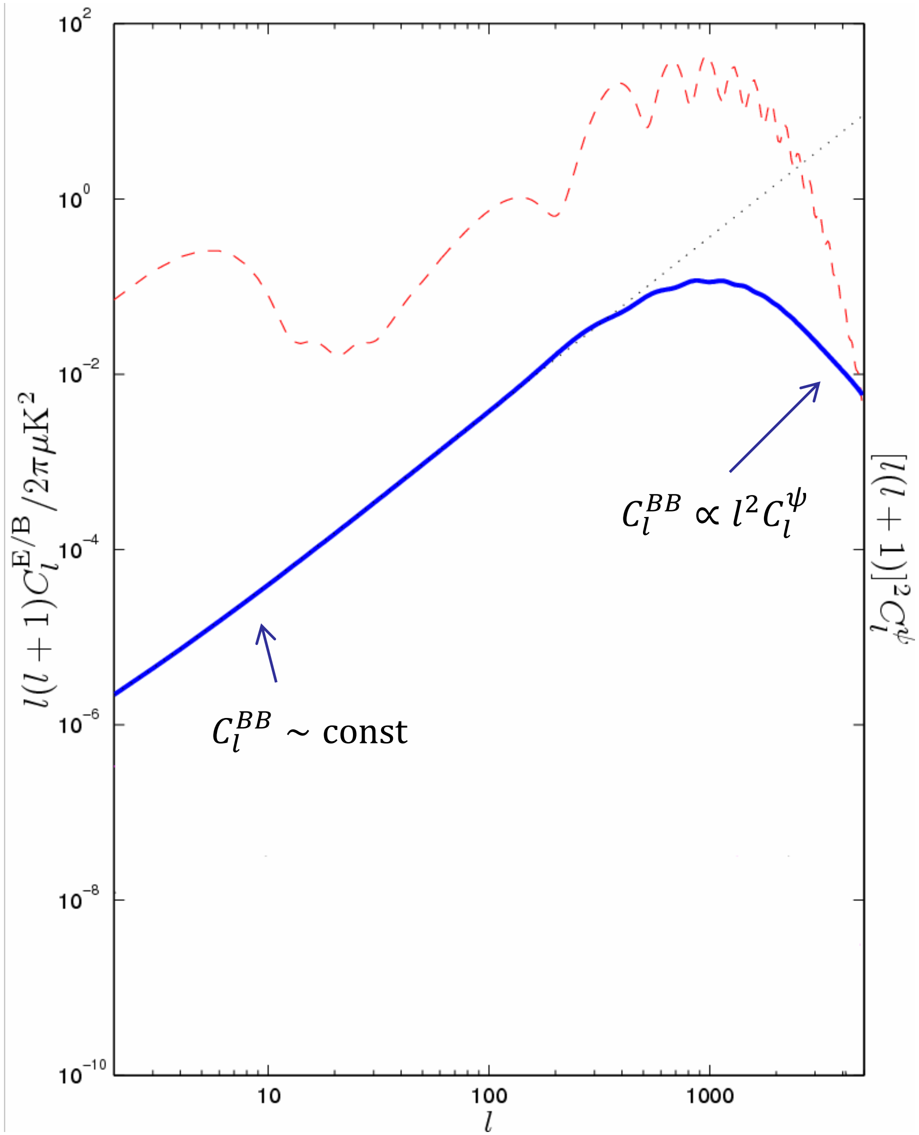
Observed

Polarization lensing: C_l^X and C_l^{EE}





Polarization lensing: C_l^{BB}



Nearly white BB spectrum on large scales

$$\tilde{C}_l^B \sim \int \frac{d^2 \mathbf{l}'}{(2\pi)^2} l'^4 C_{l'}^{\psi} C_{l'}^E \sin^2 2(\phi_{\mathbf{l}'} - \phi_{\mathbf{l}})$$
$$= \frac{1}{4\pi} \int \frac{dl'}{l'} l'^4 C_{l'}^{\psi} l'^2 C_{l'}^E,$$

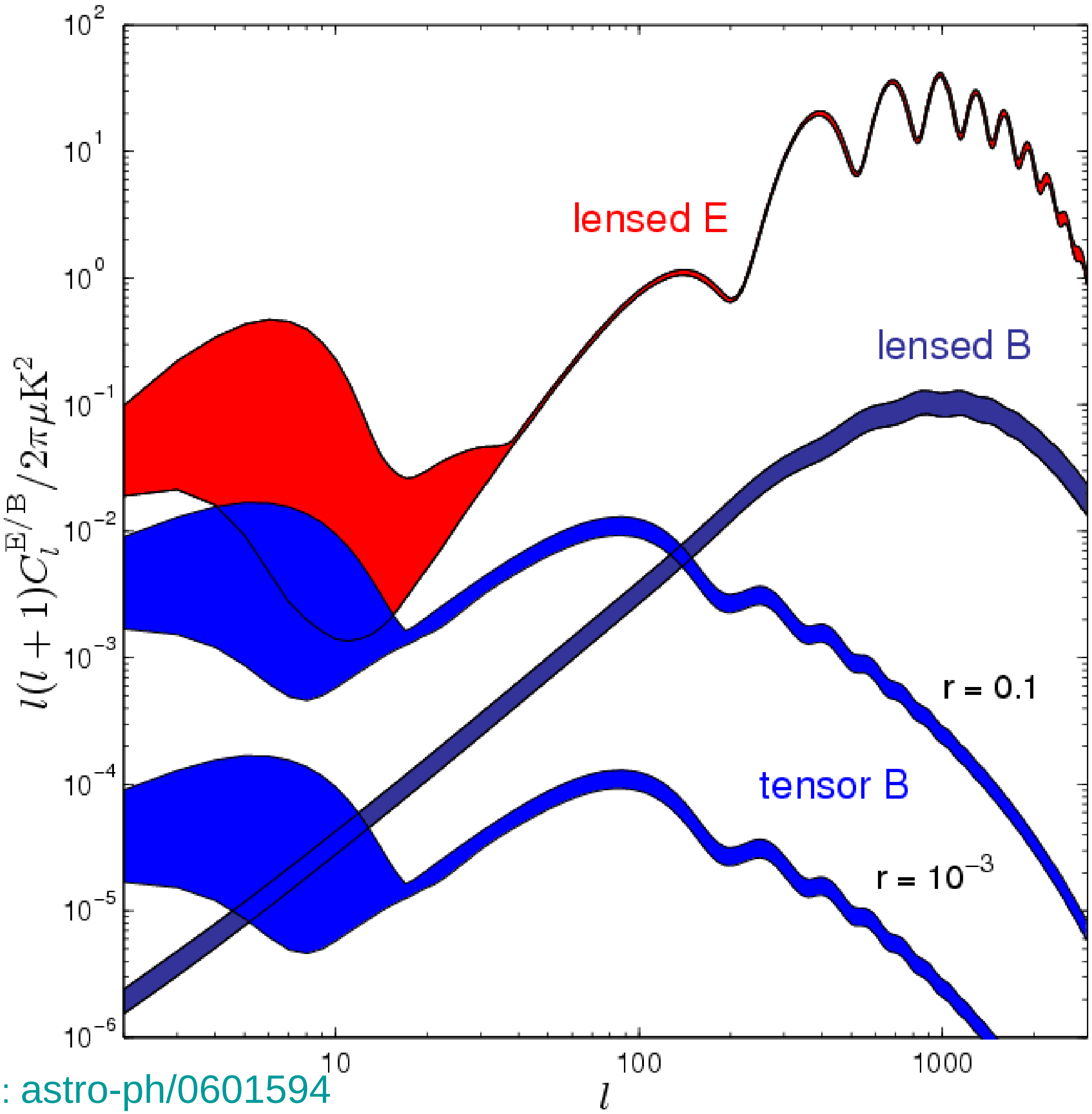
- originates from wide range of deflection angle and E modes

On very small scales little unlensed power

$$\Rightarrow \quad C_l^{BB} \sim C_l^{EE} \propto C_l^{\alpha}$$

Polarization power spectra

Current 95% indirect limits for LCDM given WMAP+2dF+HST (bit old)



Lewis, Challinor : astro-ph/0601594

Non-Gaussianity/statistical anisotropy
Reconstructing the lensing field

Marginalized over (unobservable) lensing field:

$$T \sim \int P(T, \psi) d\psi$$

- Non-Gaussian statistically isotropic temperature distribution

- Large-scale squeezed bispectrum + significant connected 4-point function

For a given lensing field :

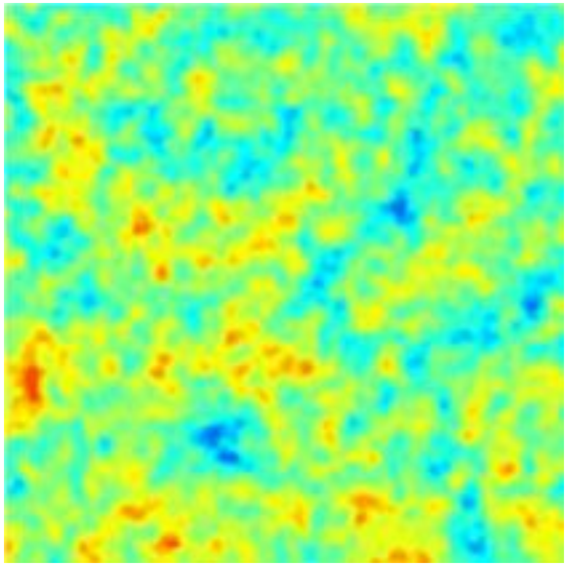
$$T \sim P(T|\psi)$$

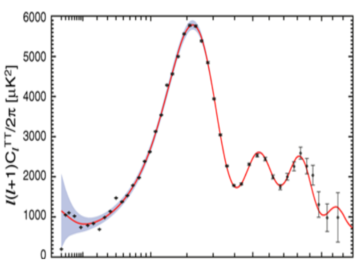
- Anisotropic Gaussian temperature distribution

- Different parts of the sky magnified or demagnified and sheared

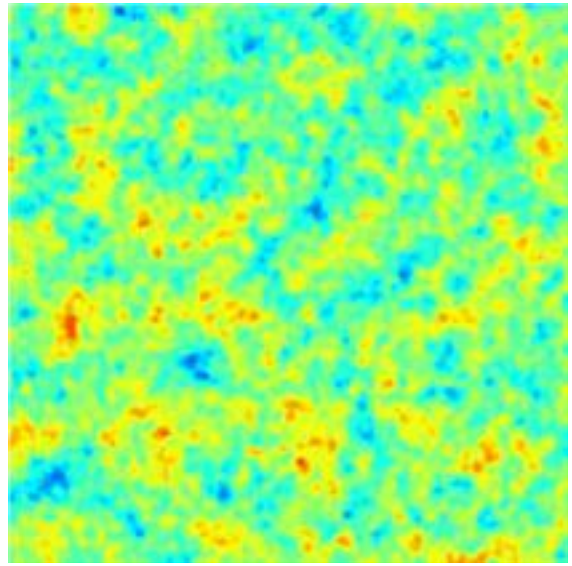
Fractional magnification ~ convergence $\kappa = -\boldsymbol{\nabla} \cdot \boldsymbol{\alpha}/2$

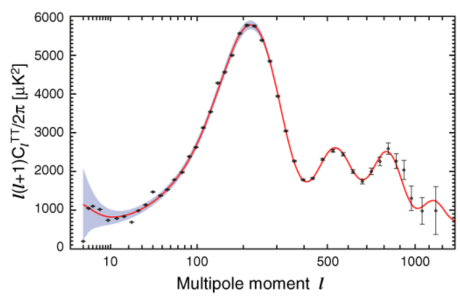
Magnified



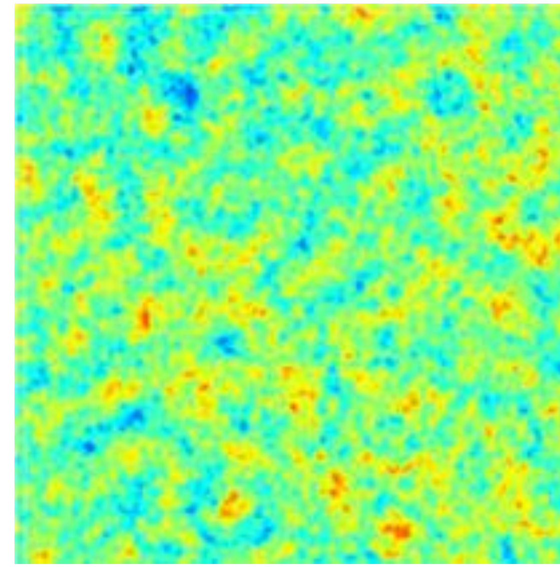


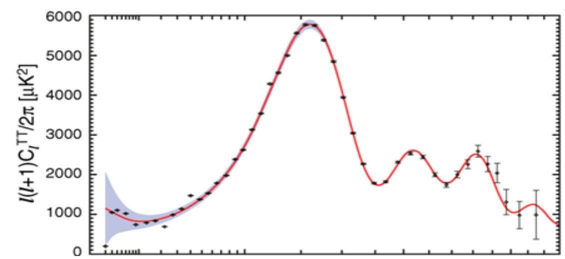
Unlensed





Demagnified

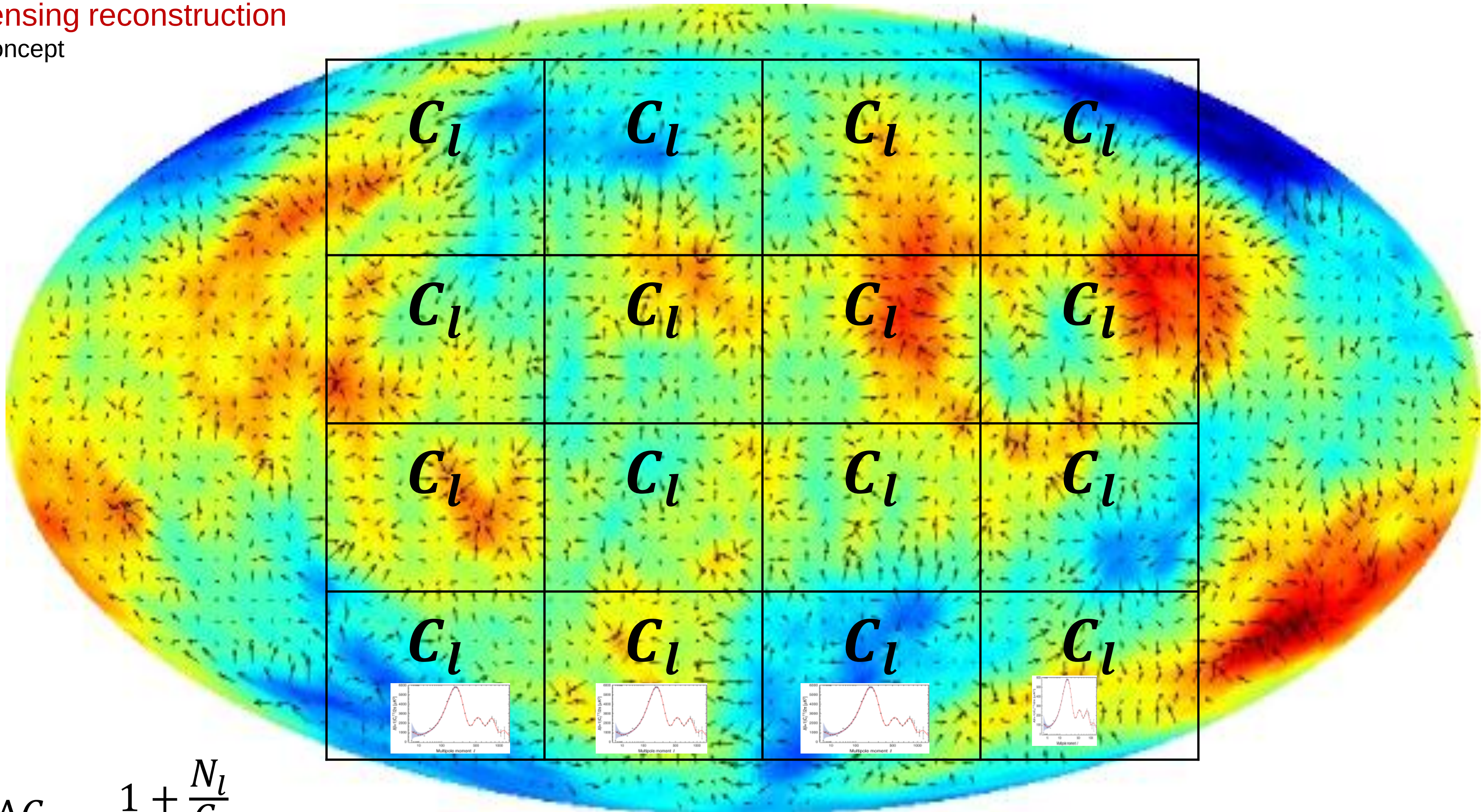




+ shear (shape) modulation [c.f. Bucher et al.]

Lensing reconstruction

-concept



$$\frac{\Delta C_l}{C_l} \sim \frac{1 + \frac{N_l}{C_l}}{\sqrt{2l+1}}$$

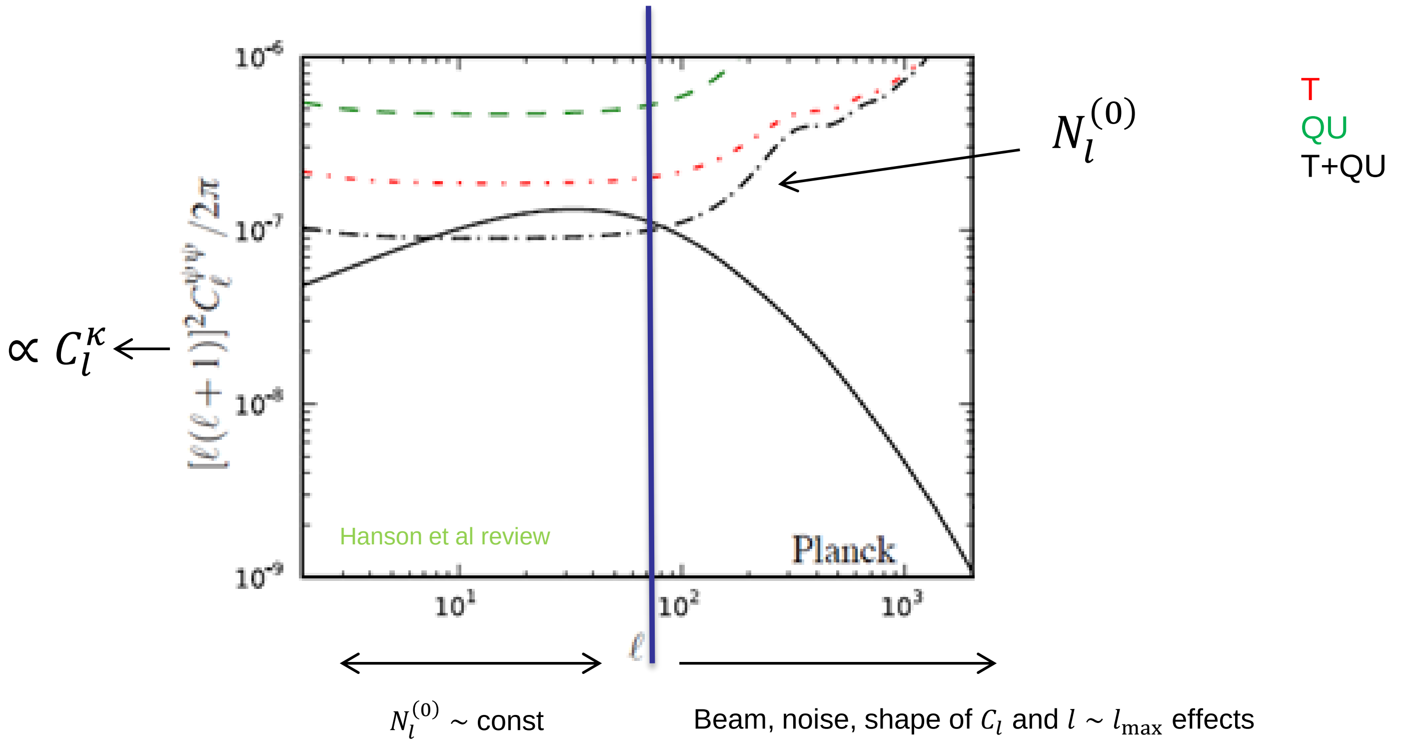
Variance in each C_l measurement $\propto 1/N_{\rm modes}$

$N_{\rm modes} \propto l_{\rm max}^2$ - dominated by smallest scales

⇒ measurement of angular scale in each box nearly independent

⇒ Uncorrelated variance on estimate of magnificantion κ in each box

⇒ Nearly white 'reconstruction noise' $N_l^{(0)}$ on κ , with $N_l^{(0)} \propto 1/l_{\rm max}^2$



Lensing reconstruction information mostly in the *smallest scales* observed

- Want high resolution and sensitivity
- Almost totally insensitive to large-scale TQU (so only *small-scale* foregrounds an issue)

Potential problems due to other effects that look partly like spatially varying magnification and shear, e.g.

- Beam asymmetries (quadrupole moment ~ shear, can be modelled)
- Boundaries and holes in observed region (can be modelled well, but degrade S/N)
- Anisotropic noise, other systematics and foregrounds
- Other 2nd-order physical effects (thought to be very small, but no full calculation)

Lensing reconstruction

- Maths and algorithm sketch

For a *given* (fixed) modulation field X, $T \sim P(T|X)$:

X here is lensing potential, deflection angle, or κ

Anisotropic Gaussian temperature distribution

Flat sky approximation: modes correlated for $\mathbf{k}_2 \neq \mathbf{k}_3$
First-order series expansion in the lensing field:

$$\langle \tilde{T}(\mathbf{k}_2)\tilde{T}(\mathbf{k}_3)\rangle_{P(\tilde{T}|X)} \approx \int \mathrm{d}\mathbf{K} X(\mathbf{K})^* \underbrace{\left\langle \frac{\delta}{\delta X(\mathbf{K})^*}\left(\tilde{T}(\mathbf{k}_2)\tilde{T}(\mathbf{k}_3)\right)\right\rangle}_{\mathcal{A}(K,k_2,k_3)\delta(K+k_2+k_3)} \approx \mathcal{A}(K,k_2,k_3)\, X(\mathbf{K})^*|_{\mathbf{K}=-\mathbf{k}_2-\mathbf{k}_3}$$

function easy to calculate for $X(\mathbf{K}) = 0$

$$A(L, l_1, l_2) \sim \left(\mathbf{l}_1 \cdot \mathbf{L}\tilde{C}_{l_1} + \mathbf{l}_2 \cdot \mathbf{L}\tilde{C}_{l_2}\right)$$

Can reconstruct the modulation field X

For small X can construct “optimal” quadratic (QML) estimator $\hat{X}(K)$ by summing filtered fields appropriately over k_2, k_3

$$\hat{X}(K) \sim N[\textstyle\sum_{\mathbf{k}_2,\mathbf{k}_3} A(K, k_2, k_3)\, \bar{T}(\mathbf{k}_2)\bar{T}(\mathbf{k}_3) - (\text{Monte carlo for zero signal})]$$

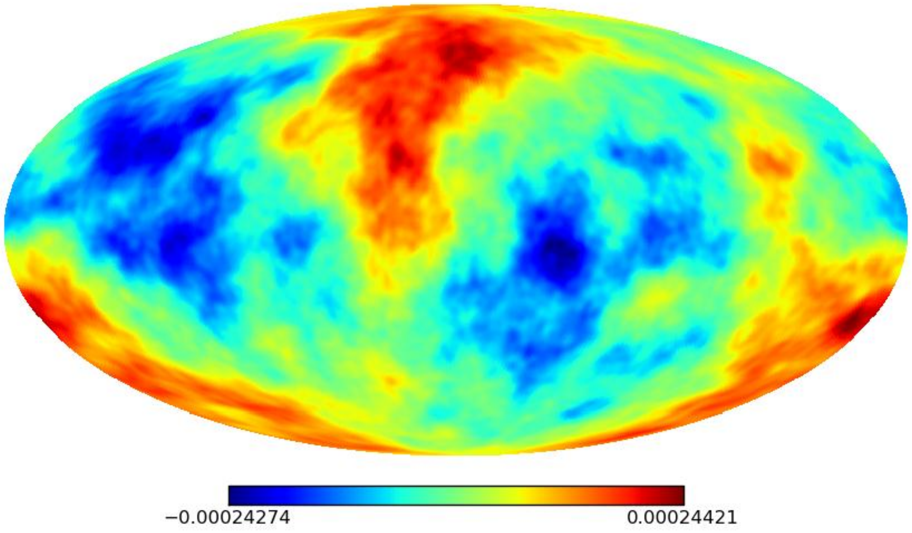
Zaldarriaga, Hu, Hanson, etc..

Can also re-write in as fast real-space estimator

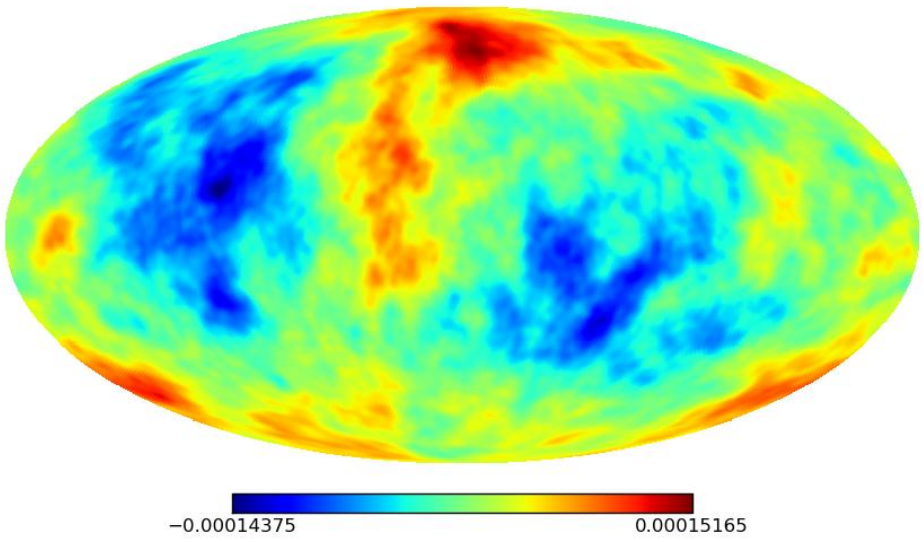
$$\hat{\alpha}_{\mathrm{LM}} \propto (F_1 \nabla F_2)_{LM} \qquad F_1 = (S+N)^{-1}T \qquad F_2 = S(S+N)^{-1}T$$

- Similar estimators for polarization (but more complicated tensor fields)

True (simulated)



Reconstructed (Planck noise, Wiener filtered)



(Credit: Duncan Hanson)

Reconstructed ψ map

⇒ can correlate with other lensing or density probes (CIB, galaxy lensing, galaxy counts, 21cm…)
⇒ estimate $C_l^{T\psi}$ - probe of ISW and dark energy, but only on large scales ($l < \sim 100$), $< 7\sigma$
⇒ estimate $C_l^{\psi\psi}$

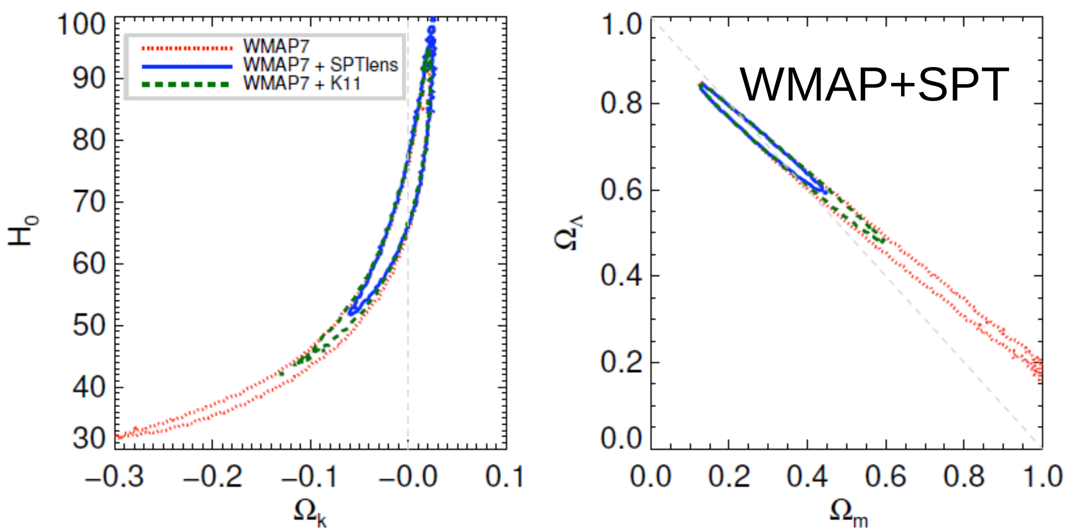
What does an estimate of $C_l^{\psi\psi}$ do for us?

Probe $0.5 \leq z \leq 6$: depends on geometry and matter power spectrum

⇒ break degeneracies in the linear CMB power spectrum

- Better constraints on neutrino mass, dark energy, Ω_K, …

Engelen et al, 1202.0546



Neutrino mass talk to come..

Reconstruction with polarization

- Expect *no* primordial small-scale B modes (r-modes only large scales $l < \sim 300$)

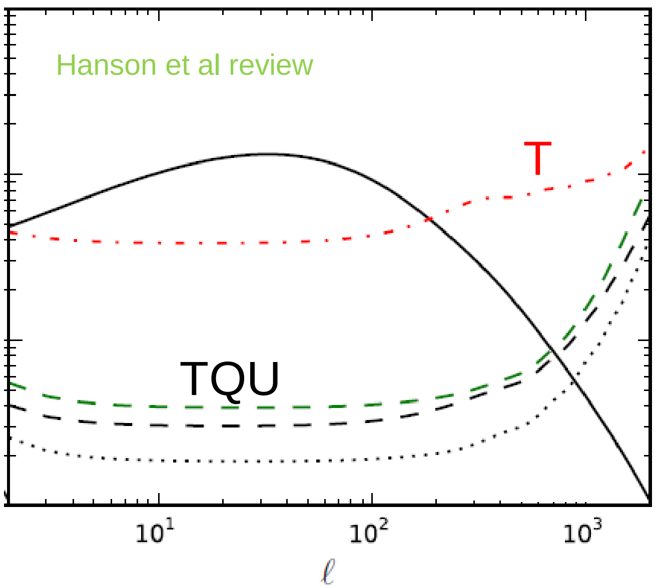
- *All* small-scale B-mode signal is lensing: *no* cosmic variance confusion with primordial signal as for E and T, in principle only limited by noise

- Ideally perfect B-mode observation ⇒ perfect lensing reconstruction
  (Hirata & Seljak)

- Polarization data does *much* better than temperature if sufficiently good S/N (mainly EB estimator).

ACTpol, POLAR-1, etc.

e.g. Planck with 27x lower $\sigma(\mathrm{TQU})$



Note: simple quadratic estimator suboptimal – need maximum likelihood or iterative scheme

CMB lensing summary

- changes power spectra at several per cent

- introduces non-Gaussian signal

- reconstruct lensing potential (0.5 <~ z <~ 7):
  Quadratic estimator: signal almost all in *small-scale* modes
  Iterative/max-likelihood estimators needed with high S/N polarization

- C_l^{ψ}: integrated probe of total matter and geometry; break parameter degeneracies

- generates large-scale B modes with white spectrum (known amplitude)
  potential confusion with primordial gravitational waves for r <~ 10^{-3}
  contributes to effective r-mode noise unless actual *realization* can be subtracted

- If *large-scale* r-modes $\ll$ *large-scale* lensing B:

-
  - use *small-scale* B to reconstruct ψ
  - use ψ and E to de-lens large-scale B: extract r-modes (lensing cleaning)
  - *Required* for $r \ll 10^{-3}$
    BUT needs very high sensitivity and quite high resolution over large f_{sky}
    ultimate limit unclear