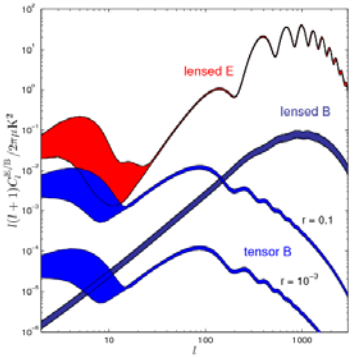
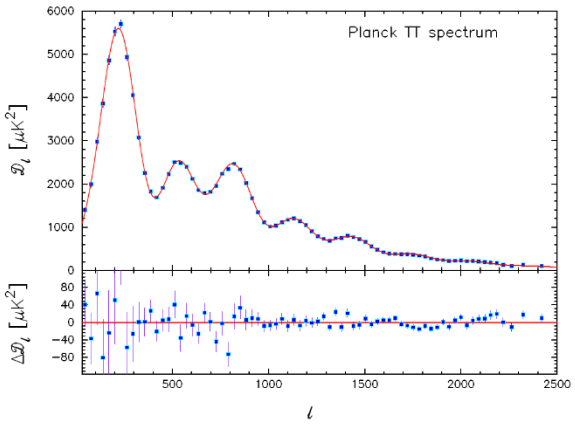
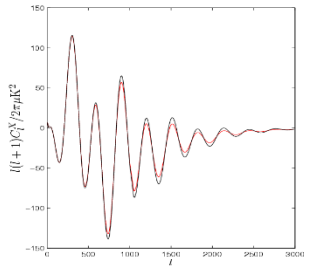
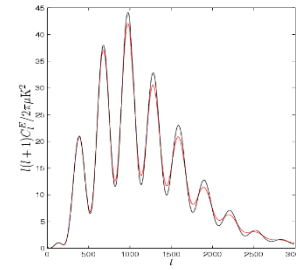
Introduction to CAMB

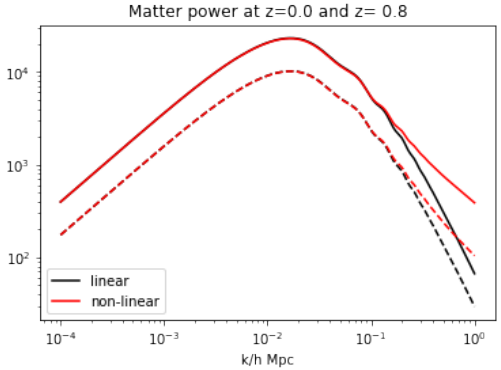
Code for **A**nisotropies in the **M**icrowave **B**ackground

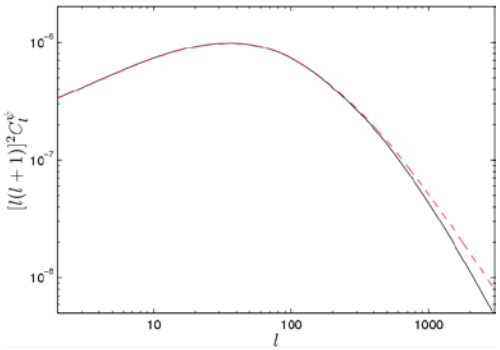












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http://cosmologist.info/

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European Research Council

Established by the European Commission

References

- http://camb.info/ (main site, register for mailing list and download)
  http://github.com/cmbant/CAMB/ (latest source, fixes and other branches)

- http://camb.readthedocs.io/en/latest/ (Python CAMB docs, “pip install camb”)

- Help and discussion, and searchable store of previous question answers:
  http://cosmocoffee.info/viewforum.php?f=11

- Python example notebooks (made with Jupyter notebook)
  http://camb.readthedocs.io/en/latest/CAMBdemo.html
  http://camb.readthedocs.io/en/latest/ScalEqs.html (equations and symbolics)

- CAMB notes: http://cosmologist.info/notes/CAMB.pdf
  (equations and definitions)

- Other lectures/introductions:
  http://icg.port.ac.uk/~jschewts/cantata/CAMB/CAMB_lecture.pdf
  http://camb.info/Work_with_CAMB_V13_for_AL.pdf

Requirements

Local installation

Python 2.7 or 3.4+
Fortran 90 (gfortran 4.9+ or ifort)

Virtual machines
(Linux with all requirements pre-installed)

http://cosmologist.info/CosmoBox/

On Windows can install (unmodified) Python wrapper using “pip install camb” without a working gfortran installation. Otherwise get 64-bit gfortran (https://gcc.gnu.org/wiki/GFortranBinaries) or use virtual machine.

Main numerical code is in Fortran, wrapped by Python module. Two main branches on github:

master: uses standard Fortran 90 and will compile on most F90 compilers
devel: is Fortran 2003 (uses classes) and requires gfortran 6+ or ifort 14+

Both branches are maintained. *devel* is a somewhat modernized update that allows things like run-time switching of the dark energy model (also supported in python wrapper). Intended to become the default once gfortran 6 becomes more widespread.

Examples here use the *master* (default) version.

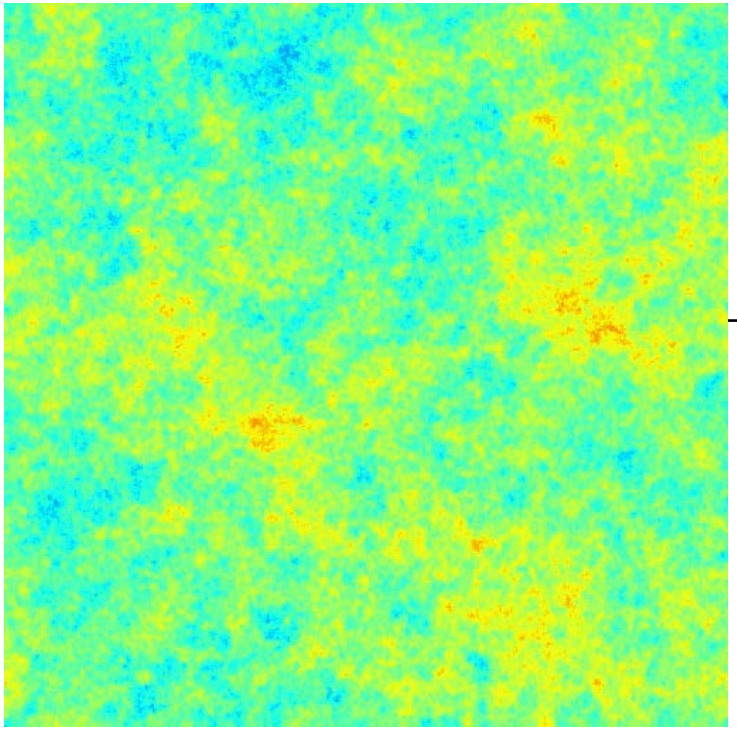
Alternative/cross-check: CLASS http://class-code.net/

CMB power spectra (C_l, temperature and polarization)

10^{-5} perturbations ⇒ Linear theory predictions very accurate

⇒ Fourier modes of perturbations evolve independently

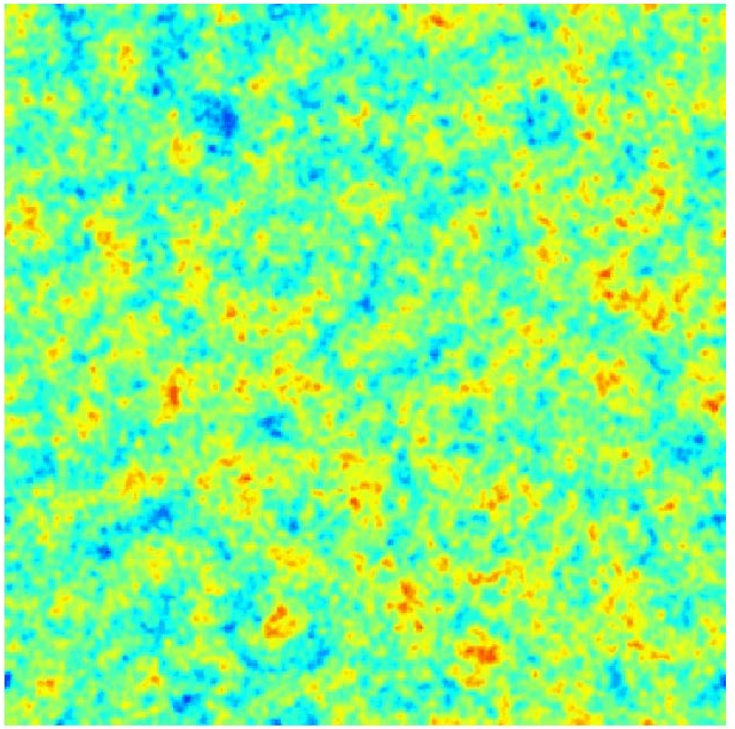
End of inflation (A_s, n_s, n_{run})



gravity+
pressure+
diffusion

$(\Omega_b h^2,$
$\Omega_c h^2 ..)$

Last scattering surface



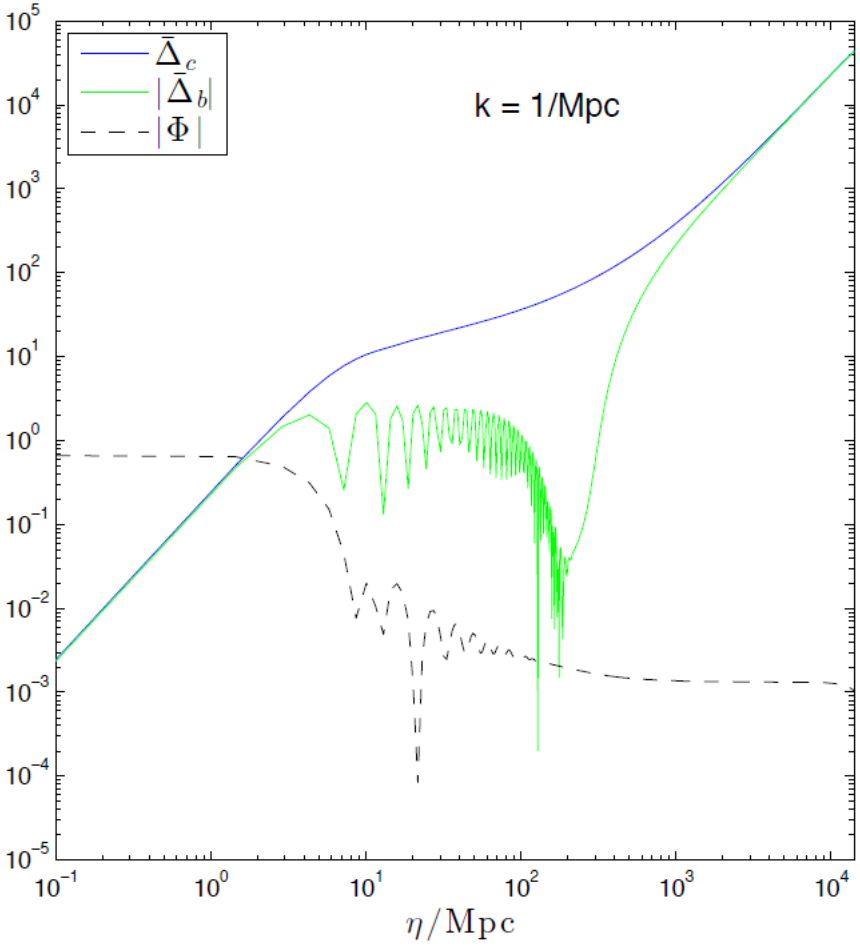
Linear, but:

relativistic (GR), Boltzmann hierarchies for relativistic species (γ, ν), photon polarization

To calculate power spectrum from statistically homogeneous perturbations do *not* need to evolve realisations (unlike in large-scale structure simulations)

Linearity: $X(\boldsymbol{k},\eta) = X(\boldsymbol{k},0)T(k,\eta)$

- only need to evolve $T(k,\eta)$, tells you how *all* perturbations with same |**k**| evolve



Transfer functions
for each perturbation:

e.g.
$\Delta_c = \delta\rho_c/\rho_c$
$\Delta_b = \delta\rho_b/\rho_b$
Φ = gravitational potential

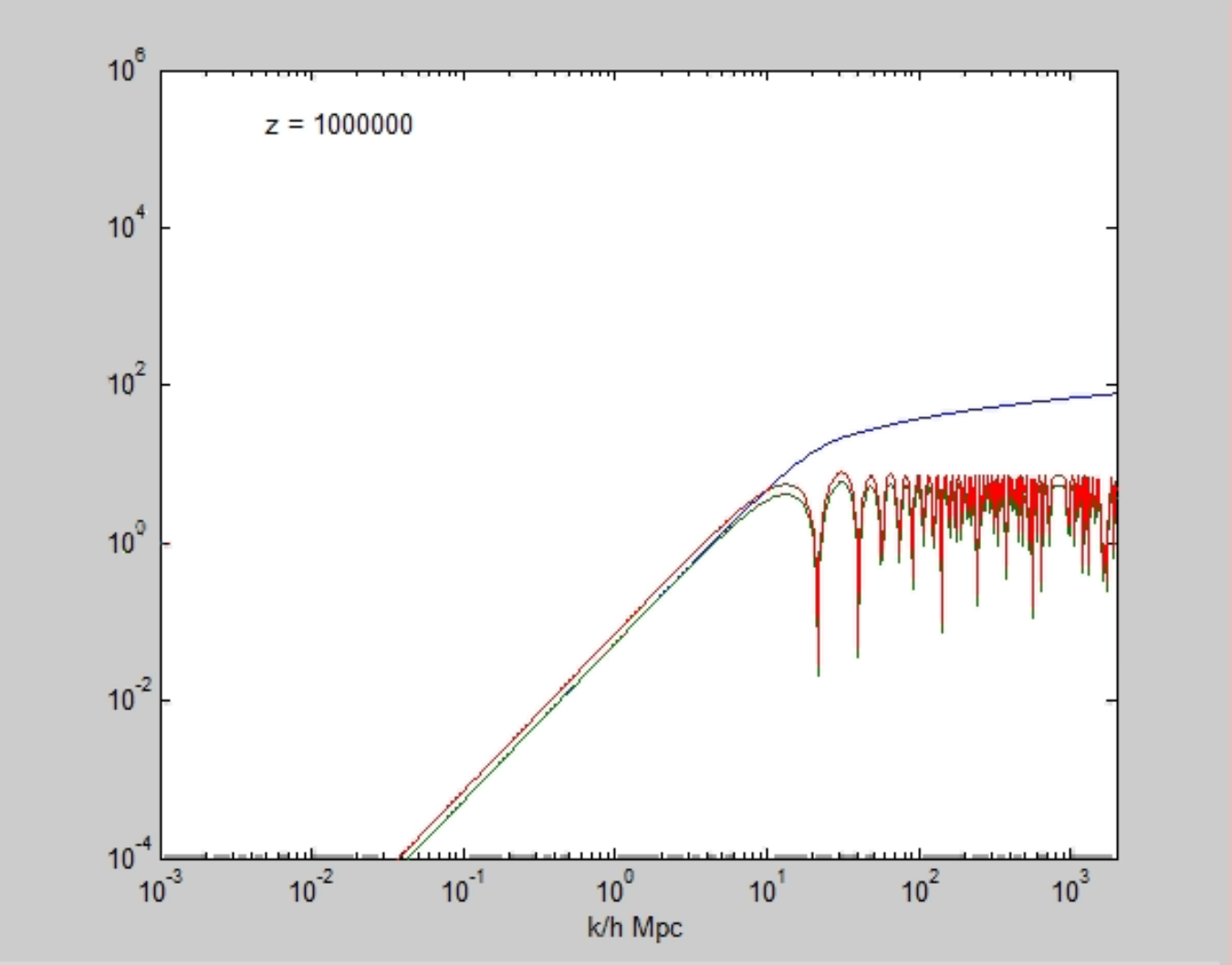
Note: CAMB integrates transfer functions for variables defined in the CDM frame/synchronous gauge.

Outside horizon
No causal contact

Acoustic oscillations
in radiation fluid

Growth of structure

Just need to evolve 1D grid of k values for each species/mutipole



Photon, Baryon and dark matter transfer functions

Integration of ~50 differential equations (Boltzmann hierarchy in angular modes l for each species)
-for each decoupled wavenumber mode (few hundred k)
- for each type of perturbation (density, vorticity, gravitational wave) if needed
- from conformal time η when modes well outside horizon (series solution) until after recombination or today

Photon temperature ($0 \leq l \leq 2-30$)

$$I_l' + \frac{k}{2l+1}\left[\beta_{l+1}^m \frac{(l+1)^2-m^2}{l+1} I_{l+1} - l I_{l-1}\right] =$$

$$- S n_e \sigma_T \left(I_l - \delta_{l0} I_0 - \frac{4}{3}\delta_{l1} v - \frac{2}{15}\zeta \delta_{l2}\right) + \frac{8}{15} k\sigma \delta_{l2} - 4h'\delta_{l0} - \frac{4}{3} k A \delta_{l1}$$

Coupling of distribution function multipoles

Thomson scattering
$\gamma + e^- \rightarrow \gamma + e^-$

Gravitational coupling

Photon polarization

$$\mathcal{E}_l^{m\pm\prime} + k\left[\beta_{l+1}^m \frac{(l+3)(l-1)}{(l+1)^3}\frac{(l+1)^2-m^2}{(2l+1)}\mathcal{E}_{l+1}^{m\pm} - \frac{l}{2l+1}\mathcal{E}_{l-1}^{m\pm} - \frac{2m}{l(l+1)}\sqrt{\beta_0^m}\mathcal{B}_l^{m\mp}\right] = -S n_e \sigma_T (\mathcal{E}_l^{m\pm} - \frac{2}{15}\zeta^{m\pm}\delta_{l2})$$

$$\mathcal{B}_l^{m\pm\prime} + k\left[\beta_{l+1}^m \frac{(l+3)(l-1)}{(l+1)^3}\frac{(l+1)^2-m^2}{(2l+1)}\mathcal{B}_{l+1}^{m\pm} - \frac{l}{2l+1}\mathcal{B}_{l-1}^{m\pm} + \frac{2m}{l(l+1)}\sqrt{\beta_0^m}\mathcal{E}_l^{m\mp}\right] = 0. \qquad (15)$$

+ neutrinos (no scattering, but massive so energy dependent)

+ Baryons, dark matter, dark energy, …

+ Gravitational perturbations

Equations are evolved (using Runge-Kutta) and used to compute "sources" $S(\eta, k)$ for each observable which are integrated over time to get the l-space "transfers":

$$T_{X,l}(k) = \int_0^{\eta_0} j_l(k(\eta_0 - \eta)) S_X(\eta, k)$$

X is T, E, B *or* ϕ (CMB lensing potential). S_X depends on lowest moments of the distribution functions, e.g. monopole, dipole, quadrupole etc.

Finally adding up all the modes integrated against primordial curvature perturbation power spectrum $P_R(k)$ gives the angular power spectra

$$C_l^{XY} \propto \int d \ln k \;\; P_R(k)\, T_{X,l}(k) T_{Y,l}(k)$$

The matter power spectrum is simpler
(though not directly observable: function of comoving k at constant time slice)

$$P(k, z) \propto P_R(k) T_\Delta(k, \eta(z))$$

(+ non-linear corrections: in CAMB fit using HALOFIT approximate model)

Δ is usually defined as the total synchronous gauge matter density perturbation.

Using CAMB: Python wrapper

Installation:
Unmodified default code: “pip install camb”
From source: in pycamb folder “python setup.py install”

See sample notebook: http://camb.readthedocs.io/en/latest/CAMBdemo.html

Load the module

```
import camb
from camb import model, initialpower
```

Make a **CAMBparams** object instance and set parameters you want

```
#Set up a new set of parameters for CAMB
pars = camb.CAMBparams()
#This function sets up CosmoMC-like settings, with one massive neutrino and helium set using BBN consistency
pars.set_cosmology(H0=67.5, ombh2=0.022, omch2=0.122, mnu=0.06, omk=0, tau=0.06)
pars.InitPower.set_params(ns=0.965, r=0)
```

Set l_{max} you want. *lens_potential_accuracy=1* gets non-linear lensing potential.

```
pars.set_for_lmax(2000, lens_potential_accuracy=1)
```

Actually do the calculation and get a results object:

```
results = camb.get_results(pars)
```

Then get things you want from the results object:

```
#get dictionary of CAMB power spectra
powers =results.get_cmb_power_spectra(pars)
for name in powers: print name
```

```
total
lens_potential
lensed_scalar
unlensed_scalar
unlensed_total
tensor
```

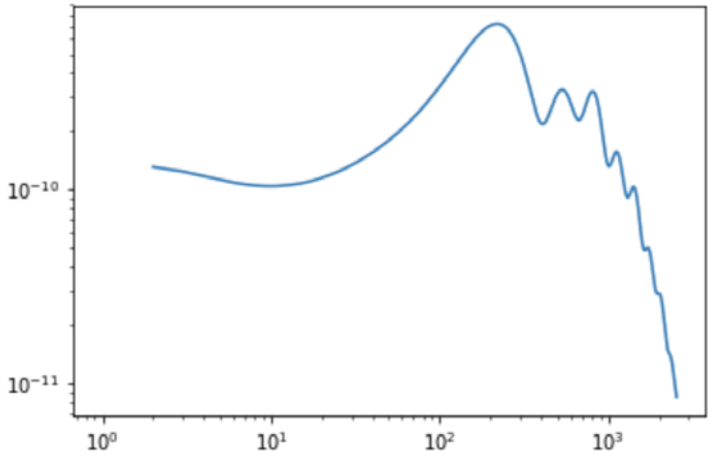
For example the total (lensed scalar+tensor) CMB power spectra are

```
totCL=powers['total']
```

totCL[L, i] is C_L for i=0 (TT),1 (EE), 2 (BB),3 (TE)

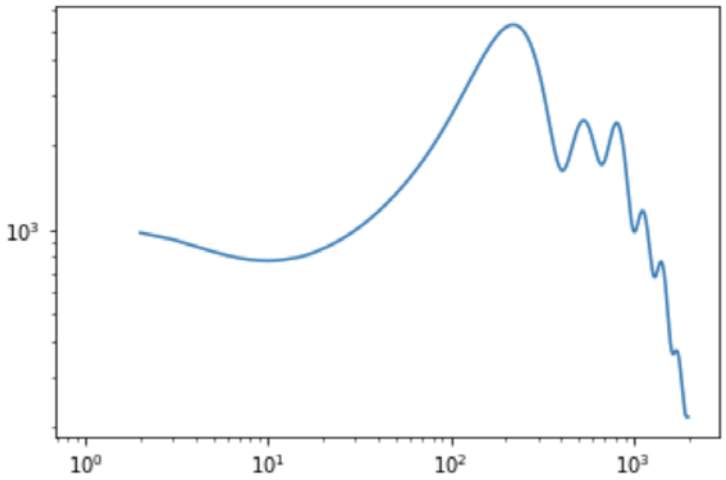
C_L result array by default are $\frac{l(l+1)C_l}{2\pi}$ in dimensionless units (i.e. $\Delta T/T$)

```
plt.loglog(np.arange(totCL.shape[0]), totCL[:,0]);
```



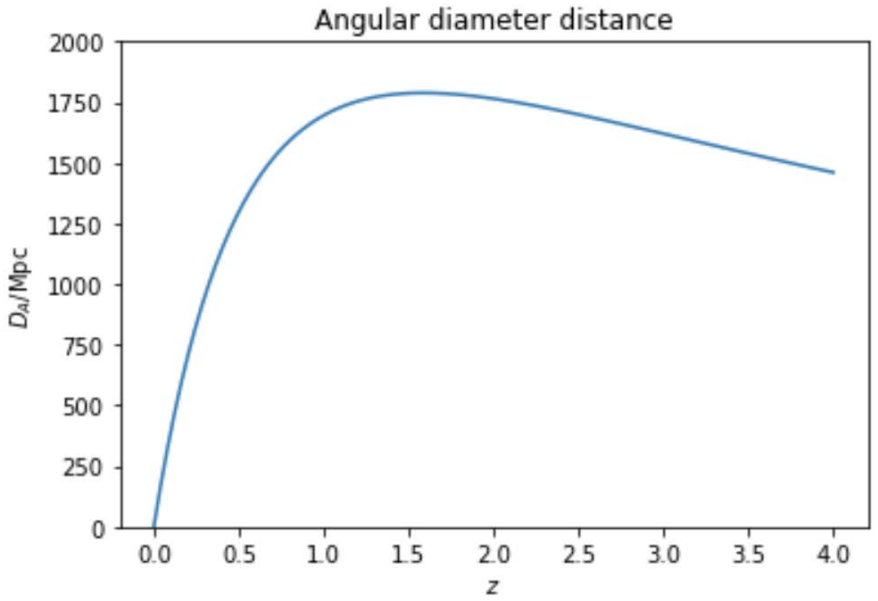

```
lensed_scalar =results.get_lensed_scalar_cls(lmax=2000, CMB_unit = 'muK')
plt.loglog(np.arange(2001), lensed_scalar[:,0]);
```

Can use other functions
to pull spectrum of interest
and change units.
E.g. in μK^2 units.



Similarly you can get the matter power spectrum, lensing potential spectrum, background functions, derives parameters and more.

```
z = np.linspace(0,4,100)
DA = results.angular_diameter_distance(z)
plt.plot(z, DA)
plt.xlabel('$z$')
plt.ylabel(r'$D_A /\rm{Mpc}$')
plt.title('Angular diameter distance')
plt.ylim([0,2000]);
```



See the [example notebook] for for further examples

camb.symbolic

Python module using **sympy** giving most of the symbolic equations being used, relations between variables, functions to change between gauges and write camb code.

```
from camb.symbolic import *
sympy.init_printing()
print 'CAMB: ', camb.__version__,'Sympy: ', sympy.__version__
```

```
CAMB:  0.1.5 Sympy:  1.0
```

```
display('background_eqs',background_eqs)
display('contraints',constraints)
display('var_subs',var_subs)
display('q_sub',q_sub)
display('pert_eqs',pert_eqs)
display('total_eqs',total_eqs)
#can use tot_eqs as combination of total_eqs + pert_eqs + background_eqs
```

'background_eqs'

$$\left[\frac{d}{dt}a(t) = H(t)a(t), \quad \frac{d}{dt}H(t) = -\frac{\kappa}{6}\left(3P(t) + \rho(t)\right)a^2(t), \quad \frac{d}{dt}\mathrm{exptau}\,(t) = \mathrm{visibility}\,(t)\right]$$

'contraints'

$$\left[Kf_1k^3\phi(t) + \frac{k\kappa}{2}\left(Kf_1\Pi(t) + \delta(t)\right)a^2(t) + \frac{3\kappa}{2}H(t)a^2(t)q(t), \quad k^2\eta(t) + 2kH(t)z(t) - \kappa a^2(t)\delta(t), \quad \frac{2k^2}{3a^2(t)}\left(-Kf_1\sigma(t) + z(t)\right) + \kappa q(t), \quad -\frac{k}{3}z(t) + A(t)H(t) + \dot{h}(t)\right]$$

'var_subs'

$$\left\{\dot{h}(t) : \frac{1}{6H(t)}\left(-k^2\eta(t) + \kappa a^2(t)\delta(t) - 6A(t)H^2(t)\right), \quad \phi(t) : -\frac{\kappa a^2(t)}{2Kf_1k^3}\left(Kf_1k\Pi(t) + k\delta(t) + 3H(t)q(t)\right), \quad \sigma(t) : \frac{1}{2Kf_1k^2H(t)}\left(k\left(-k^2\eta(t) + \kappa a^2(t)\delta(t)\right) + 3\kappa H(t)a^2(t)q(t)\right), \quad z(t) : \frac{1}{2kH(t)}\left(-k^2\eta(t) + \kappa a^2(t)\delta(t)\right)\right\}$$

'q_sub'

CAMB uses covariant perturbation variables in the 3+1 formulation.

camb.symbolic defines equations that are valid in any gauge. They can be written in specific gauges by specific choices of restrictions on variables.

```
#Fluid components
display('density_eqs',density_eqs)
display('delta_eqs', delta_eqs)
display('vel_eqs',vel_eqs)
#can use component_eqs as combination of density_eqs + delta_eqs + vel_eqs
```

'density_eqs'

$$\left[\frac{d}{dt}\rho_b(t) = -3\left(\mathrm{p_b}\left(t\right) + \rho_b(t)\right)H(t), \quad \frac{d}{dt}\rho_c(t) = -3H(t)\rho_c(t), \quad \frac{d}{dt}\rho_g(t) = -4H(t)\rho_g(t), \quad \frac{d}{dt}\rho_r(t) = -4H(t)\rho_r(t),\right.$$
$$\left.\frac{d}{dt}\rho_\nu(t) = -3\left(\mathrm{p_\nu}\left(t\right) + \rho_\nu(t)\right)H(t), \quad \frac{d}{dt}\rho_{de}(t) = -3\left(\mathrm{w_{de}}\left(t\right) + 1\right)H(t)\rho_{de}(t)\right]$$

'delta_eqs'

$$\left[\frac{d}{dt}\Delta_r(t) = -k\,\mathrm{q_r}\left(t\right) - 4\dot{h}(t), \quad \frac{d}{dt}\Delta_g(t) = -k\,\mathrm{q_g}\left(t\right) - 4\dot{h}(t),\right.$$
$$\frac{d}{dt}\Delta_b(t) = \left(k\,\mathrm{v_b}\left(t\right) + 3\dot{h}(t)\right)\left(-\frac{\mathrm{p_b}\left(t\right)}{\rho_b(t)} - 1\right) + \left(-3\,\mathrm{c_{sb}^2}\left(t\right) + \frac{3\,\mathrm{p_b}\left(t\right)}{\rho_b(t)}\right)\Delta_b(t)H(t), \quad \left.\frac{d}{dt}\Delta_c(t) = -k\,\mathrm{v_c}\left(t\right) - 3\dot{h}(t),\right.$$

...

Convert between Newtonian gauge, synchronous/CDM frame results, and other possible frame choices.

```
#e.g. can check we recover standard Newtonian gauge equations
#(note all equations above are valid in any frame)
newtonian_gauge(delta_eqs)
```

$$\left[\frac{d}{dt}\Delta_r(t) = -k\,\mathrm{q_r}\,(t) + 4\frac{d}{dt}\Phi_N(t), \quad \frac{d}{dt}\Delta_g(t) = -k\,\mathrm{q_g}\,(t) + 4\frac{d}{dt}\Phi_N(t),\right.$$

$$\frac{d}{dt}\Delta_b(t) = -\frac{1}{\rho_b(t)}\left(\left(k\,\mathrm{v_b}\,(t) - 3\frac{d}{dt}\Phi_N(t)\right)\left(\mathrm{p_b}\,(t) + \rho_b(t)\right) + 3\left(\mathrm{c_{sb}^2}\,(t)\rho_b(t) - \mathrm{p_b}\,(t)\right)\Delta_b(t)H(t)\right),$$

$$\frac{d}{dt}\Delta_c(t) = -k\,\mathrm{v_c}\,(t) + 3\frac{d}{dt}\Phi_N(t),$$

...

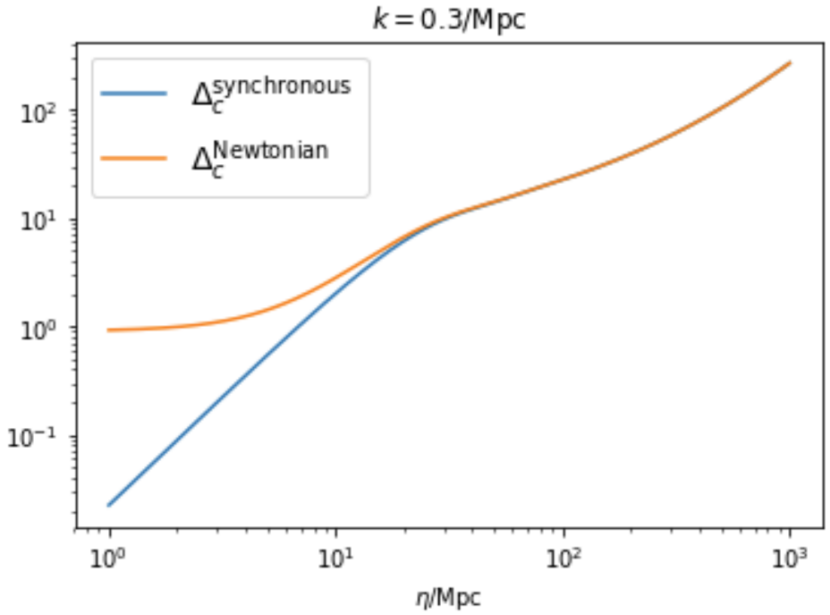
camb_fortran can convert symbolic expressions into CAMB-variable Fortran source code

```
#These are definitions used in CAMB to get the various sources for the temperature
print camb_fortran(dphi, 'phidot')
print camb_fortran(dsigma, 'sigmadot')
print camb_fortran(diff(polter_t,t), 'polterdot')
print camb_fortran(polterddot, 'polterddot')
print camb_fortran(2*diff(phi,t)*exptau,'ISW')
print camb_fortran(monopole_source, 'monopole_source')
print camb_fortran(doppler, 'doppler')
print camb_fortran(quadrupole_source, 'quadrupole_source')
```

```
phidot = (1.0d0/2.0d0)*(-adotoa*dgpi - 2*adotoa*k**2*phi + dgq*k &
    -diff_rhopi + k*sigma*(gpres + grho))/k**2
sigmadot = -adotoa*sigma - 1.0d0/2.0d0*dgpi/k + k*phi
polterdot = (1.0d0/10.0d0)*pigdot + (3.0d0/5.0d0)*Edot(2)
polterddot = -2.0d0/25.0d0*adotoa*dgq/(k*Kf(1)) - &
    4.0d0/75.0d0*adotoa*k*sigma - 4.0d0/75.0d0*dgpi - &
    2.0d0/75.0d0*dgrho/Kf(1) + dopacity*(-1.0d0/10.0d0*pig + &
    (7.0d0/10.0d0)*polter - 3.0d0/5.0d0*E(2)) &
    -3.0d0/50.0d0*k*octgdot*Kf(2) + (1.0d0/25.0d0)*k*qgdot - &
    1.0d0/5.0d0*k*Edot(3)*Kf(2) + opacity*(-1.0d0/10.0d0*pigdot + &
```

Pass symbolic expressions to some camb functions to get numerical results directly
(behind the scenes it converts to Fortran, compiles, loads and then calls it from the main code)

```
data = camb.get_transfer_functions(pars)
#For example, this plots the Newtonian gauge density compared to the synchronous gauge one
import camb.symbolic as cs
Delta_c_N = cs.make_frame_invariant(cs.Delta_c, 'Newtonian')
ev=data.get_time_evolution(k, eta, ['delta_cdm',Delta_c_N])
plt.figure(figsize=(6,4))
plt.loglog(eta,ev[:,0])
plt.loglog(eta,ev[:,1])
plt.title(r'$k= %s/\rm{Mpc}$'%k)
plt.xlabel(r'$\eta/\rm{Mpc}$');
plt.legend([r'$\Delta_c^{\rm synchronous}$', r'$\Delta_c^{\rm Newtonian}$'], fontsize=14);
```



See further symbolic examples at
http://camb.readthedocs.io/en/latest/ScalEqs.html

Using CAMB: Compiling and using command line

Recommend to clone from git repository

```
git clone https://github.com/cmbant/CAMB.git
```

Compile Fortran: `make` **(see Fortran compiler requirements earlier)**

Run: `./camb params.ini` **(params.ini is text file of input parameter values)**

Output: some parameter values and a set of .dat text files with results

```
-rw-rw-r--  1 aml1005 aml1005    1737 Oct 23 17:26 test_params.ini
-rw-rw-r--  1 aml1005 aml1005  180401 Oct 23 17:26 test_scalCls.dat
-rw-rw-r--  1 aml1005 aml1005  312401 Oct 23 17:26 test_scalCovCls.dat
-rw-rw-r--  1 aml1005 aml1005  140701 Oct 23 17:26 test_lensedCls.dat
-rw-rw-r--  1 aml1005 aml1005  246401 Oct 23 17:26 test_lenspotentialCls.dat
```

(the xxx_params.ini file logs the parameters actually read and used for the run.)