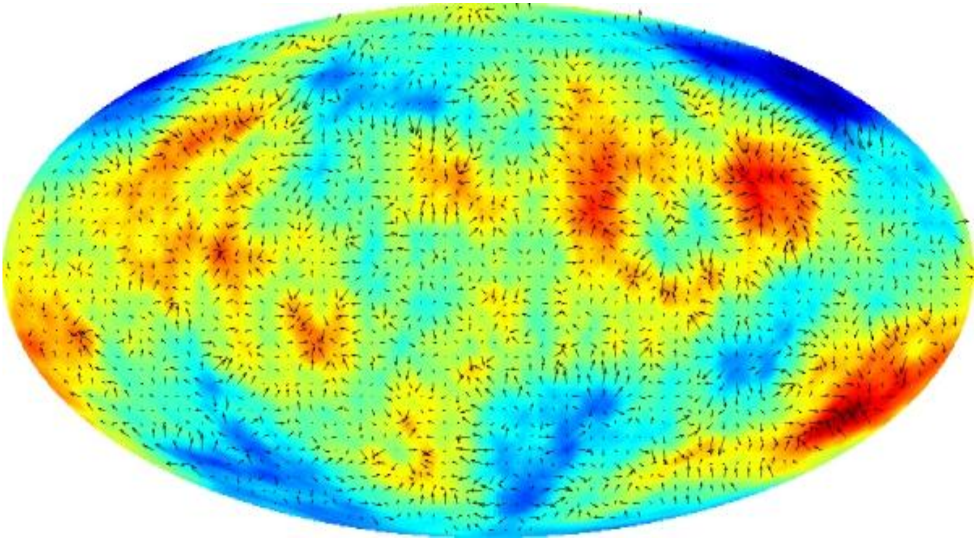
Primordial squeezed non-Gaussianity and observables in the CMB



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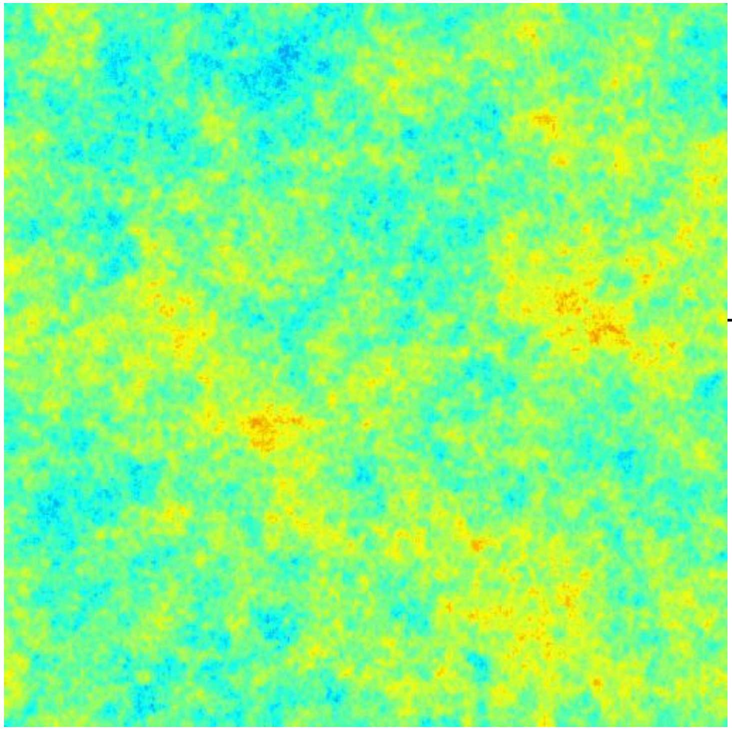
Lewis ***arXiv:1204.5018***
see also Creminelli et al astro-ph/0405428, arXiv:1109.1822, Bartolo et al arXiv:1109.2043

Lewis ***arXiv:1107.5431***
Lewis, Challinor & Hanson ***arXiv:1101.2234***
Pearson, Lewis & Regan ***arXiv:1201.1010***

Benasque August 2012

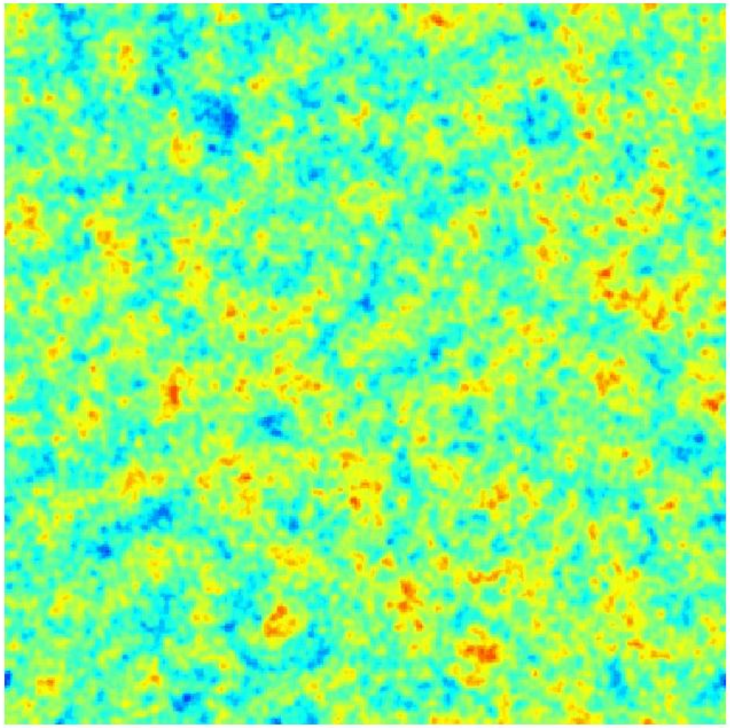
CMB temperature

End of inflation



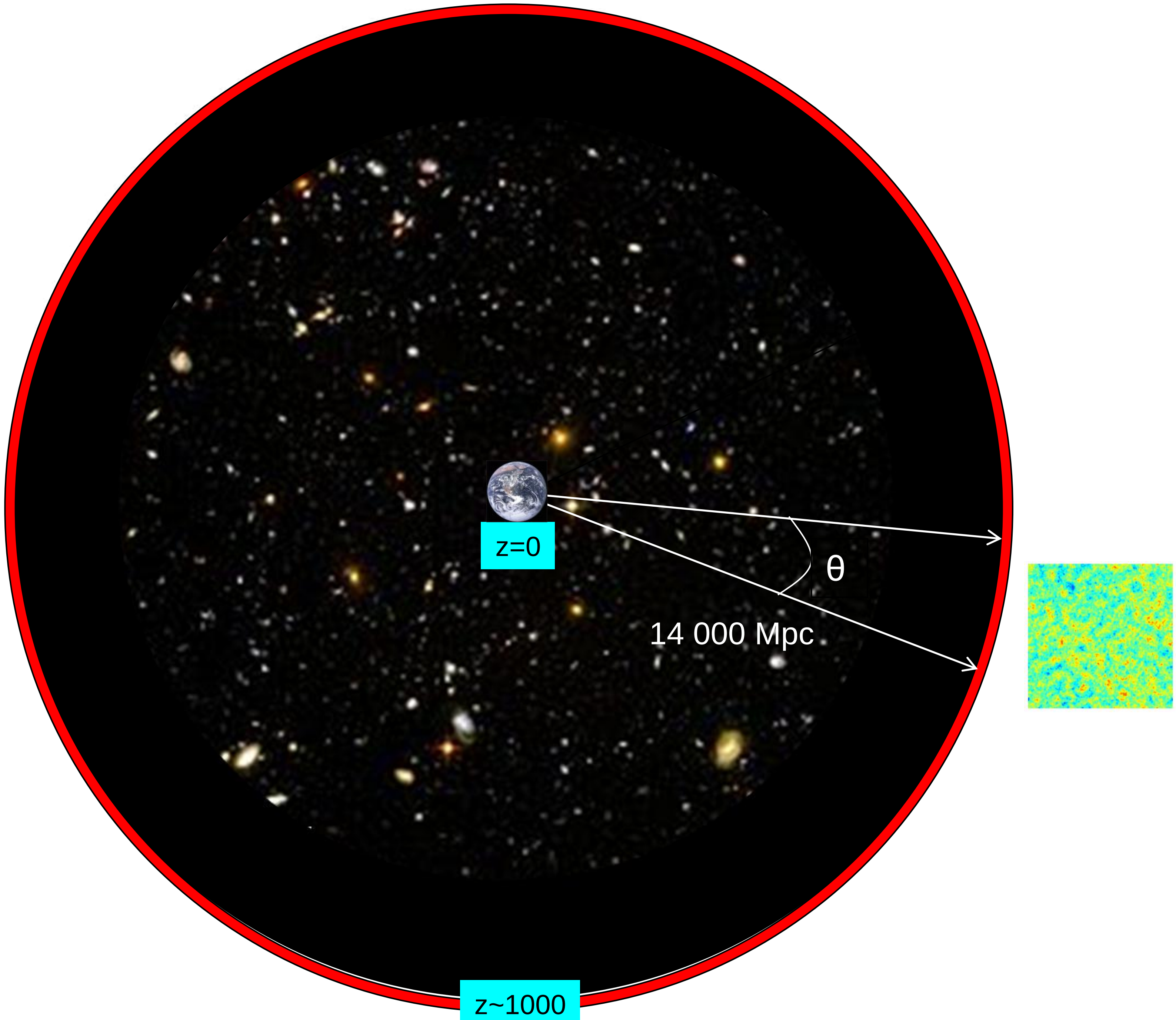
gravity+
pressure+
diffusion

Last scattering surface

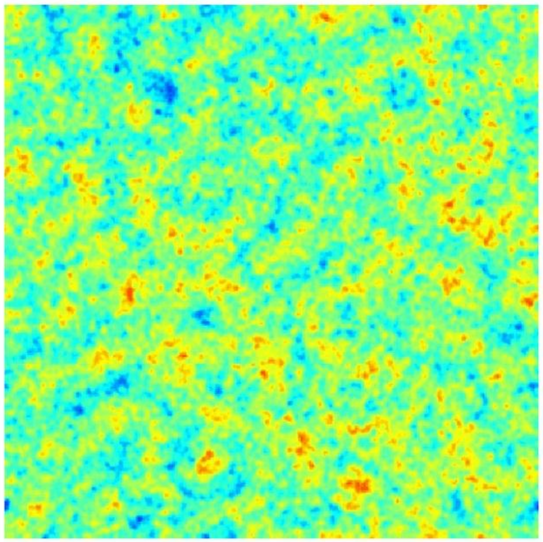


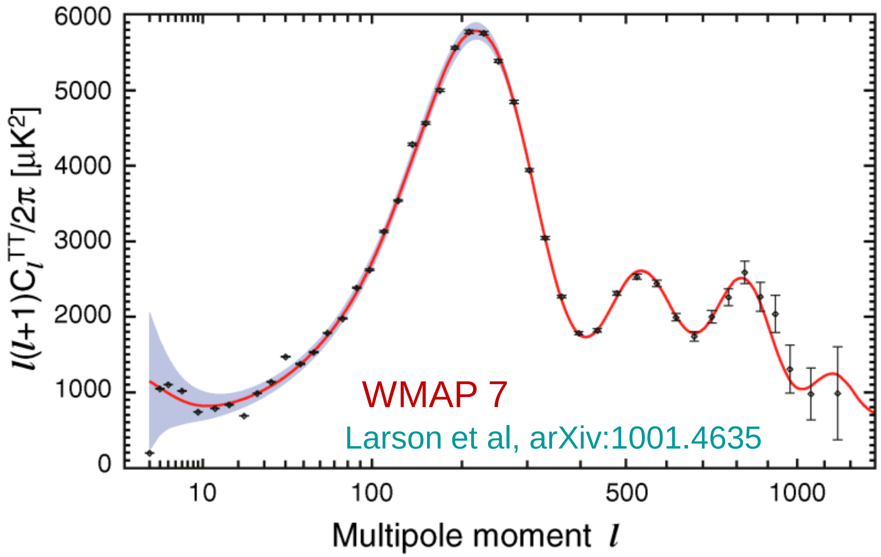
Perturbations super-horizon

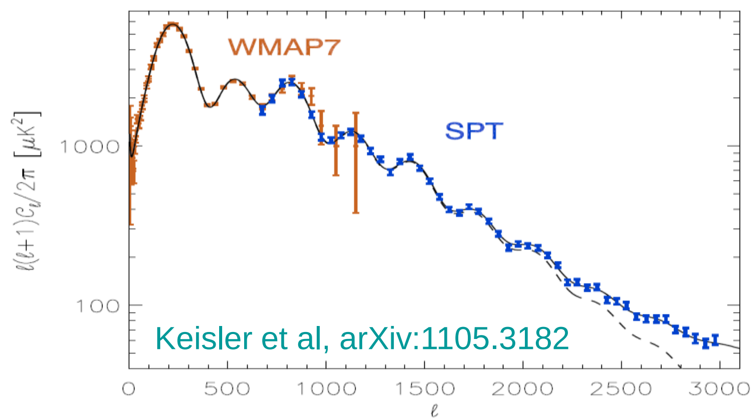
Sub-horizon acoustic oscillations
+ modes that are still super-horizon


z=0
θ
14 000 Mpc
z~1000

Observed CMB temperature power spectrum







Observations

Constrain theory of early universe
+ evolution parameters and geometry

Beyond Gaussianity – general possibilities

Flat sky approximation: $\Theta(x) = \frac{1}{2\pi}\int d^2l\,\Theta(l)e^{ix\cdot l}$ $(\Theta = T)$

Gaussian + statistical isotropy

$$\langle\Theta(l_1)\Theta(l_2)\rangle = \delta(l_1 + l_2)C_l$$

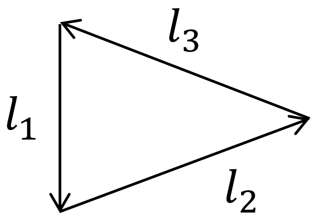
- power spectrum encodes all the information
- modes with different wavenumber are independent

Higher-point correlations

Gaussian: can be written in terms of C_l

Non-Gaussian: non-zero connected n-point functions

Bispectrum



$$\boldsymbol{l_1} + \boldsymbol{l_2} + \boldsymbol{l_3} = \boldsymbol{0}$$

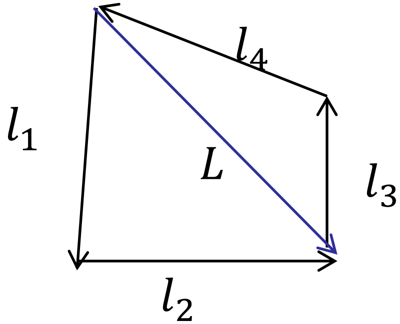
Flat sky approximation: $\langle \Theta(l_1)\Theta(l_2)\Theta(l_3)\rangle = \frac{1}{2\pi}\delta(l_1 + l_2 + l_3) b_{l_1 l_2 l_3}$

If you know $\Theta(l_1), \Theta(l_2)$, sign of $b_{l_1 l_2 l_3}$ tells you which sign of $\Theta(l_3)$ is more likely

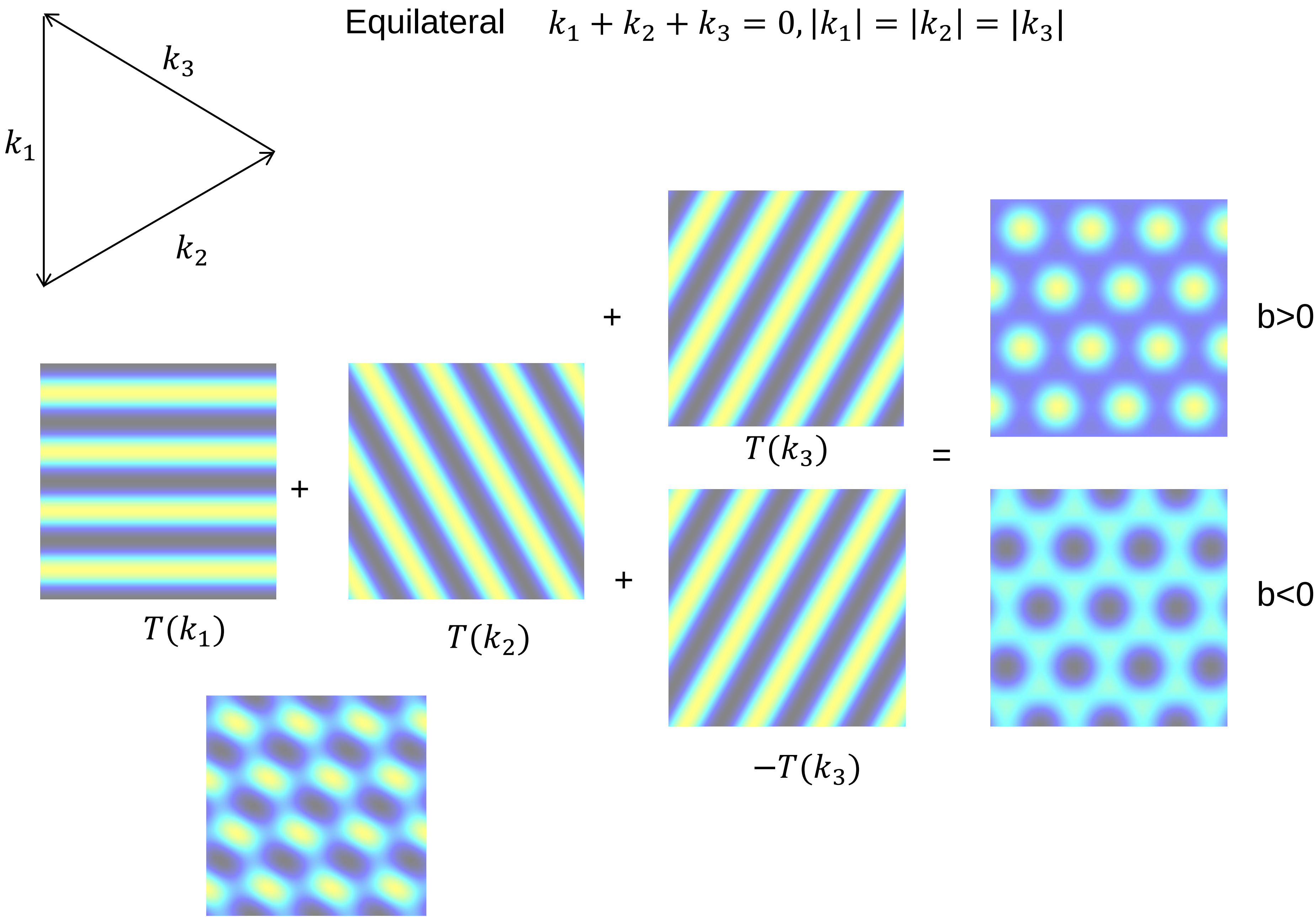
Trispectrum

$$\langle \Theta(\mathbf{l}_1)\Theta(\mathbf{l}_2)\Theta(\mathbf{l}_3)\Theta(\mathbf{l}_4)\rangle_C = (2\pi)^{-2}\delta(\mathbf{l}_1 + \mathbf{l}_2 + \mathbf{l}_3 + \mathbf{l}_4) T(\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3, \mathbf{l}_4)$$

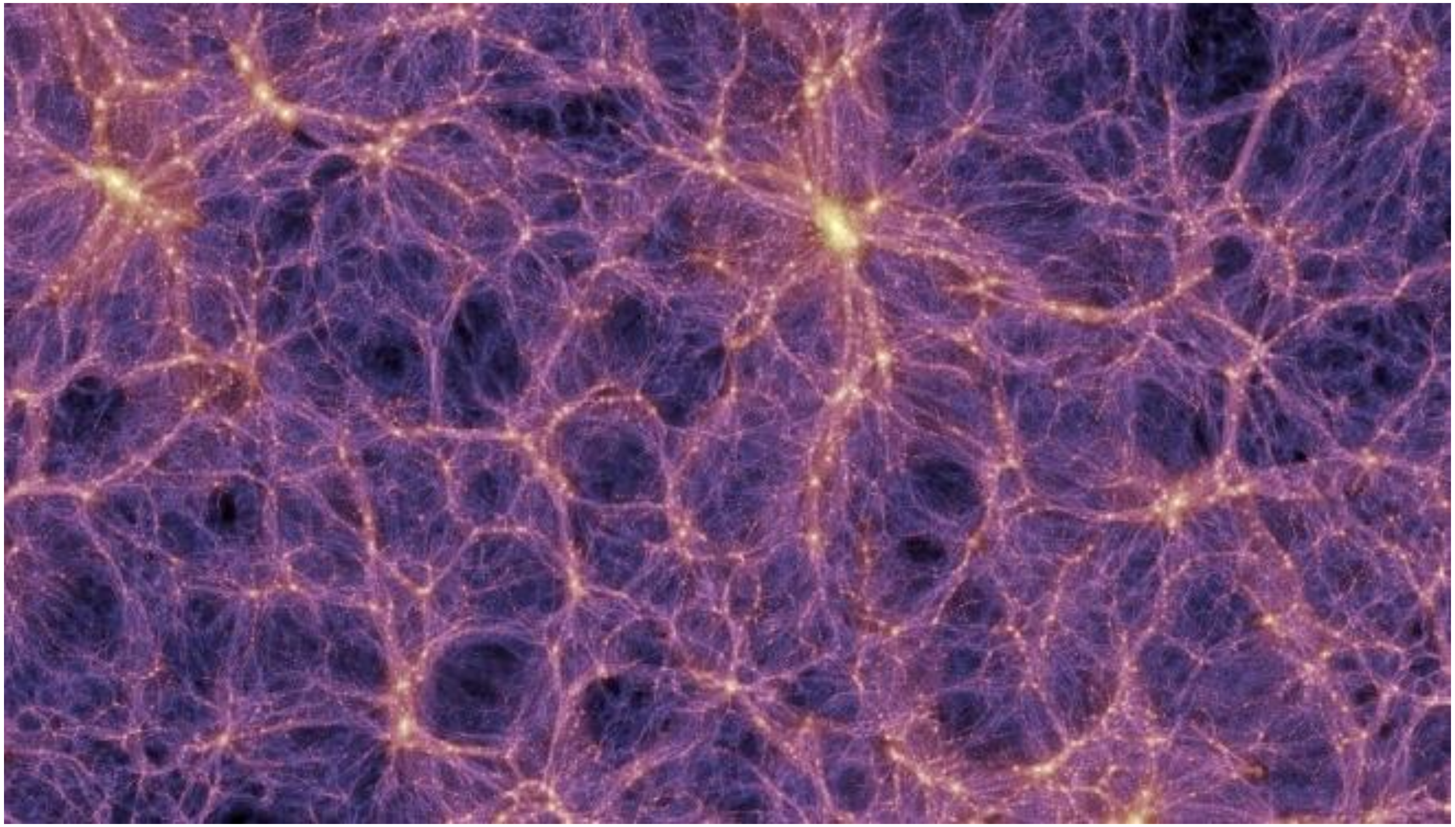
$$\langle \Theta(\mathbf{l}_1)\Theta(\mathbf{l}_2)\Theta(\mathbf{l}_3)\Theta(\mathbf{l}_4)\rangle_C = \frac{1}{2}\int \frac{d^2\mathbf{L}}{(2\pi)^2}\delta(\mathbf{l}_1 + \mathbf{l}_2 + \mathbf{L})\delta(\mathbf{l}_3 + \mathbf{l}_4 - \mathbf{L})\mathbb{T}^{(\ell_1 \ell_2)}_{(\ell_3 \ell_4)}(L) + \text{perms.}$$



N-spectra…

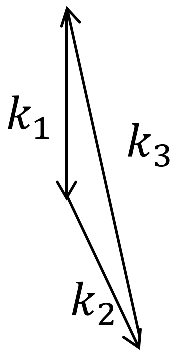


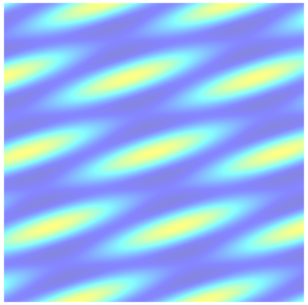
Equilateral $k_1 + k_2 + k_3 = 0, |k_1| = |k_2| = |k_3|$
k_1
k_2
k_3
+
+
+
=
$T(k_1)$
$T(k_2)$
$T(k_3)$
$-T(k_3)$
b>0
b<0

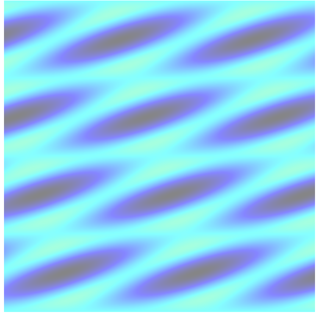


Millennium simulation

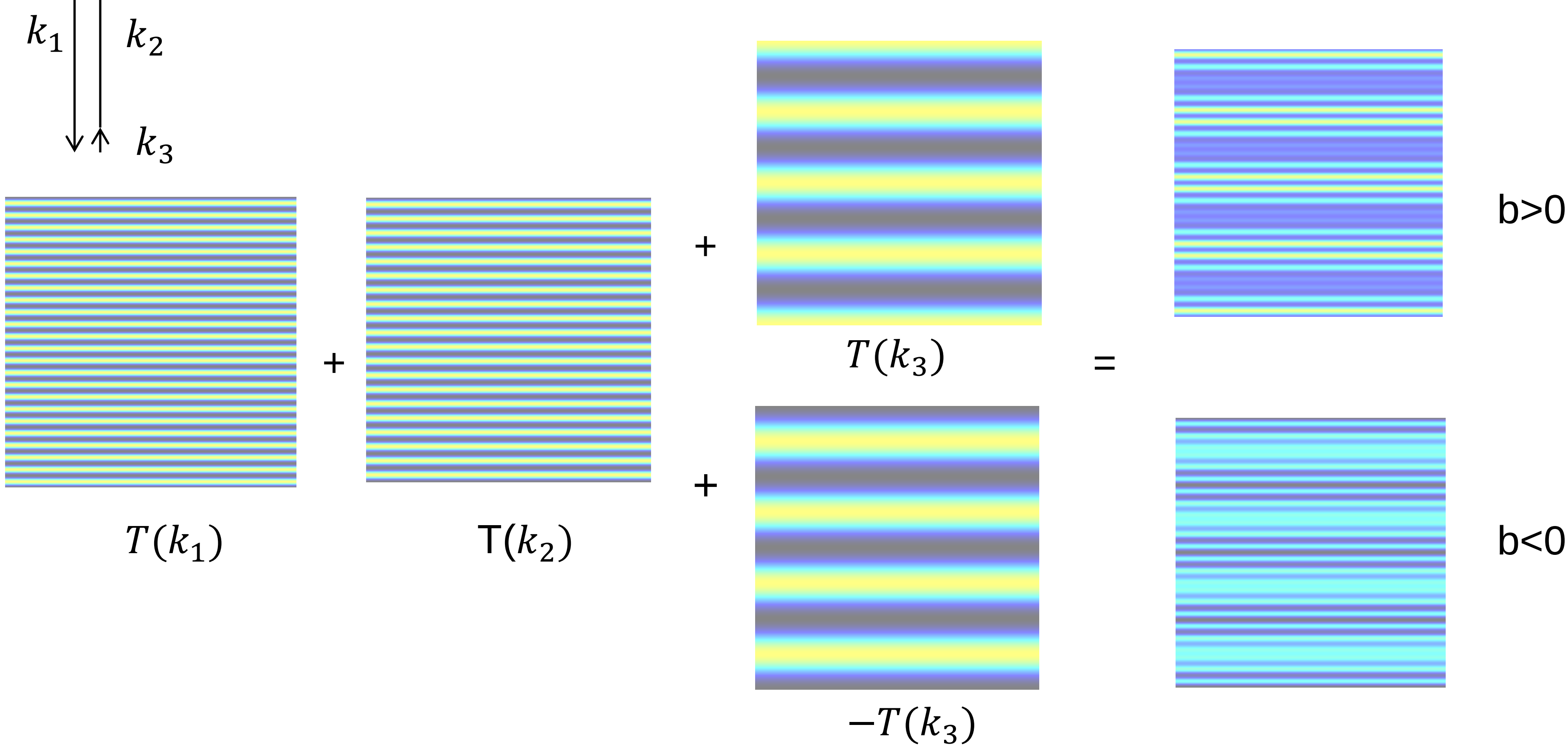
Near-equilateral to flattened:







Local (squeezed) $k_1 + k_2 + k_3 = 0, \quad k_1 \ll k_2, k_3 \quad k_2 \sim -k_3$



Squeezed bispectrum is a *correlation* of small-scale power with large-scale modes

For more pretty pictures and trispectrum see: The Real Shape of Non-Gaussianities, arXiv:1107.5431

Squeezed bispectrum

'Linear-short leg' approximation very accurate for large scales where cosmic variance is large

$l_1 \ll l_2 \leq l_3$ with modulation field(s) X_i

$$\langle \tilde{T}_{l_1 m_1} \tilde{T}_{l_2 m_2} \tilde{T}_{l_3 m_3} \rangle \approx C_{l_1}^{T X_i} \left\langle \frac{\delta}{\delta X^*_{i, l_1 m_1}} \left(\tilde{T}_{l_2 m_2} \tilde{T}_{l_3 m_3} \right) \right\rangle$$

Correlation of the modulation with the large-scale field

Response of the small-scale power to changes in the modulation field (non perturbative)

Note: uses *only* the linear short leg approximation, otherwise non-perturbatively exact

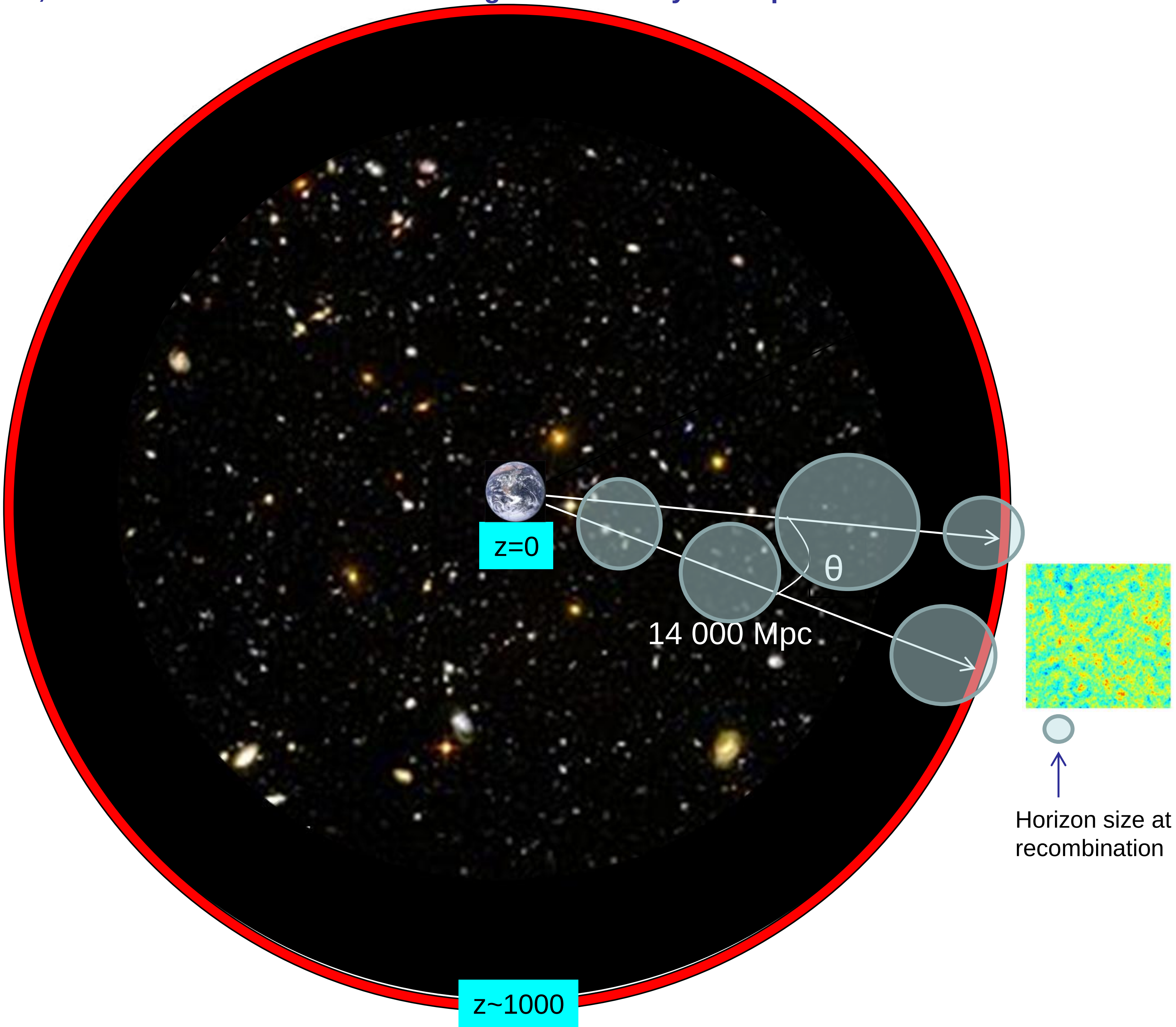
Example: local primordial non-Gaussianity

Primordial curvature perturbation is modulated as $\zeta = [1 + (3 f_{\mathrm{NL}}/5)\zeta_g]\zeta_g$

$l_1 \ll 100$: modulation super-horizon and constant through last-scattering, $\zeta_g = \zeta_0^*$

$$\Rightarrow \quad b_{l_1 l_2 l_3} \approx \frac{6}{5} f_{\mathrm{NL}} C_{l_1}^{T \zeta_0^*} (\tilde{C}_{l_2} + \tilde{C}_{l_3})$$

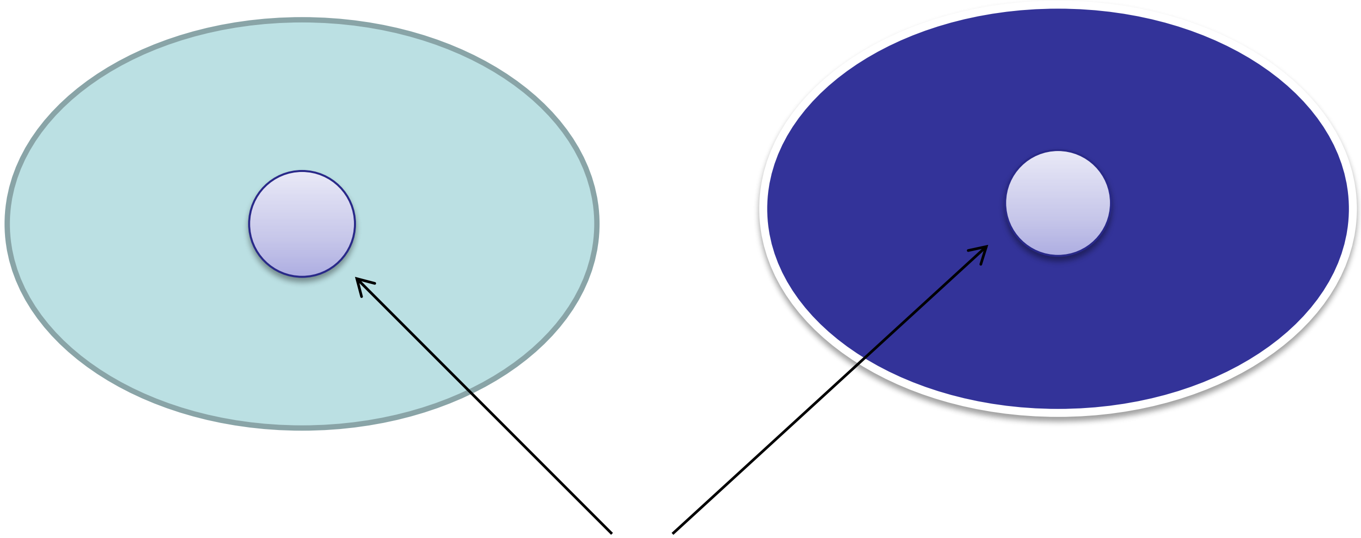
Even with $f_{NL} = 0$, we observe CMB at last scattering modulated by other perturbations



What is the modulating effect of large-scale super-horizon perturbations?

Single-field inflation: only one degree of freedom, e.g. everything determined by local temperature (density) on super-horizon scales

Cannot locally observe super-horizon perturbations (to $O(\frac{k^2}{H^2})$)



Observers in different places on LSS will see statistically exactly the same thing (at given fixed temperature/time from hot big bang)

- local physics is identical in Hubble patches that differ only by super-horizon modes

BUT: a distant observer *will* see modulations due to the large modes <~ horizon size today
- can see and compare multiple different Hubble patches

- Super-horizon modes induce linear perturbations on all scales

  linear CMB anisotropies on large scales ($l < 100$)

- Sub-horizon perturbations are observed in perturbed universe:
  -small-scale perturbations are modulated by the effect large-scale modes

  squeezed-shape non-Gaussianities

Linear CMB anisotropies

Linear perturbation theory with $ds^2 = a(\eta)^2 \left[(1+2\Psi)d\eta^2 - (1-2\Phi)d\mathbf{x}^2\right]$

Using the geodesic equation in the Conformal Newtonian Gauge:

$$E(\eta_0) = a(\eta)E(\eta)\left[1 + \Psi(\eta) - \Psi_0 + \int_\eta^{\eta_0} d\eta(\Psi' + \Phi')\right]$$

All photons redshift the same way, so $kT \sim E$.

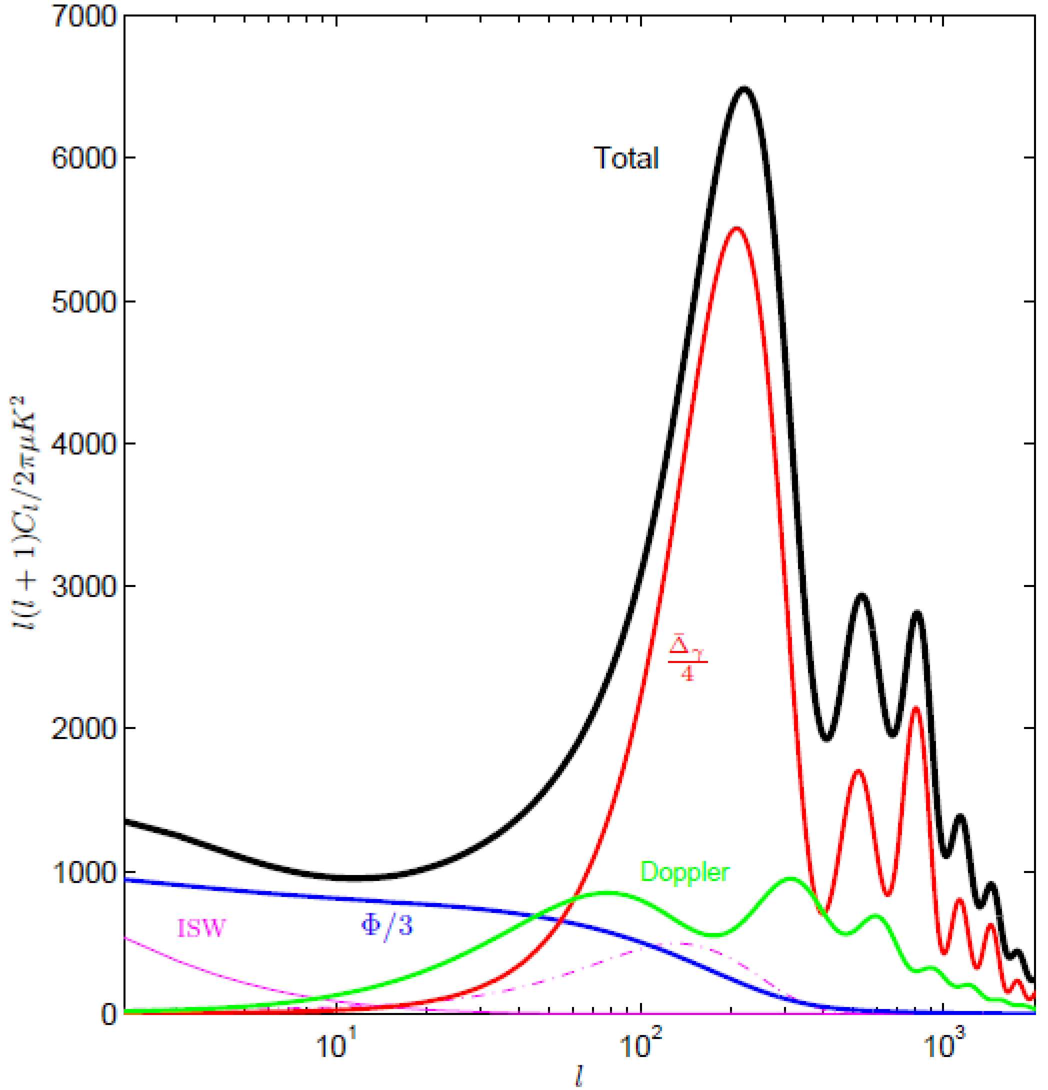
Recombination fairly sharp at background time η_*: ~ *constant temperature* surface.
Also add Doppler effect:

$$\begin{aligned} T(\hat{\mathbf{n}}, \eta_0) &= (a_* + \delta a)T_* \left[1 + \Psi(\eta_*) - \Psi_0 + \hat{\mathbf{n}} \cdot (\mathbf{v}_o - \mathbf{v}) + \int_{\eta_*}^{\eta_0} d\eta(\Psi' + \Phi')\right] \\ &= T_0 \left[1 + \frac{\delta a}{a_*} + \Psi(\eta_*) - \Psi_0 + \hat{\mathbf{n}} \cdot (\mathbf{v}_o - \mathbf{v}) + \int_{\eta_*}^{\eta_0} d\eta(\Psi' + \Phi')\right] \end{aligned}$$

$$\rho_\gamma \propto T^4 \propto a^4$$

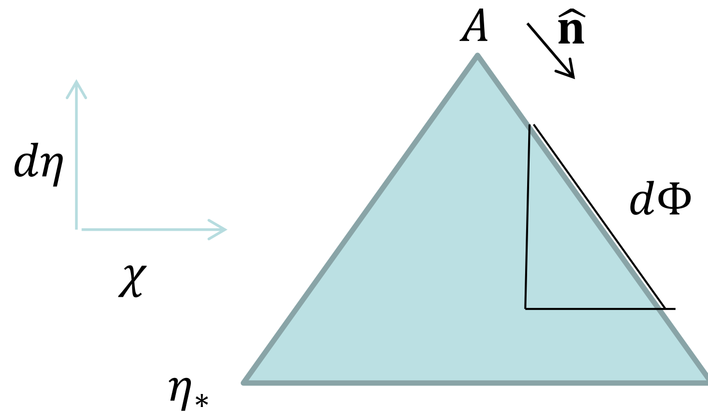
$$\Rightarrow \quad \frac{\Delta T_0}{T}(\hat{\mathbf{n}}) = \frac{\Delta_\gamma(\eta_*)}{4} + \underbrace{\Psi(\eta_*) - \Psi_0}_{\text{Sachs-Wolfe}} + \underbrace{\hat{\mathbf{n}} \cdot (\mathbf{v}_o - \mathbf{v})}_{\text{Doppler}} + \underbrace{\int_{\eta_*}^{\eta_0} d\eta (\Psi' + \Phi')}_{\text{ISW}}$$

Temperature perturbation at recombination



Note: no scale on which Sachs-Wolfe $\Phi/3$ result is accurate
Doppler dominates at $l \sim 60$ because other terms cancel

Alternative



$$d\Phi = \frac{\partial\Phi}{\partial\eta}\, d\eta + \frac{\partial\Phi}{\partial\chi}\, d\chi$$

$$a_A E_A = a(\eta)E(\eta)\left[1 + \Psi(\eta) - \Psi_A + \int_\eta^{\eta_A} \mathrm{d}\eta \partial_\eta(\Psi + \Phi)\right]$$

$$= a(\eta)E(\eta)\left[1 - \Phi(\eta) + \Phi_A + \int_\eta^{\eta_A} \mathrm{d}\eta \partial\chi(\Psi + \Phi)\right]$$

$$\Delta T(\hat{\mathbf{n}}) = \frac{\Delta_\gamma}{4} - \Phi + \hat{\mathbf{n}}\cdot(\mathbf{v}_A - \mathbf{v}) + \int_\eta^{\eta_A} \mathrm{d}\eta\, \hat{\mathbf{n}}\cdot\boldsymbol{\nabla}(\Psi + \Phi)$$

$$= \zeta_\gamma + \hat{\mathbf{n}}\cdot(\mathbf{v}_A - \mathbf{v}) + \int_\eta^{\eta_A} \mathrm{d}\eta\, \hat{\mathbf{n}}\cdot\boldsymbol{\nabla}(\Psi + \Phi)$$

Gauge-invariant 3-curvature on constant temperature hypersurfaces;
Redshifting from δN expansion of the beam makes the δN expansion from inflation observable
(but line of sight integral is larger on large-scales: overdensity looks colder)

Non-linear effect due to redshifting by large-scale modes?

Large-scale *linear* anisotropies are due to the *linear* anisotropic redshifting of the otherwise uniform (zero-order) temperature last scattering surface

$$T \to (1 + \Delta T)T$$

Also *non-linear* effect due to the *linear* anisotropic redshifting of the *linear* last scattering surface

$$\Delta T_{\text{small}} \to (1 + \Delta T_{\text{large}})\Delta T_{\text{small}}$$

Reduced bispectrum $b_{l_1 l_2 l_3} \approx C_{l_1} \left(\tilde{C}_{l_2} + \tilde{C}_{l_3}\right)$

Large-scale power spectrum

Small-scale (non-perturbative) power spectrum

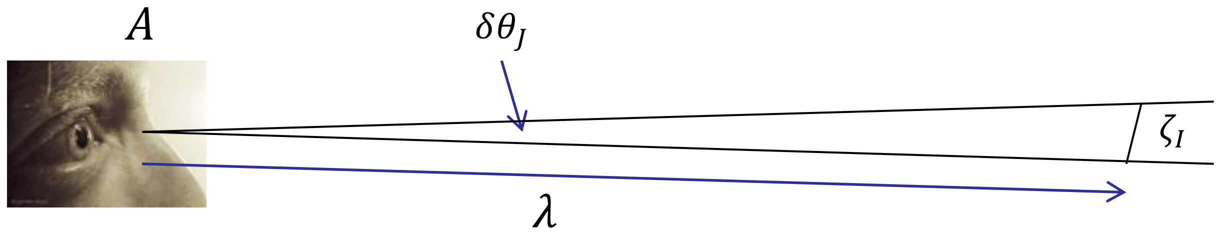
(Actually very small, so not very important)

Linear effects of large-scale modes

- Redshifting as photons travel through perturbed universe

- Transverse directions also affected:
  perturbations at last scattering are distorted as well as anisotropically redshifted

Jabobi map relates observed angle to physical separation of pair of rays



$$\xi_I(\lambda) = \mathcal{D}_{IJ}(\lambda)\delta\theta_J$$

Physical separation vector orthogonal to ray

Jacobi map

Angular separation seen by observer at A

Evolution of Jacobi map: $\dfrac{\mathrm{d}^2\mathcal{D}_{IJ}}{\mathrm{d}\lambda^2} = \mathcal{T}_{IK}\mathcal{D}_{KJ}$

Optical tidal matrix depends on the Riemann tensor: $\mathcal{T}_{IJ} \equiv -E_I^b E_J^c k^a k^d R_{abcd}$

(k^a is wave vector along ray, E_I^a projects into ray-orthogonal basis)

‘Riemann = Weyl + Ricci’

Non-local part (does not depend on local density):
- e.g. determined by Weyl (Newtonian) potential $\frac{1}{2}(\Phi+\Psi)$

differential deflection of light rays ⇒ convergence and shear of beam

- (Weyl) **lensing**

(can be modelled as transverse deflection angle)

Einstein equations relate Ricci to stress-energy tensor: depends on local density

⇒ ray area changes due to expansion of spacetime as the light propagates

- **Ricci focussing**

(cannot be modelled by deflection angle)

FRW background universe has Weyl=0, Ricci gives standard angular diameter distance

At radial distance χ_*, trace of Jacobi map determines physical areas: $\mathcal{D}/2 = \chi_* a_*$

Beam propagation in a perturbed universe, e.g. Conformal Newtonian Gauge

$$ds^2 = a^2(\eta)[(1+2\Psi)d\eta^2 - (1-2\Phi)\delta_{ij}dx^i dx^j]$$

$$\xi_I(\lambda) = \mathcal{D}_{IJ}(\lambda)\delta\theta_J$$

Trace-free part of Jacobi map depends on the shear: $\mathcal{D}_{\langle IJ\rangle} = a_*\chi_*\gamma_{IJ}$

$$\gamma_{IJ} = \nabla_{\langle I}\nabla_{J\rangle}\psi \qquad \psi \equiv -2\int_0^{\chi_*} \mathrm{d}\chi \frac{\chi_* - \chi}{\chi\chi_*}\Psi_W(\chi\hat{\mathbf{n}}, \eta_A - \chi)$$

Area of beam determined by trace of Jacobi map:

$$\mathcal{D}(\hat{\mathbf{n}},\eta)/2 = \chi(\hat{\mathbf{n}},\eta)a(\eta)[1 + \Phi_A - \Phi - \kappa + \hat{\mathbf{n}}\cdot\mathbf{v}_A]$$

CMB is constant temperature surface:

$$\eta = \eta_* + \delta\eta$$

$$\rho_\gamma \propto T^4 \propto a^4$$

$$\mathcal{D}/2 = \chi_* a_*\left[1 + \frac{\delta\chi}{\chi_*} + \underbrace{\frac{\Delta_\gamma}{4} - \Phi}_{\zeta_\gamma \equiv \Delta_\gamma/4 - \Phi} - \kappa + \hat{\mathbf{n}}\cdot\mathbf{v}_A\right]$$

Radial displacement (small, $\delta\chi \ll \chi_*$)

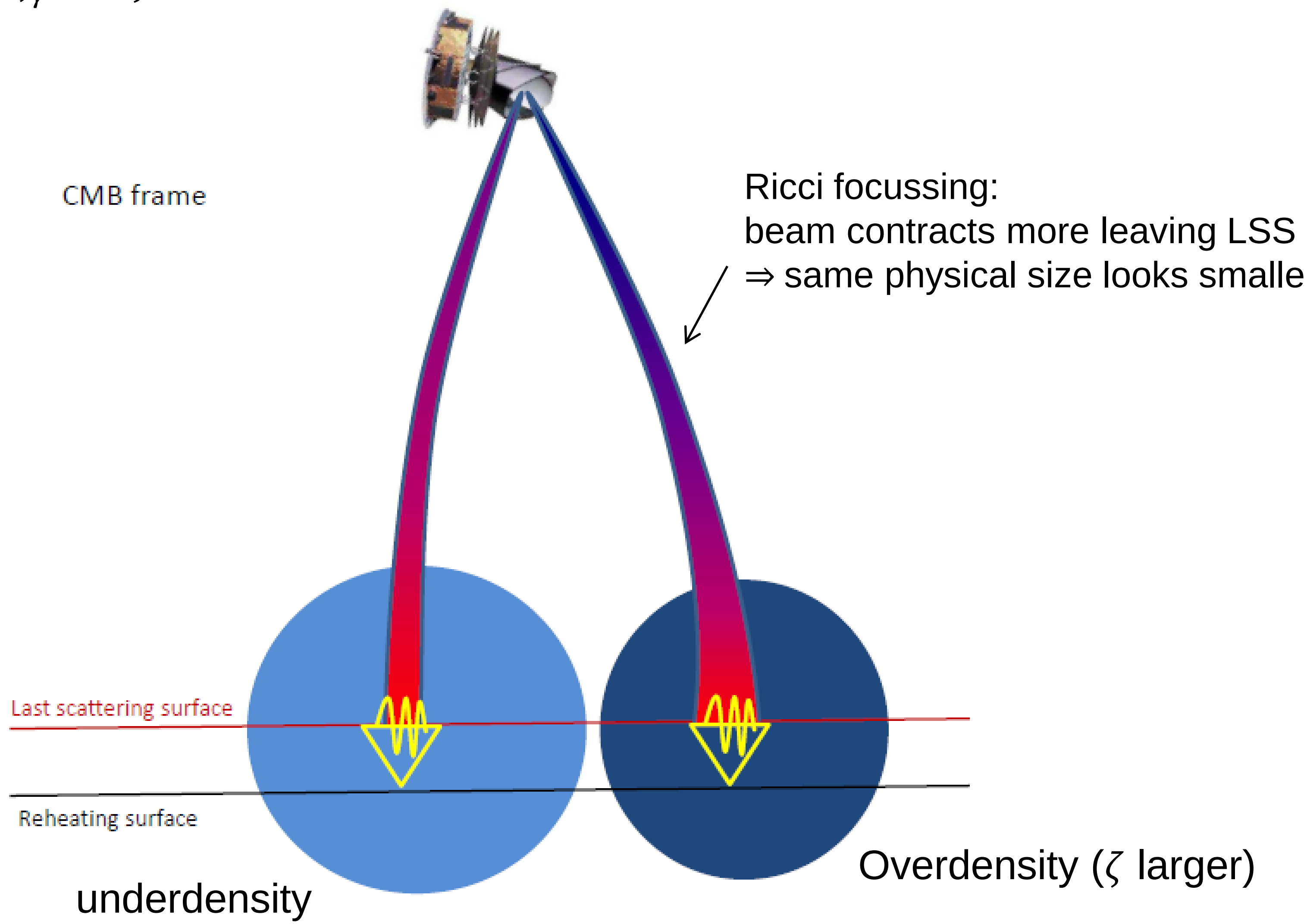
$\zeta_\gamma \equiv \Delta_\gamma/4 - \Phi$

Ricci focussing

(Weyl) convergence

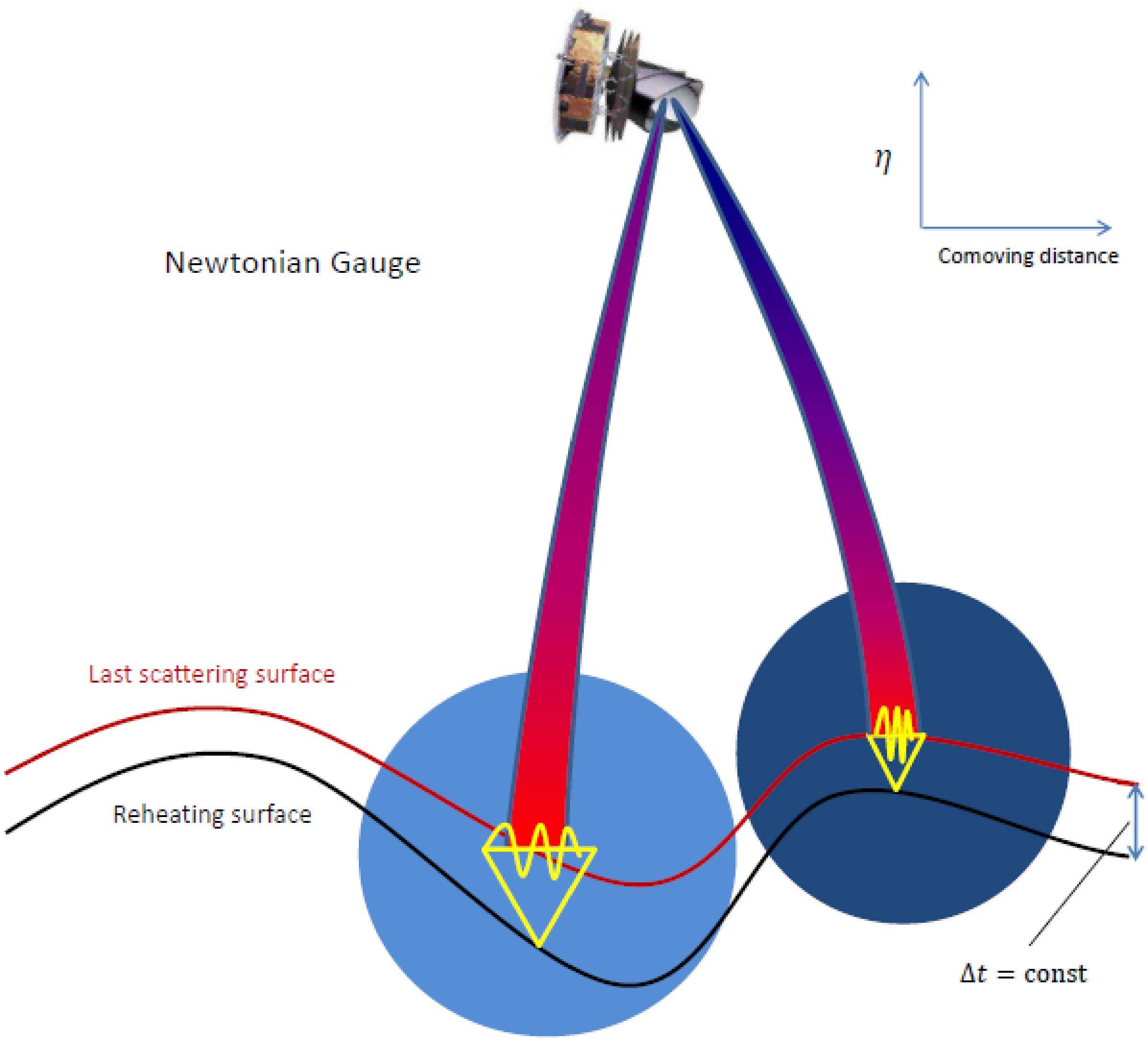
Local aberration

$$\frac{D}{2} \approx \chi_* a_* (1 + \zeta_\gamma - \kappa)$$



(Weyl lensing effect not shown and partly cancels area effect)

Gauge-invariant Ricci focussing $\zeta_\gamma \equiv \Delta_\gamma/4 - \Phi$



Observable CMB bispectrum from single-field inflation

Linear-short leg approximation for nearly-squeezed shapes:

$$\langle \tilde{T}_{l_1 m_1} \tilde{T}_{l_2 m_2} \tilde{T}_{l_3 m_3} \rangle \approx C_{l_1}^{T X_i} \left\langle \frac{\delta}{\delta X^*_{i, l_1 m_1}} \left(\tilde{T}_{l_2 m_2} \tilde{T}_{l_3 m_3} \right) \right\rangle$$

Where X_i here is δT, κ and ζ_γ, with $\frac{D}{2} \approx \chi_* a_* (1 + \zeta_\gamma - \kappa)$. For super-horizon adiabatic modes $\zeta_\gamma = \zeta_0$.

Weyl lensing bispectrum

$$b_{l_1 l_2 l_3} = \frac{1}{2} \left[(l_1(l_1+1) + l_2(l_2+1) - l_3(l_3+1) \right] C_{l_1}^{T\psi} \tilde{C}_{l_2} \text{ + perms}$$

Squeezed limit ($l_1 \ll l$) $\quad b_{l_1 l_2 l_3} \approx C_{l_1}^{T\kappa} \left[\frac{1}{l^2} \frac{\mathrm{d}(l^2 \tilde{C}_l)}{d \ln l} + \cos 2\phi_{l_1 l} \frac{\mathrm{d}\tilde{C}_l}{\mathrm{d} \ln l} \right]$ $\quad \mathbf{l} \equiv (\mathbf{l}_2 - \mathbf{l}_3)/2$

Ricci focussing bispectrum

$$b_{l_1 l_2 l_3} \approx C_{l_1}^{T \zeta_0^*} \frac{1}{2} \frac{\mathrm{d}}{\mathrm{d} \ln \chi_*} \left[\tilde{C}_{l_2} + \tilde{C}_{l_3} \right]$$

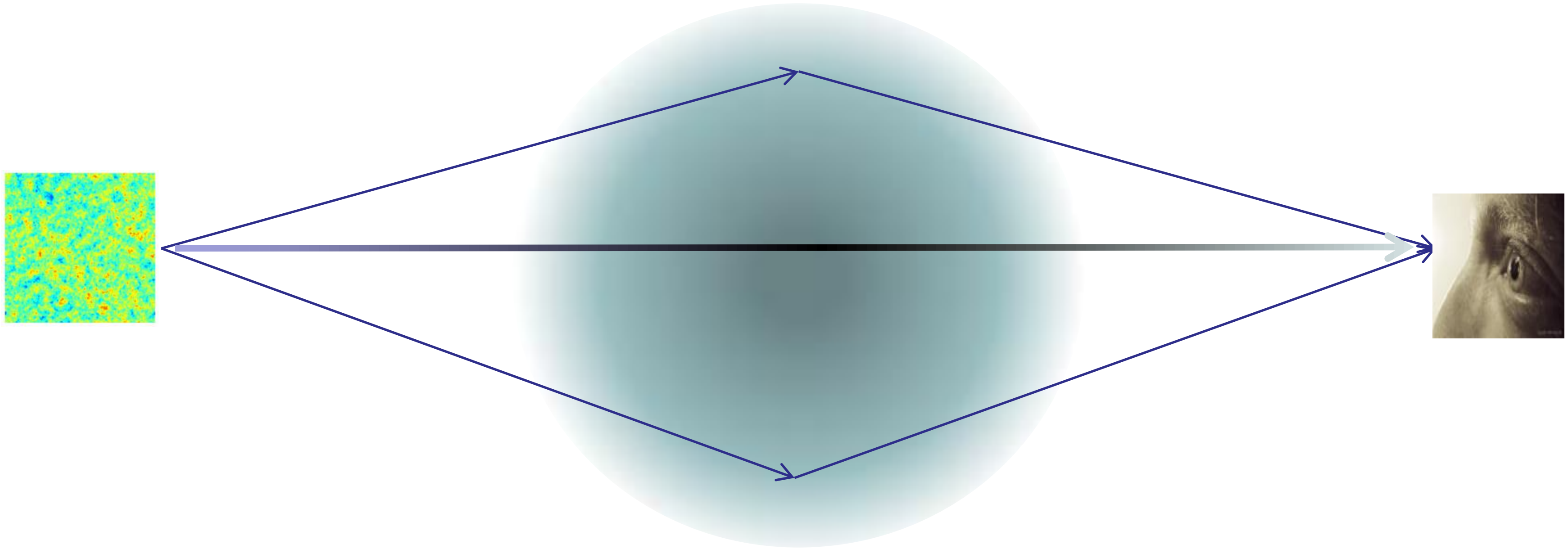
Squeezed limit ($l_1 \ll l$) $\quad b_{l_1 l_2 l_3} \approx -C_{l_1}^{T \zeta_0^*} \frac{1}{l^2} \frac{\mathrm{d}}{\mathrm{d} \ln l} (l^2 \tilde{C}_l)$

+ anisotropic redshifting bispectrum (from before) $\quad b_{l_1 l_2 l_3} \approx C_{l_1} \left(\tilde{C}_{l_2} + \tilde{C}_{l_3} \right)$

Weyl lensing bispectrum

$C_l^{T\kappa}$: Correlation between lenses and CMB temperature?

$$\Delta T_{\rm ISW}(\hat{\mathbf{n}}) = 2\int_0^{\chi_*} \mathrm{d}\chi\, \dot{\Psi}(\chi\hat{\mathbf{n}}; \eta_0 - \chi).$$



Overdensity: magnification correlated with positive Integrated Sachs-Wolfe (net blueshift)

Underdensity: demagnification correlated with negative Integrated Sachs-Wolfe (net redshift)

$C_l^{T\kappa}$: Correlation between lenses and CMB temperature?

- The late Integrated Sachs Wolfe effect (late ISW) at low redshift from decaying potentials

+

- Large-scale modes that span recombination and also act as lenses
- The early Integrated Sachs Wolfe effect (early ISW) due to the transition from radiation to matter domination, and decaying modes
- Lenses close to last-scattering being correlated to density perturbations that have infall giving a Doppler signal in the CMB
- Doppler signal from scattering at reionization
- Lenses at last-scattering that directly correlate perturbations to lensing at the recombination surface

Linear effects,
All included in self-consistent linear calculation with CAMB

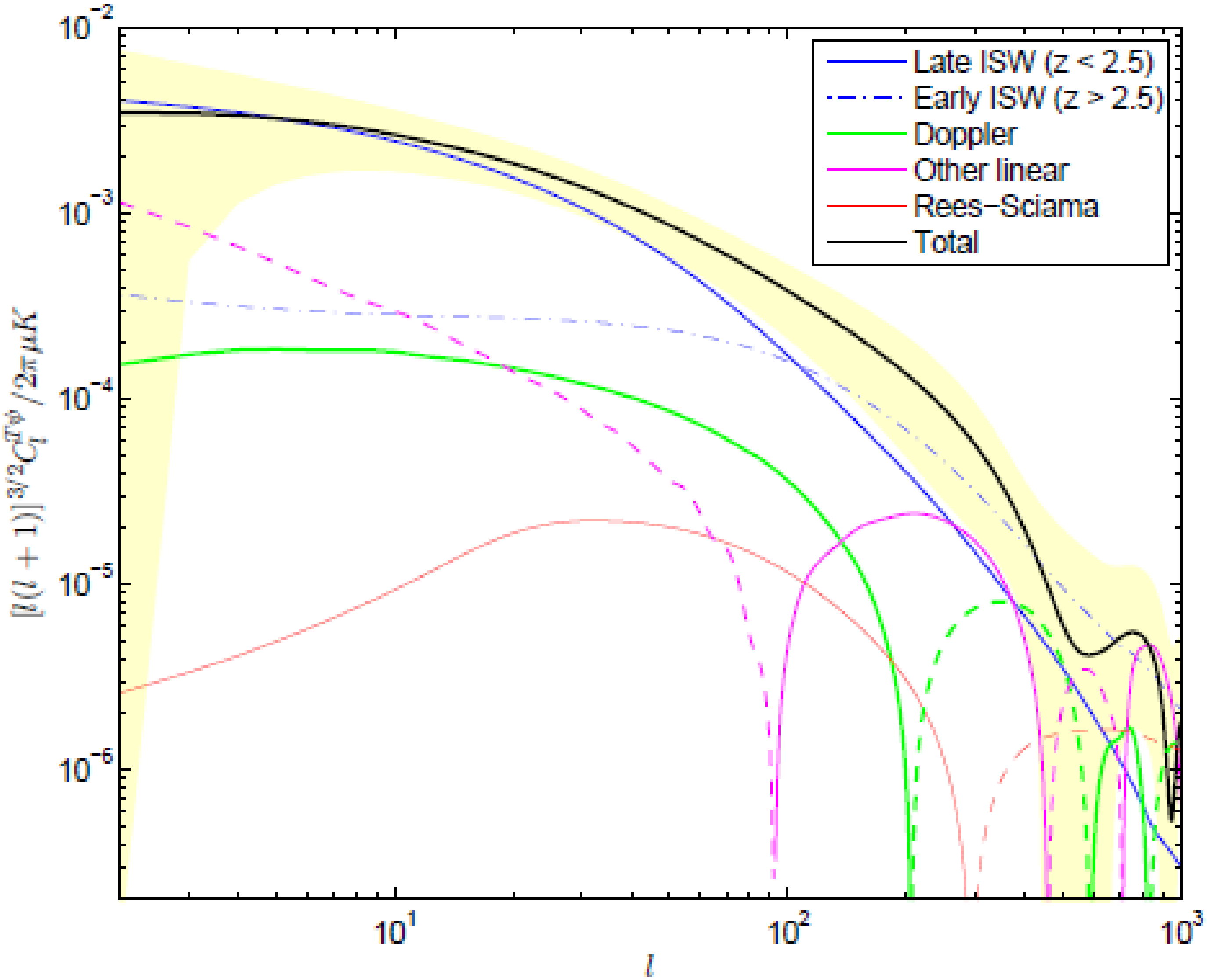
- Non-linear Rees-Sciama signal at low redshift from non-linear gravitational clustering

Non-linear growth effect
- estimate using e.g. Halofit

- Non-linear SZ signal from scattering in clusters
- Correlations due to foreground contaminants

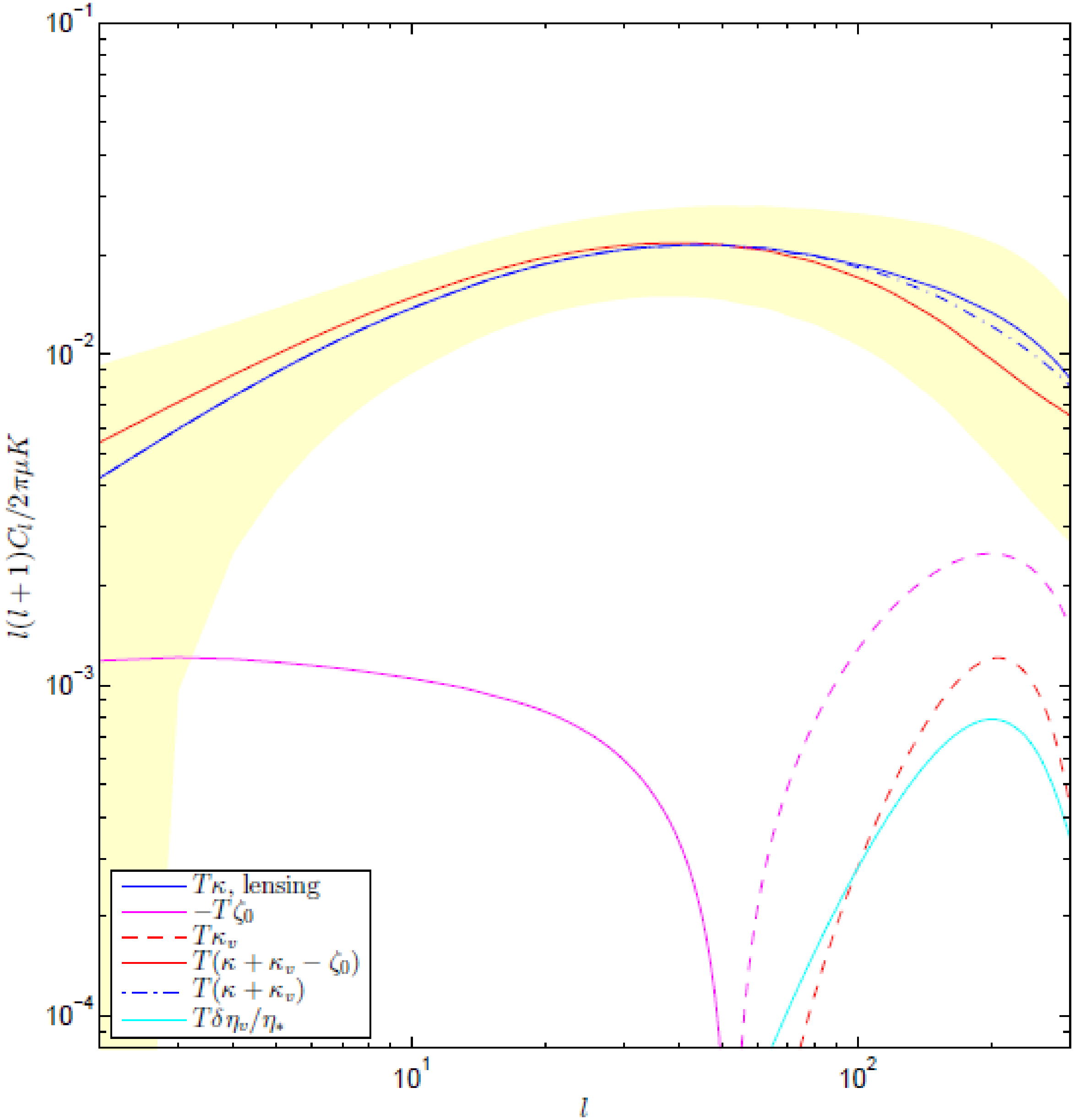
Potentially important,
but frequency dependent
- ‘foregrounds’

Contributions to the lensing-CMB cross-correlation, $C_l^{T\psi}$



(note Rees-Sciama contribution is small, numerical problem with much larger result of Verde et al, Mangilli et al.;
see also Junk et al. 2012 who agree with me)

Weyl lensing total + Ricci focussing (+ estimates of sub-horizon dynamics)



Does this look like squeezed non-Gaussianity f_{NL} from multi-field inflation
(local modulation of small scale perturbation amplitudes in each Hubble patch)?

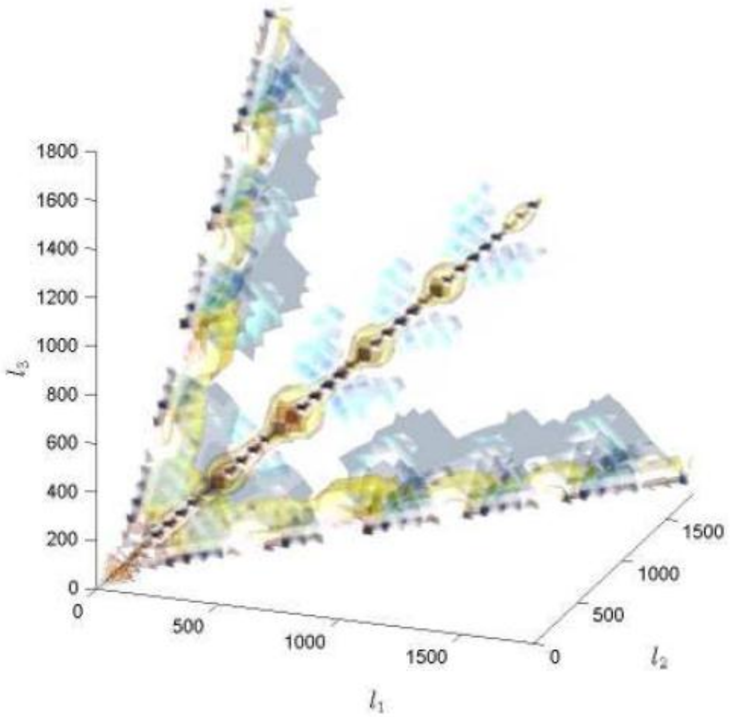
		bias on $f_{\rm NL}$			
Data used	$\sigma_{f_{\rm NL}}$	Weyl	Ricci	Redshift	Total
T	4.3	9.5	1.5	-0.22	10.7
Planck T	5.9	6.4	1.0	-0.22	7.1
T ($l_1 < 60$)	4.6	10.6	1.7	-0.25	12.0
Planck T ($l_1 < 60$)	6.2	7.0	1.1	-0.25	7.9
T+E	2.1	2.6	1.1	-0.05	3.7
Planck T+E	5.2	4.3	1.0	-0.15	5.2

TABLE I: Individual and total biases on primordial local-model non-Gaussianity parameterized by $f_{\rm NL}$ for CMB temperature and E-polarization data with Planck-like noise (assuming isotropic coverage over the full sky with sensitivity $\Delta T = \Delta Q/2 = \Delta U/2 = 50\,\mu\text{K arcmin}$ $[N_l^T = N_l^E/4 = 2\times 10^{-4}\mu\text{K}^2]$ and a beam FWHM of 7 arcmin) or cosmic-variance limited data with $l_{\rm max} = 2000$. Results are assuming that non-$f_{\rm NL}$ contributions are only significant at $l_1 \le 300$ and negligible dynamical effects; the $l_1 < 60$ results are filtered to only use large scale modulations and are therefore immune to small-scale modulation effects. The bias is the systematic error on $f_{\rm NL}$ if the given contribution is neglected, which can be compared to $\sigma_{f_{\rm NL}}$ which is the Fisher error estimate (including lensing signal variance).

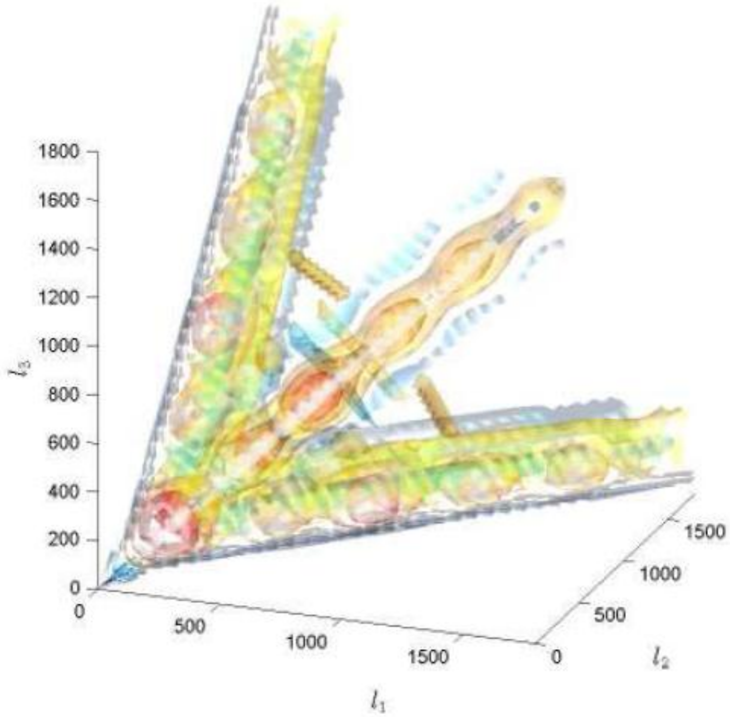
Dominated by lensing $f_{NL} \sim 6 - 10$
Ricci is an $O(1)$ correction

Calculation reliable for $l_1 < 60$ where dynamical effects suppressed by small $\frac{k^2}{H^2}$: do not need fully non-linear dynamical calculation of bispectrum a la Pitrou et al to make reliable f_{NL} constraint

Signal easily modelled
Squeezed shape but different phase, angle and scale dependence



Lensing



f_{NL}

Lewis, Challinor, Hanson 1101.2234

Note: 'Maldacena' bispectrum

$$\langle\tilde{\zeta}(\mathbf{k}_1)\tilde{\zeta}(\mathbf{k}_2)\tilde{\zeta}(\mathbf{k}_3)\rangle \approx -\frac{1}{(2\pi)^{3/2}}\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)P_{\zeta\zeta}(k_1)\frac{1}{2}\left[\frac{1}{k_3^3}\frac{\mathrm{d}}{\mathrm{d}\ln k_3}(k_3^3P_{\zeta\zeta}(k_3))+\frac{1}{k_2^3}\frac{\mathrm{d}}{\mathrm{d}\ln k_2}(k_2^3P_{\zeta\zeta}(k_2))\right]$$

Consistency relation: $f_{NL} \sim O(n_s - 1)$

is *not* an observable

- cannot measure comoving curvature perturbations on scales larger than the horizon directly
- $d/d(\ln k)$ and CMB transfer functions do not commute: cannot get correct result from primordial $f_{NL} \sim (n_s - 1)$

Observable CMB analogue is Ricci focussing bispectrum

$$b_{l_1l_2l_3} \approx -C_{l_1}^{T\zeta_0^*}\frac{1}{l^2}\frac{\mathrm{d}}{\mathrm{d}\ln l}(l^2\tilde{C}_l)$$

- larger because of acoustic oscillations, non-zero for $n_s = 1$
- different shape to f_{NL} in CMB, but projects as $f_{NL} = O(1)$

Question: primordial bispectrum calculations includes time shift $\frac{d}{d\ln k}$ terms
- not correct to calculate effective f_{NL} at end of inflation, what to do? (e.g. features)

Lensing of primordial non-Gaussianity

General case at leading order: Hanson et al. arXiv:0905.4732
Fast non-perturbative method: Pearson, Lewis, Regan arXiv:1201.1010

Bispectrum slices are smoothed by lensing, just like power spectrum

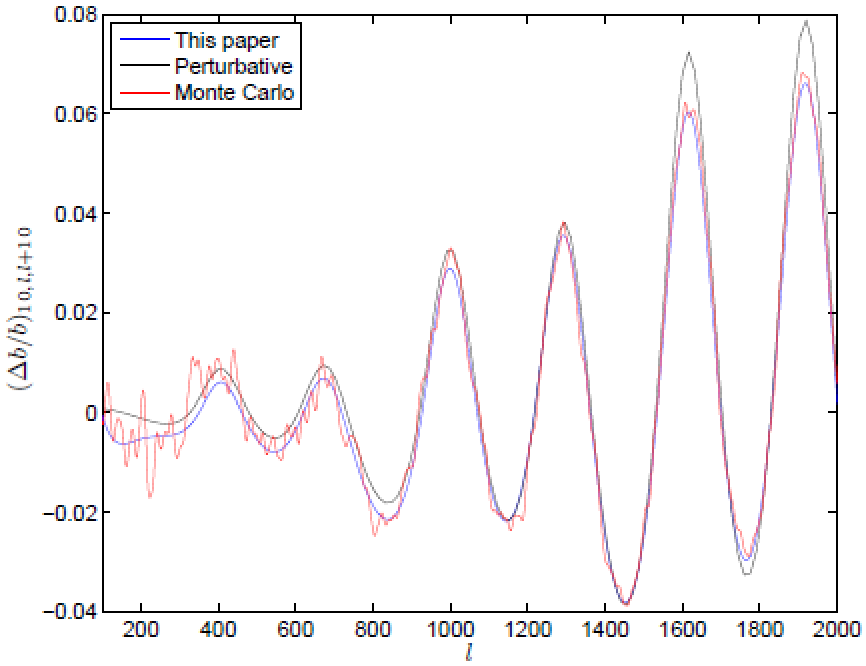


FIG. 5: The fractional change in the reduced bispectrum slice $b_{10,l,l+10}$ due to lensing. The blue line shows the non-perturbative approximation of this paper, the black line shows the leading-order perturbative result from Ref. [1]. The red lines show the result of 1000 Monte Carlo simulations of Ref. [1] smoothed over $\Delta l = 5$. The new approximation only needs to lens the isotropic component of the bispectrum, and then is both significantly more accurate on small scales and faster to compute.

BUT lensing preserves total power: expect ~ 0 bias on primordial f_{NL} estimators

Squeezed trispectrum

- Lensing gives large trispectrum, this is what is used for lensing reconstruction
- Also want to look for primordial trispectrum

e.g. from primordial modulation $\zeta(\mathbf{x}) = \zeta_0(\mathbf{x})[1+\phi(\mathbf{x})]$

Squeezed shape, constant modulation $\Rightarrow T(\hat{\mathbf{n}}) \approx T_g(\hat{\mathbf{n}})[1+\phi(\hat{\mathbf{n}}, r_*)]$

Easy accurate estimator for τ_{NL} is $\tau_{\rm NL}(L) \equiv \frac{C_L^\phi}{C_L^{\zeta_\star}}$

$$\Rightarrow \quad \hat{\tau}_{\rm NL} \approx L_{\rm min}^2 \sum_{L=L_{\rm min}}^{\infty} \frac{2L+1}{L^2(L+1)^2} \frac{\hat{C}_L^\phi}{C_L^{\zeta_\star}}$$

(optimal to percent level)

Lensing bias on τ_{NL}

$$\langle \hat{\tau}_{NL} \rangle \sim L_{\rm min}^2 \sum_{L=L_{\rm min}}^{\infty} \frac{2L+1}{L^2(L+1)^2} \frac{\alpha_L^2 C_L^\kappa}{C_L^\zeta}$$

All τ_{NL} signal at low $l < \sim 10$: cut to avoid blue lensing signal at higher l

Then fairly small, 17 to 40 depending on data: small compared to $\sigma_{\tau_{NL}} \geq 150$

<u>Lensing not a problem for τ_{NL} constraints</u> (because they are so weak!)

Conclusions

- Single field inflation predicts significant non-Gaussianity in the observed CMB

  - mostly due to (Weyl) lensing
  - total projects onto $f_{NL} \sim 7$ for Planck temperature
  - Ricci focussing expansion of beam recovers the δN from inflation,
    : gives equivalent of consistency relation, but larger
    : small and not quite observable, projects on to $f_{NL} \sim 1$
  - Squeezed calculation reliable at $l_1 < 60$
    : robust constraints on f_{NL} without 2nd order dynamics
  - effect on trispectrum is small

- Lensing bispectrum signal important but distinctive shape
  - dominated by late ISW correlation, but other term important (eg. early ISW)
  - predicted accurately by linear theory (Rees-Sciama is tiny)

- On smaller scales, and non-squeezed shapes, need full numerical calculation of non-linear dynamical effects in CMB

Question: for numerical calculation of squeezed non-linear effects, how to you handle/separate the large lensing signal?
($f_{NL} \sim 3$ sounds like mostly lensing to me)