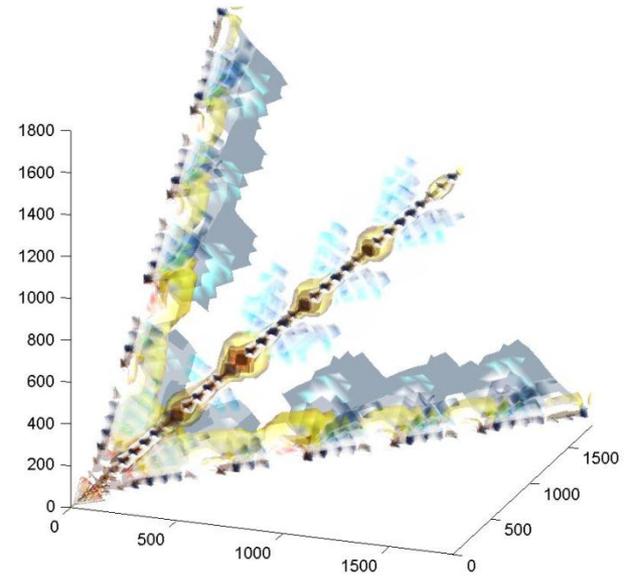
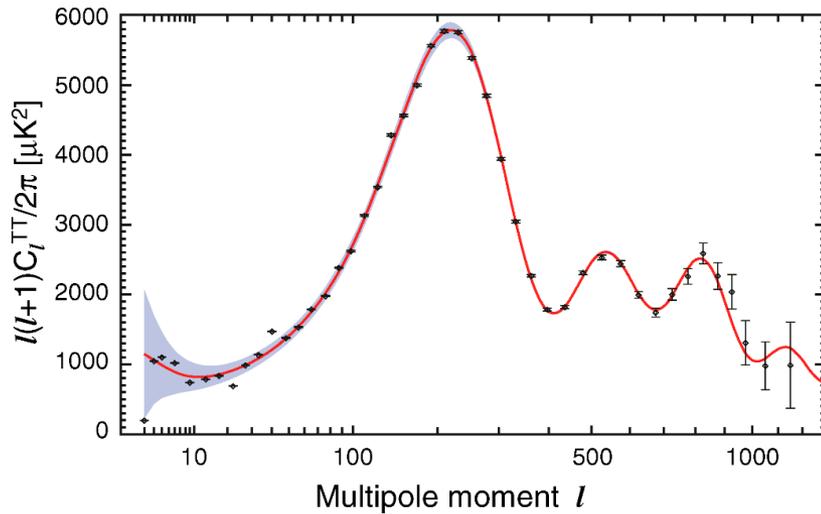
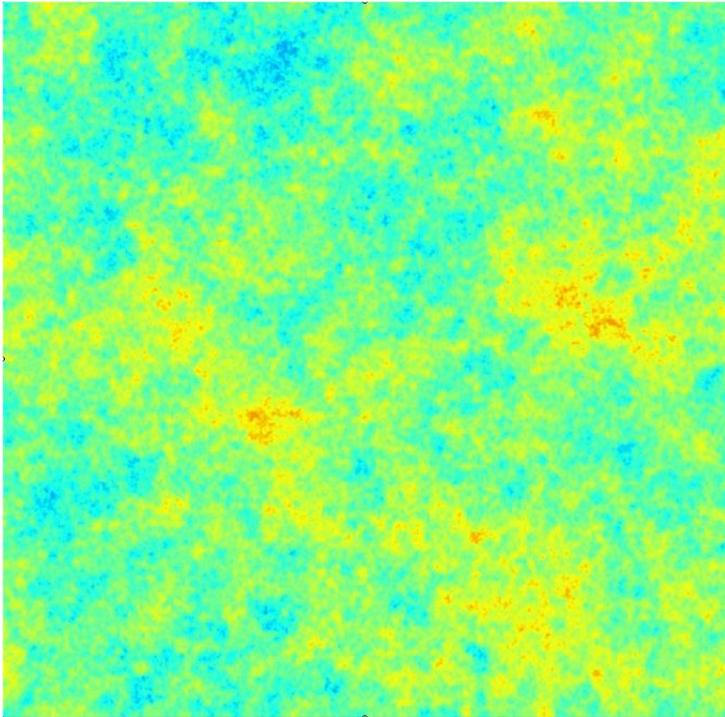


CMB Prospects



CMB temperature

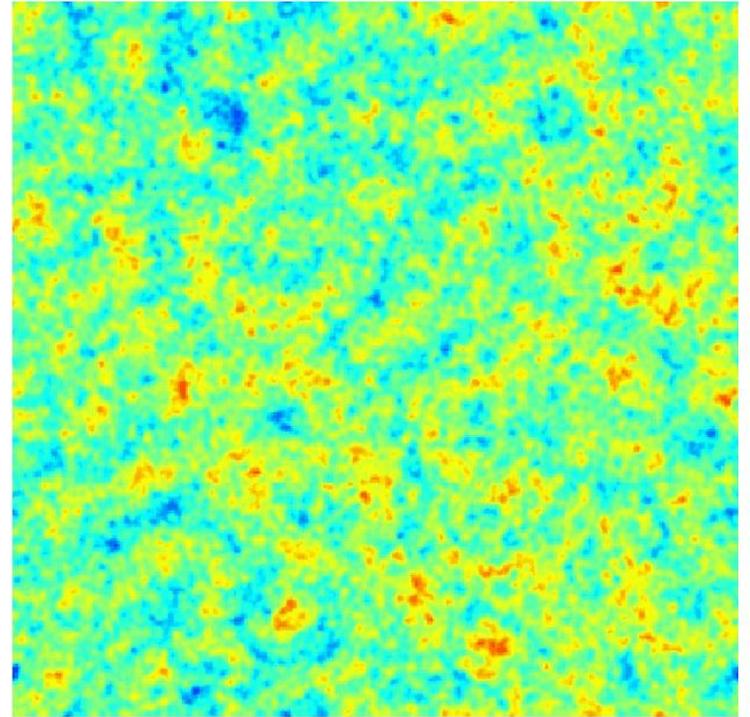
Hot big bang

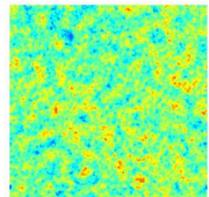
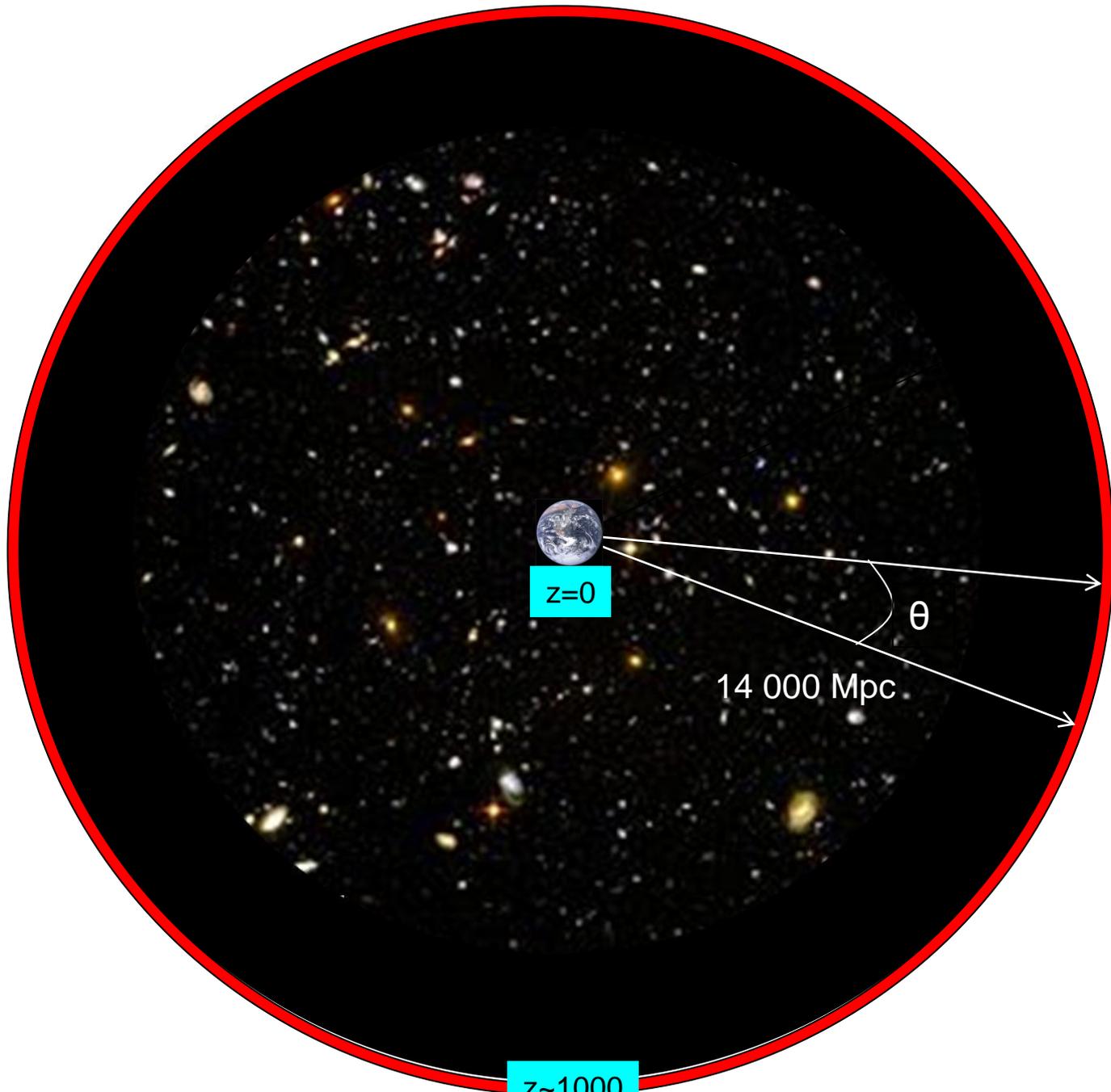


gravity+
pressure+
diffusion

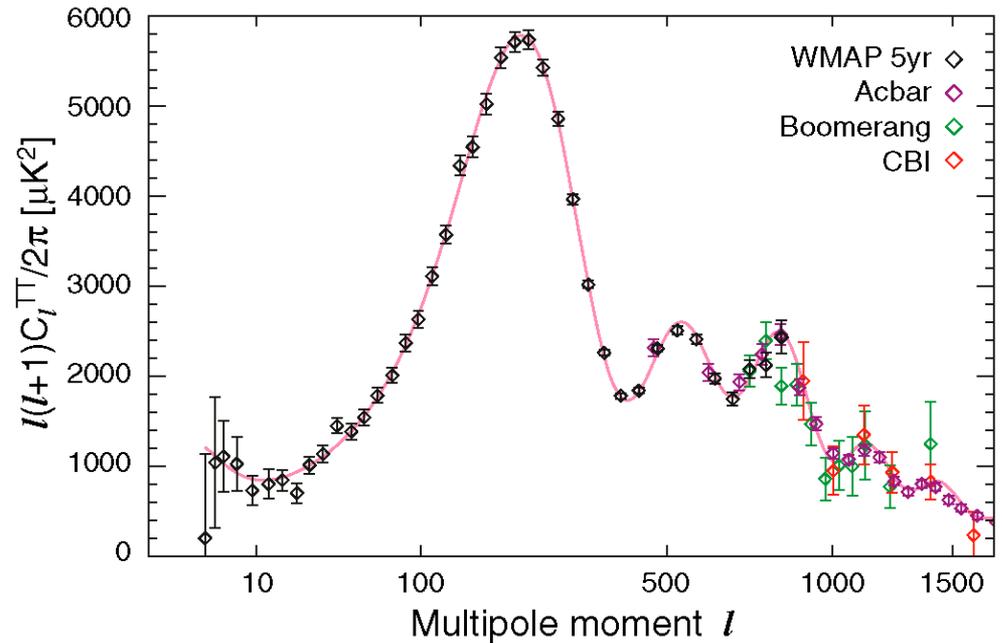
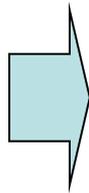
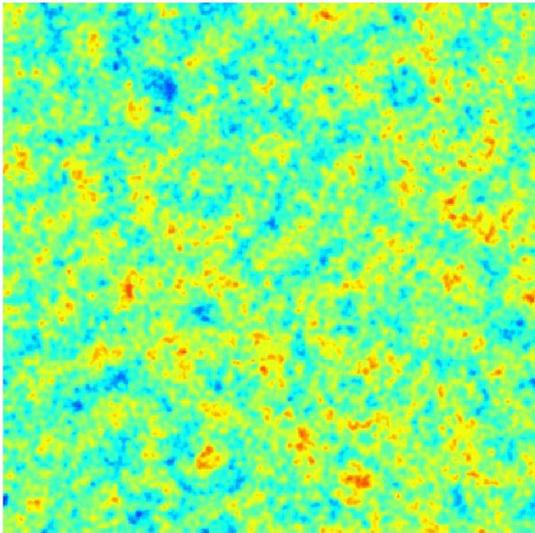


Last scattering surface





Observed CMB temperature power spectrum



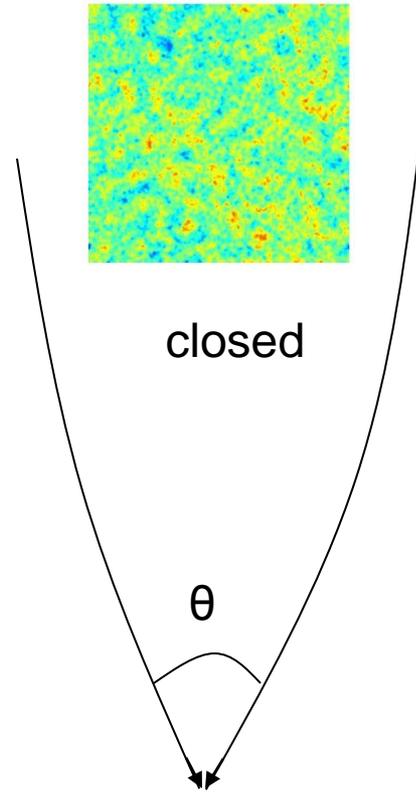
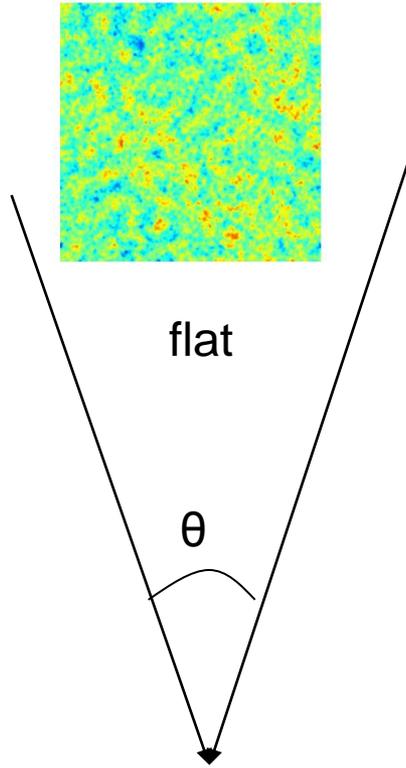
WMAP team

Observations

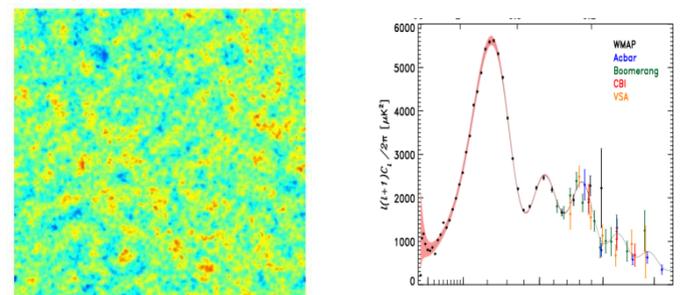
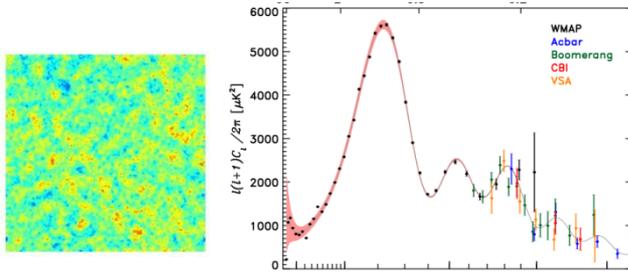


**Constrain theory of early universe
+ evolution parameters and geometry**

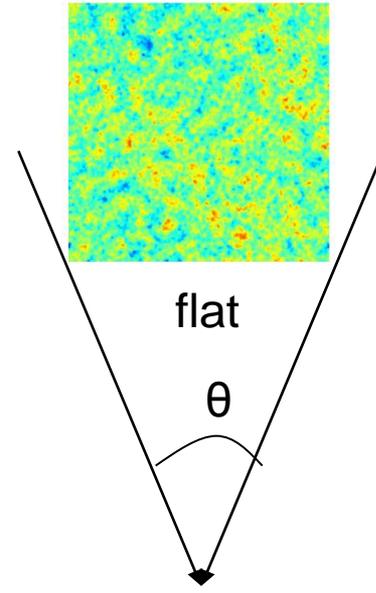
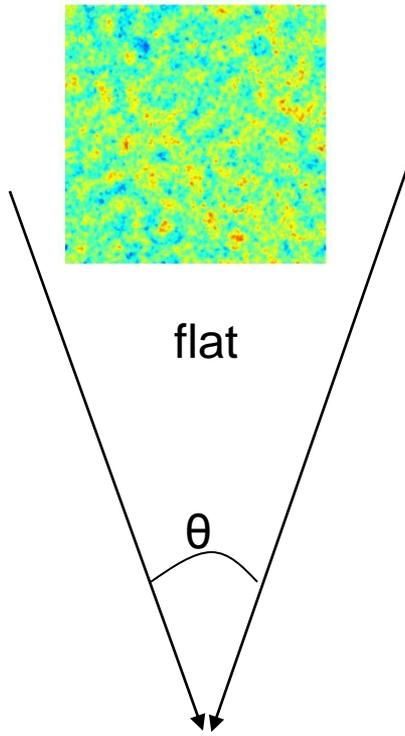
e.g. Geometry: curvature



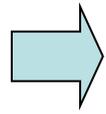
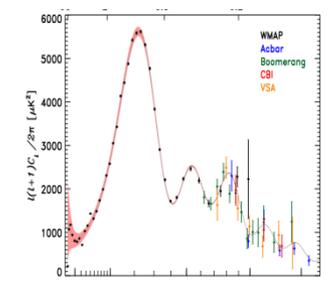
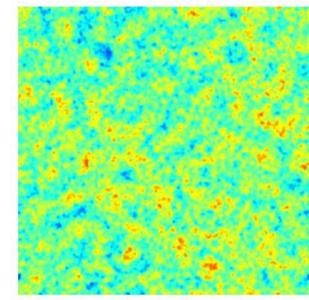
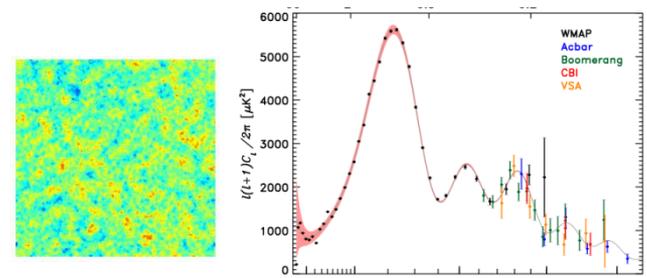
We see:



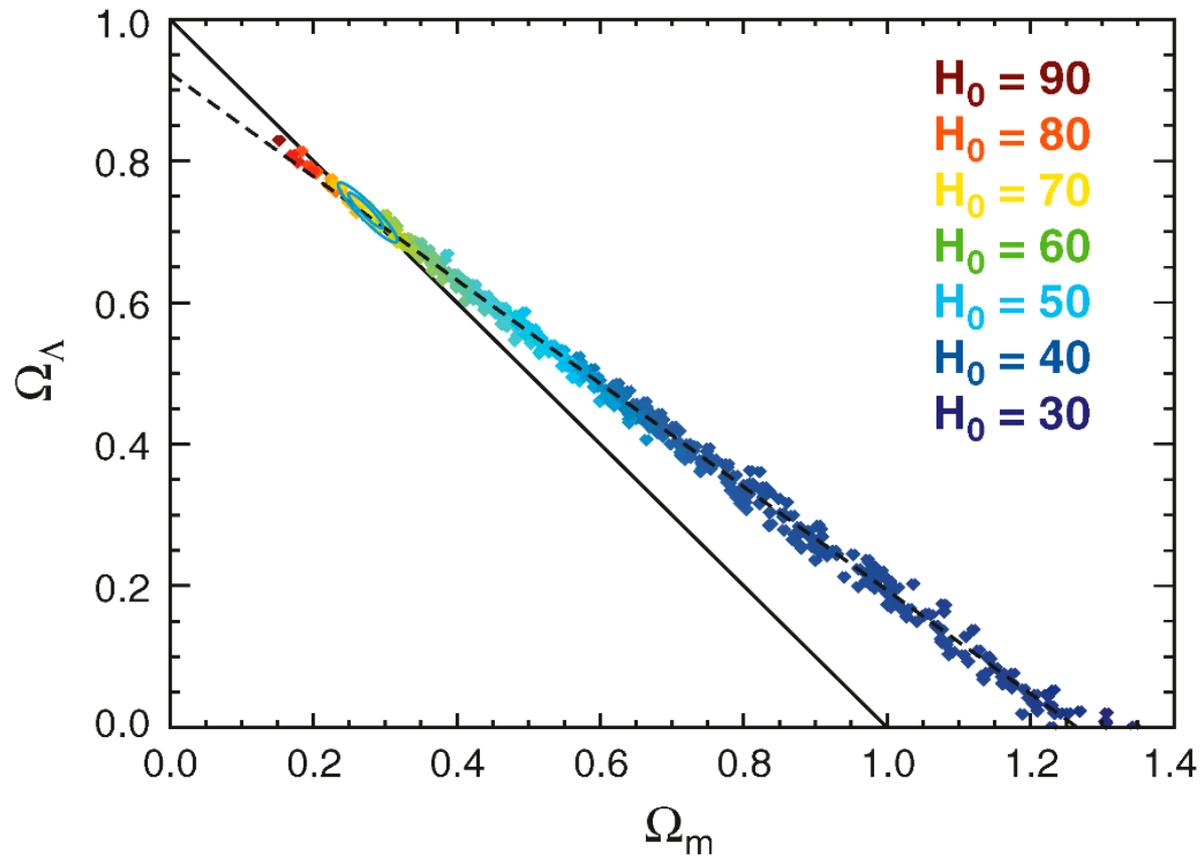
or is it just closer??



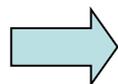
We see:



Degeneracies between parameters



WMAP 7



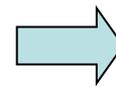
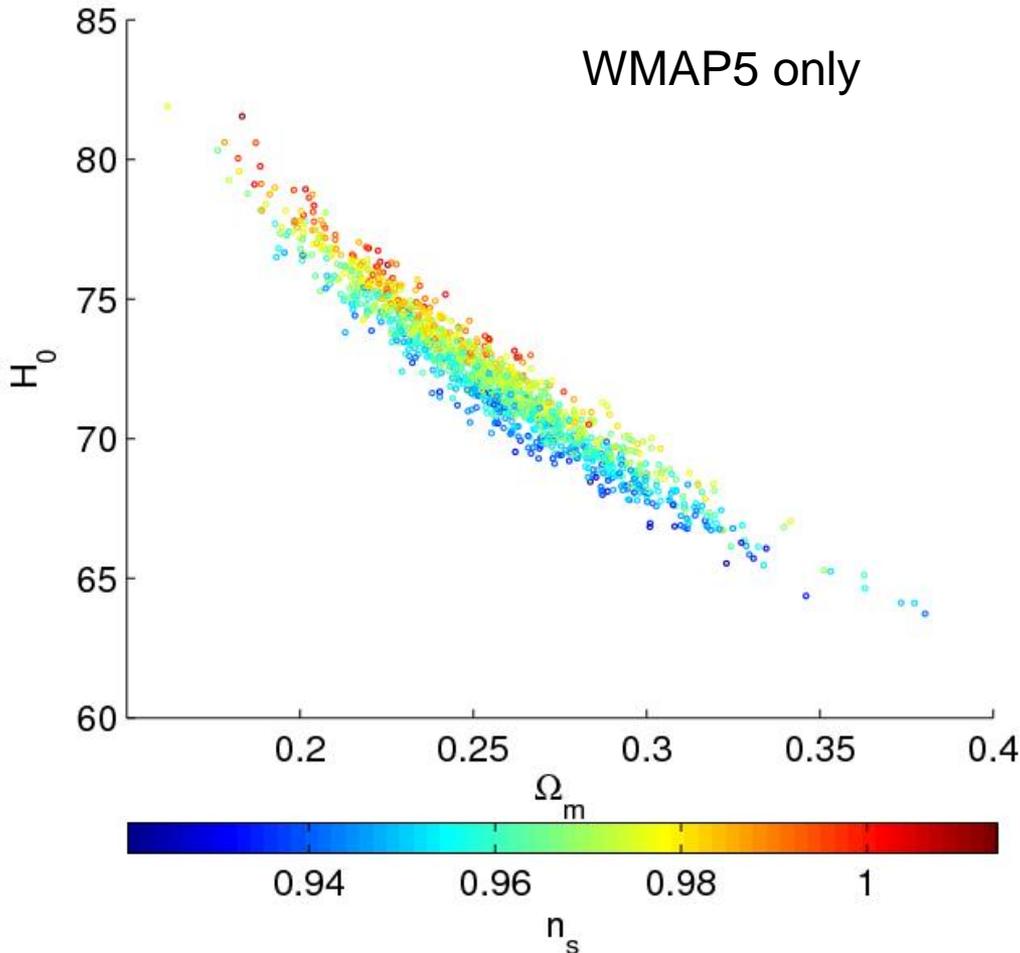
Need other information to break remaining degeneracies

Constrain *combinations* of parameters accurately

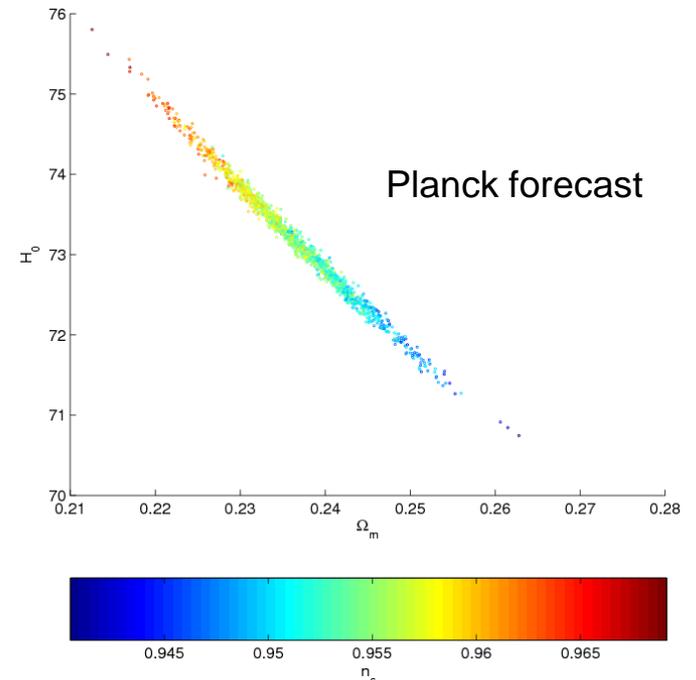
Assume Flat, $w=-1$

$$\left(\frac{\Omega_m}{0.254}\right) \left(\frac{h}{0.72}\right)^{3.15} = 1.00 \pm 0.03$$

$$\left(\frac{\Omega_m}{0.254}\right) \left(\frac{h}{0.72}\right)^{-3.15} = 1.03 \pm 0.23$$



Use other data to break remaining degeneracies



CMB anisotropies: theory

Linear perturbation theory with $ds^2 = a(\eta)^2 [(1 + 2\Psi)d\eta^2 - (1 - 2\Phi)dx^2]$

Using the geodesic equation in the Conformal Newtonian Gauge:

$$E(\eta_0) = a(\eta)E(\eta) \left[1 + \Psi(\eta) - \Psi_0 + \int_{\eta}^{\eta_0} d\eta(\Psi' + \Phi') \right]$$

All photons redshift the same way, so $kT \sim E$.

Recombination fairly sharp at background time η_* : \sim *constant temperature* surface

$$\begin{aligned} T(\hat{\mathbf{n}}, \eta_0) &= (a_* + \delta a)T_* \left[1 + \Psi(\eta_*) - \Psi_0 + \hat{\mathbf{n}} \cdot (\mathbf{v}_o - \mathbf{v}) + \int_{\eta_*}^{\eta_0} d\eta(\Psi' + \Phi') \right] \\ &= T_0 \left[1 + \frac{\delta a}{a_*} + \Psi(\eta_*) - \Psi_0 + \hat{\mathbf{n}} \cdot (\mathbf{v}_o - \mathbf{v}) + \int_{\eta_*}^{\eta_0} d\eta(\Psi' + \Phi') \right] \end{aligned}$$

$$\rho_\gamma \propto T^4 \propto a^4.$$

$$\Rightarrow \frac{\Delta T_0}{T}(\hat{\mathbf{n}}) = \frac{\Delta_\gamma(\eta_*)}{4} + \underbrace{\Psi(\eta_*) - \Psi_0}_{\text{Sachs-Wolfe}} + \underbrace{\hat{\mathbf{n}} \cdot (\mathbf{v}_o - \mathbf{v})}_{\text{Doppler}} + \underbrace{\int_{\eta_*}^{\eta_0} d\eta (\Psi' + \Phi')}_{\text{ISW}}$$

Temperature perturbation at recombination

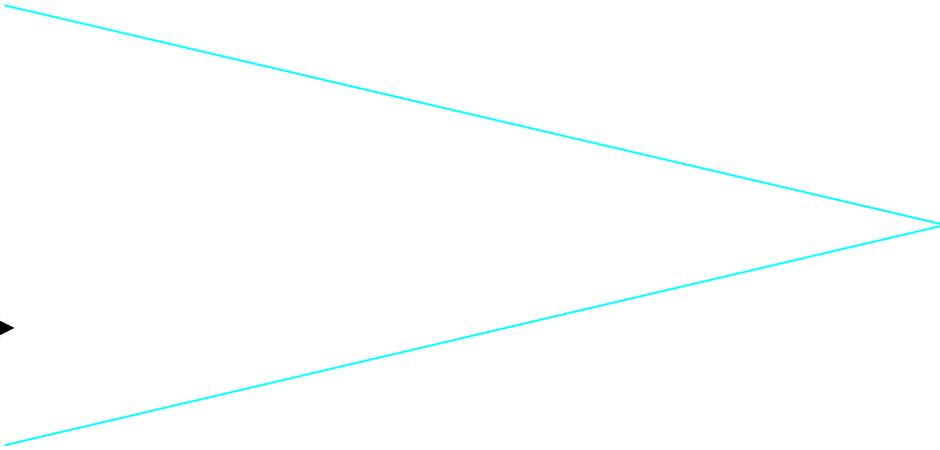
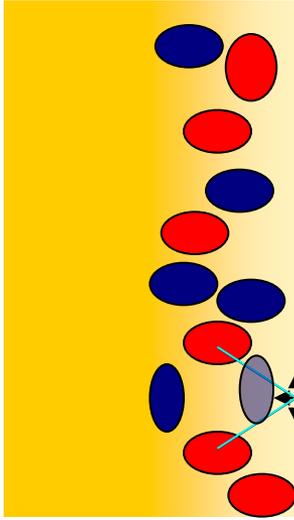
Poisson Eq: $\Delta_\gamma \sim -\left(\frac{k}{H}\right)^2 \Psi + \dots$

Big overdensities – see cold

Small overdensities – see hot

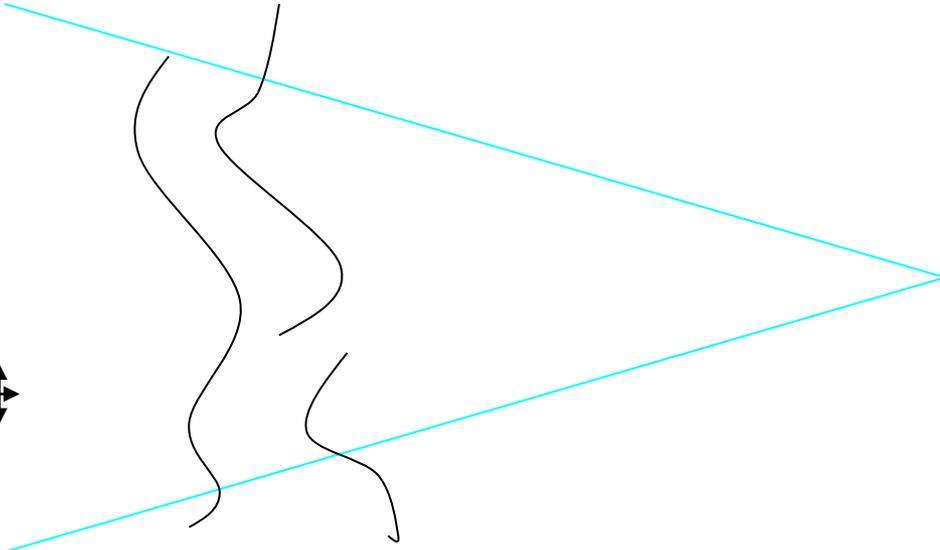
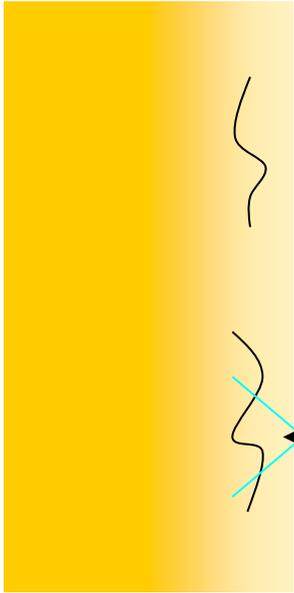
Complication: recombination is not sharp

Scalars



Tensors
(unknown amplitude)

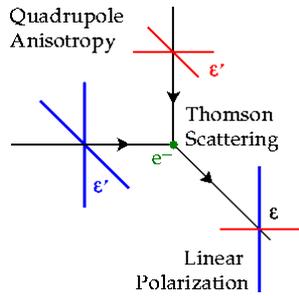
$(h \propto \frac{1}{a}$
for $k > aH$)



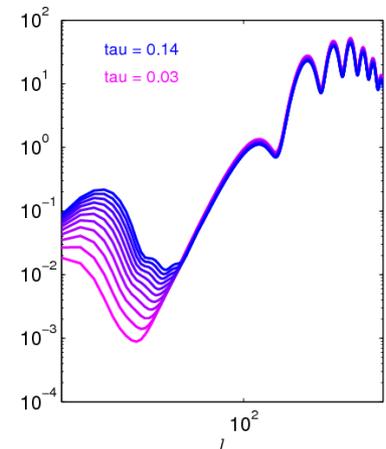
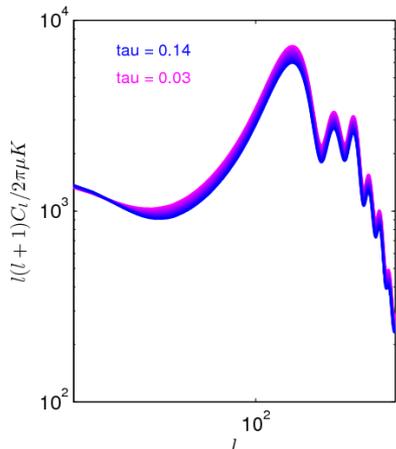
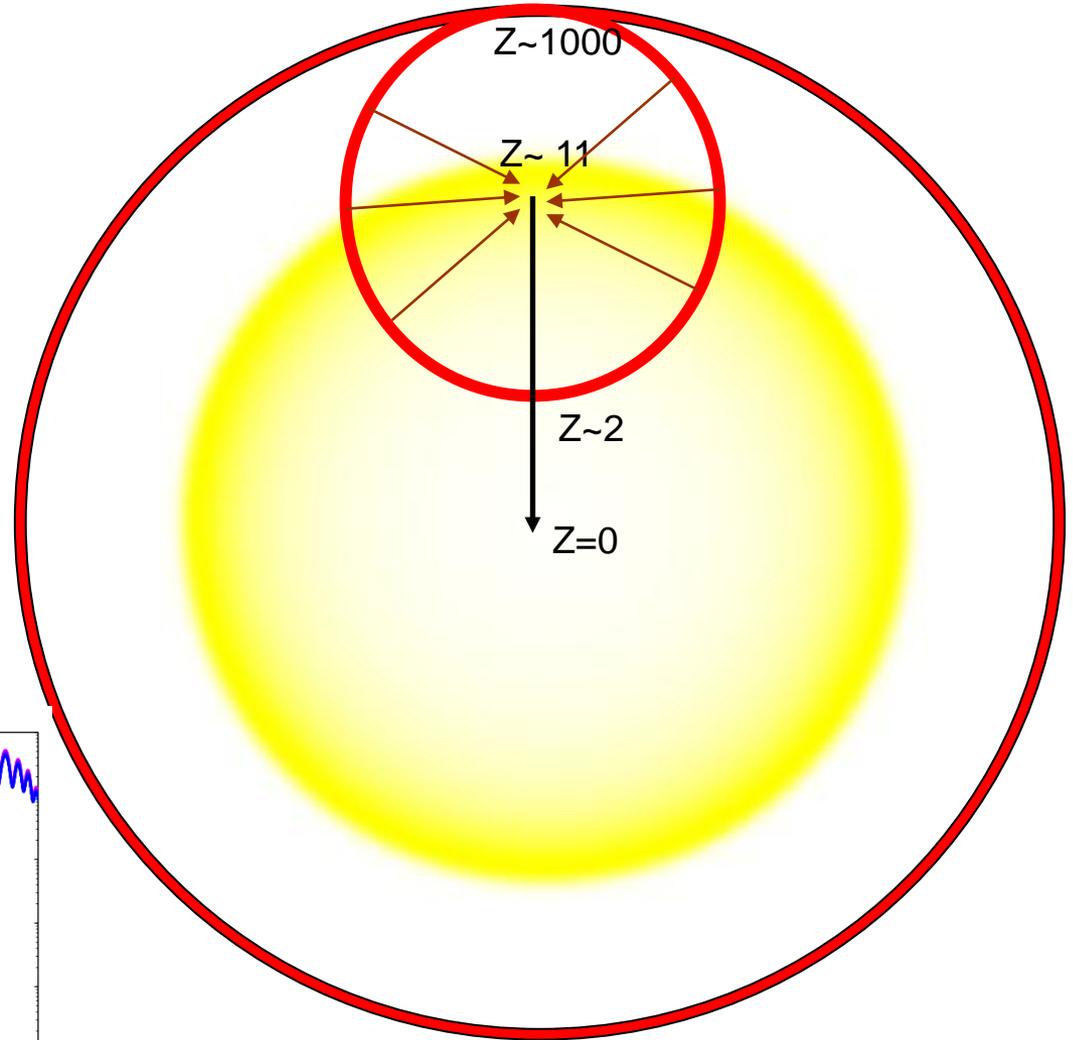
Line-of-sight averaging; Silk damping; polarization

Also reionization

- Damping by $e^{-\tau}$
- Large-scale polarization



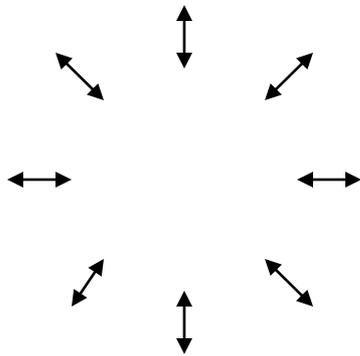
Hu astro-ph/9706147



CMB polarization: E and B modes

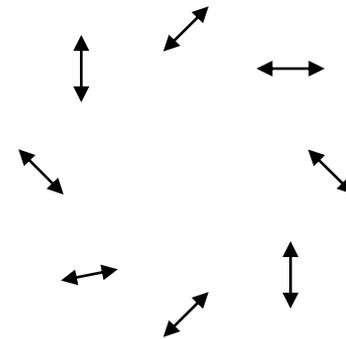
“gradient” modes
E polarization

e.g.

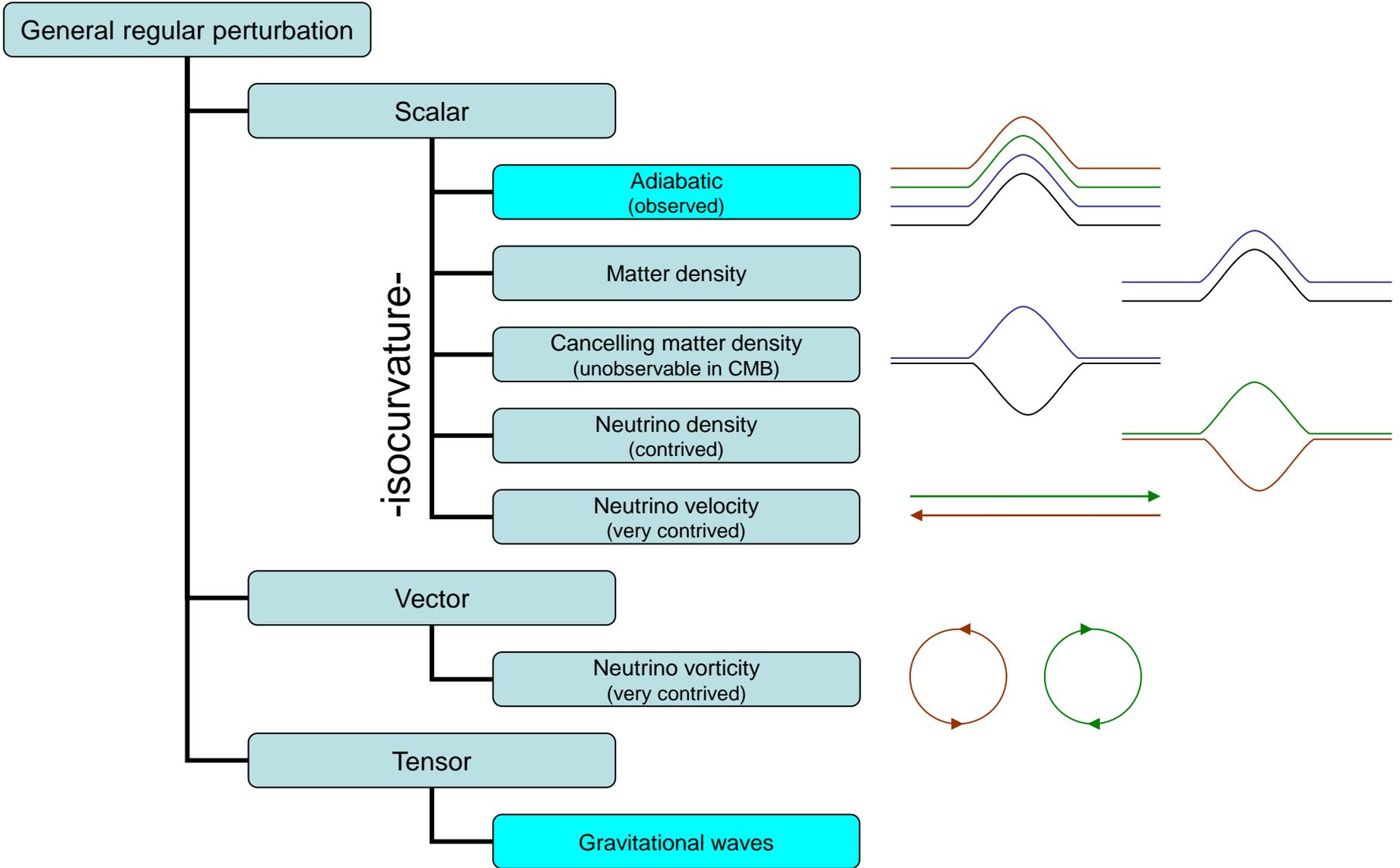


e.g. cold spot

“curl” modes
B polarization



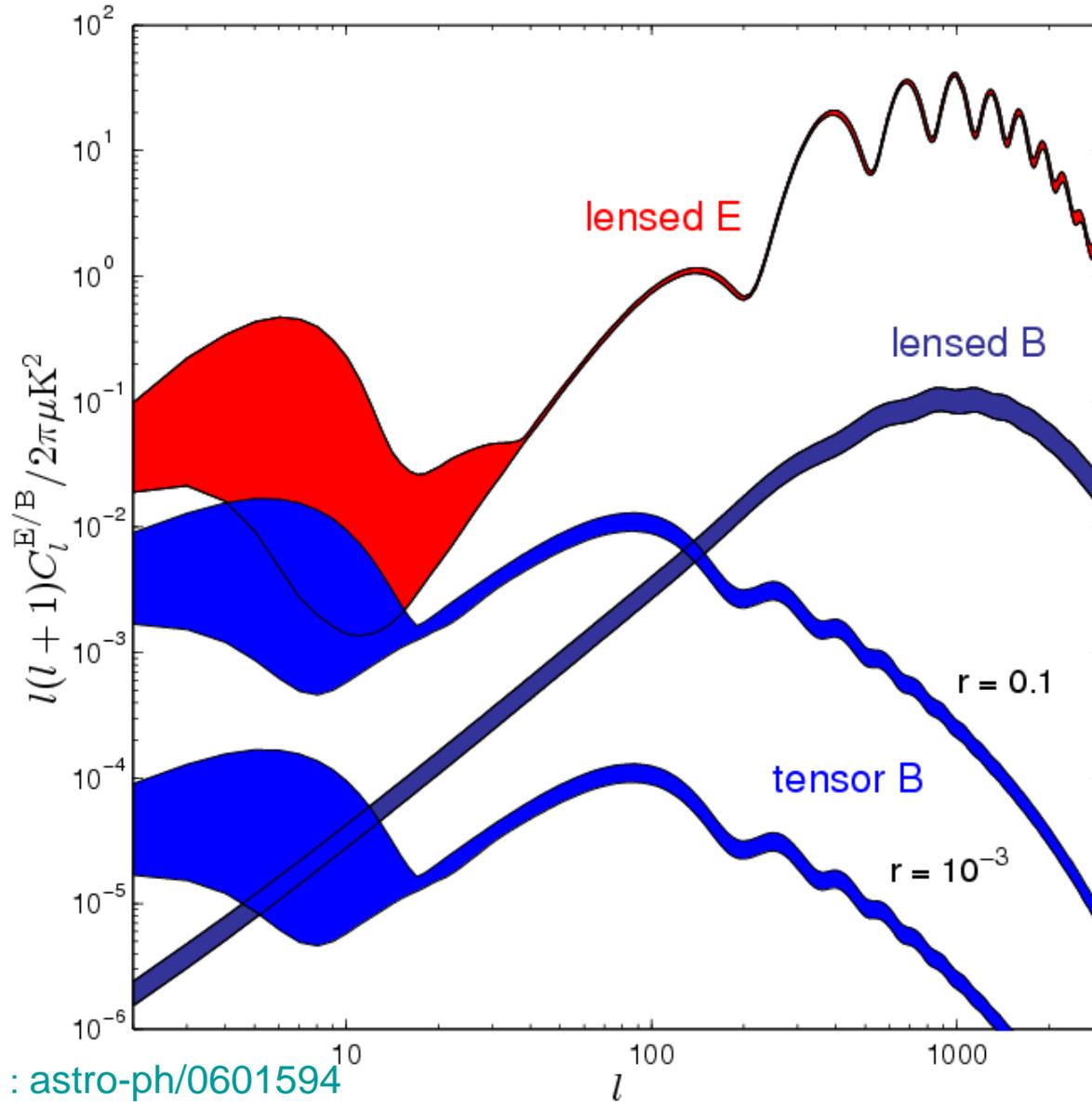
Possible regular initial perturbations



+ possible sources, e.g. strings

Polarization power spectra

Current 95% indirect limits for LCDM given WMAP+2dF+HST



Can we calculate the power spectra accurately enough?

- Linear theory (+ lensing) very well understood
- But depends on background evolution of x_e

Recombination:

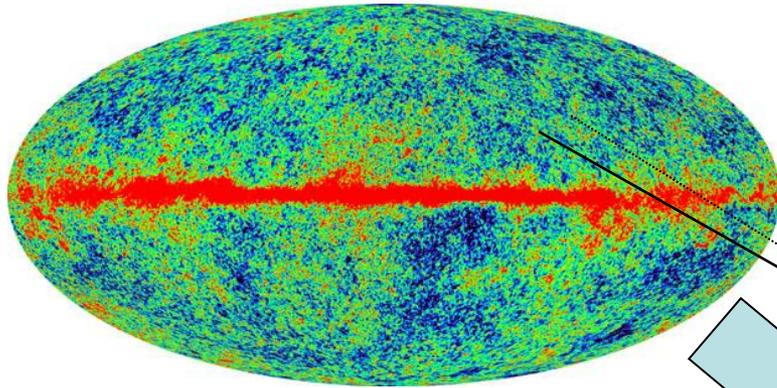
- Leading uncertainty, potentially percent-level errors
- Now independent codes, HyRec and CosmoRec
([Chluba et al 2010](#), [Ali-Hamoud et al 2011](#))
- Is there plausibly anything important that is still forgotten?

Reionization:

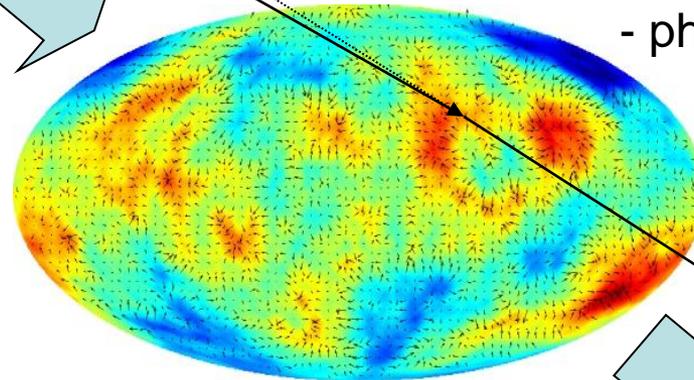
- Detailed shape unknown and not predictable in detail
- But E polarization not really sensitive, not a big issue
- Can constrain models from data

CMB Lensing

Last scattering surface



Inhomogeneous universe
- photons deflected



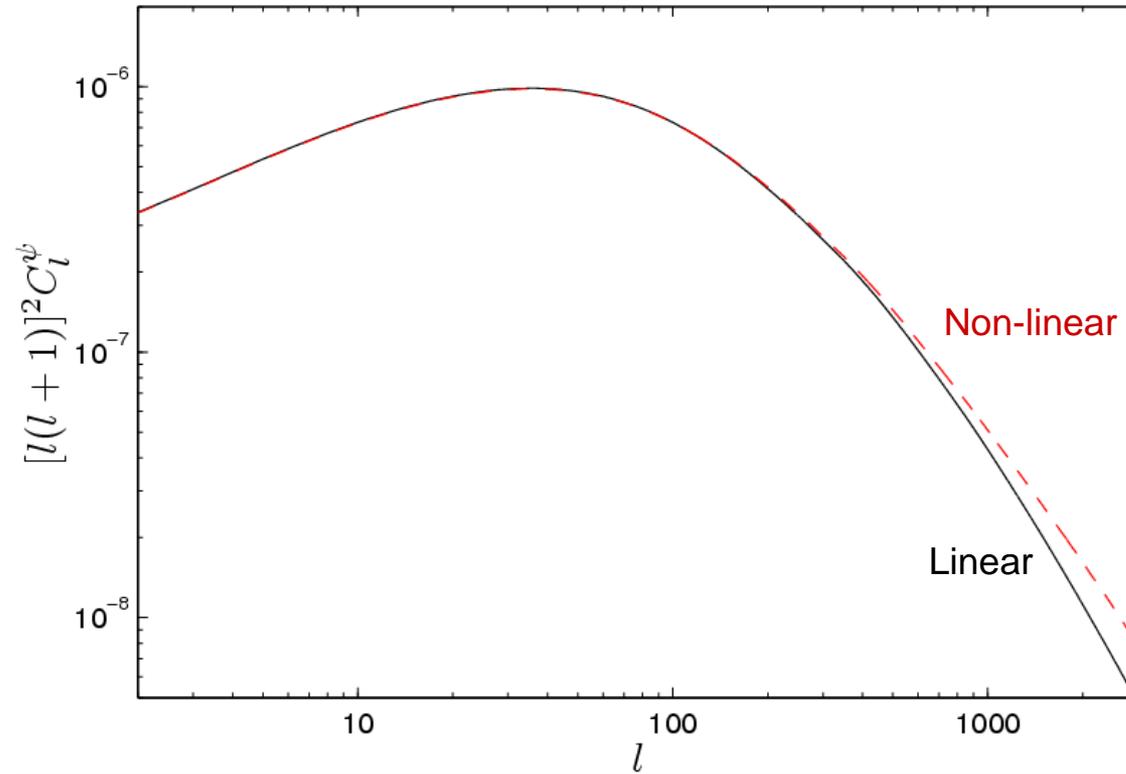
Observer

$$\psi(\hat{\mathbf{n}}) \equiv -2 \int_0^{\chi_*} d\chi \frac{f_K(\chi_* - \chi)}{f_K(\chi_*) f_K(\chi)} \Psi(\chi \hat{\mathbf{n}}; \eta_0 - \chi)$$

$$\tilde{T}(\hat{\mathbf{n}}) = T(\hat{\mathbf{n}}') = T(\hat{\mathbf{n}} + \hat{\boldsymbol{\alpha}}) \quad \hat{\boldsymbol{\alpha}} = \nabla \psi$$



Deflection angle power spectrum



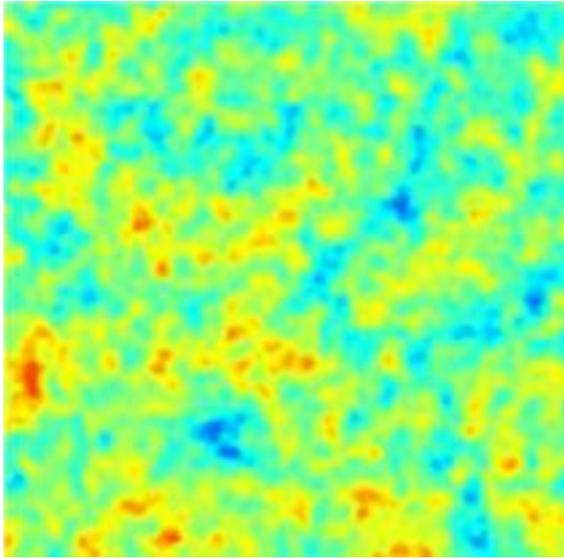
Deflections $O(10^{-3})$, but coherent on degree scales \rightarrow important!

Why lensing is important

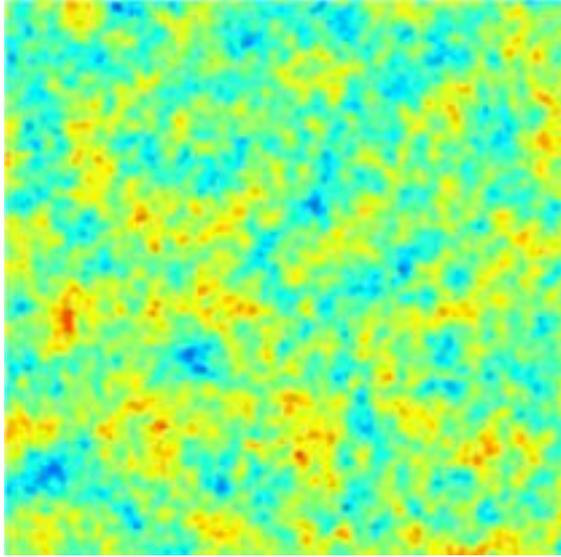
- Known effect, significant amplitude ($\sim 10^{-3}$)
- Modifies the power spectra on small-scales ($\sim 10^{-2}$)
- Lensing of E gives B-mode polarization (confusion for tensors/strings)
- Produces significant squeezed-shape bispectrum
- Large squeezed-shape trispectrum
- Non-Gaussianities measure lensing potentials
 - break degeneracies, constraint dark energy, $m_\nu, \Omega_K \dots$

Beyond the power spectrum

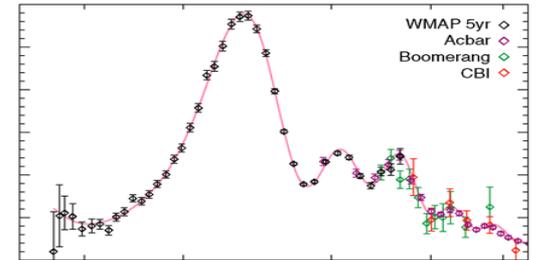
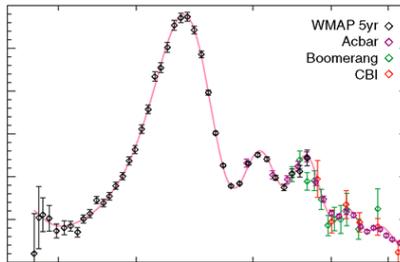
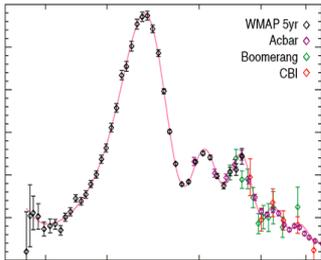
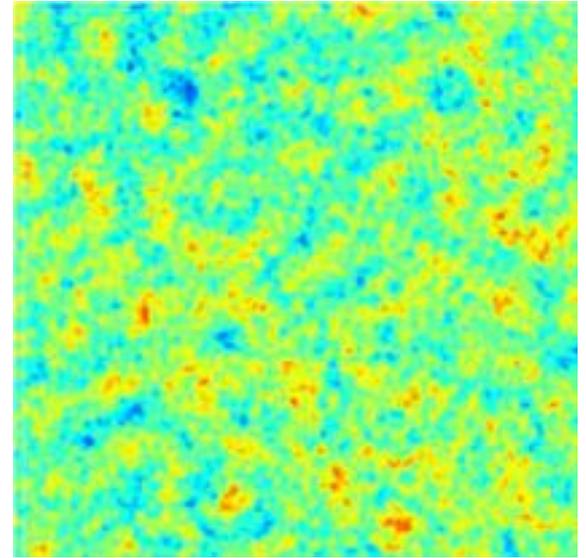
Magnified



Unlensed

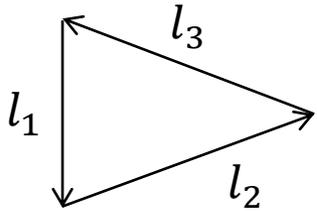


Demagnified



Beyond Gaussianities – general possibilities

Bispectrum



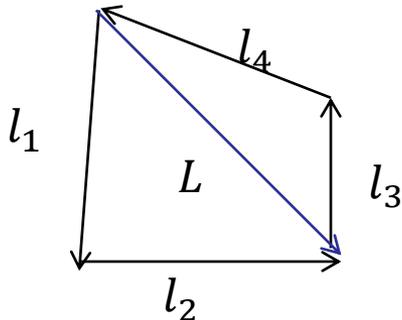
Flat sky approximation: $\langle \Theta(l_1)\Theta(l_2)\Theta(l_3) \rangle = \frac{1}{2\pi} \delta(l_1 + l_2 + l_3) b_{l_1 l_2 l_3}$

If you know $T(l_1), T(l_2)$, sign of $b_{l_1 l_2 l_3}$ tells you which sign of $T(l_3)$ is more likely

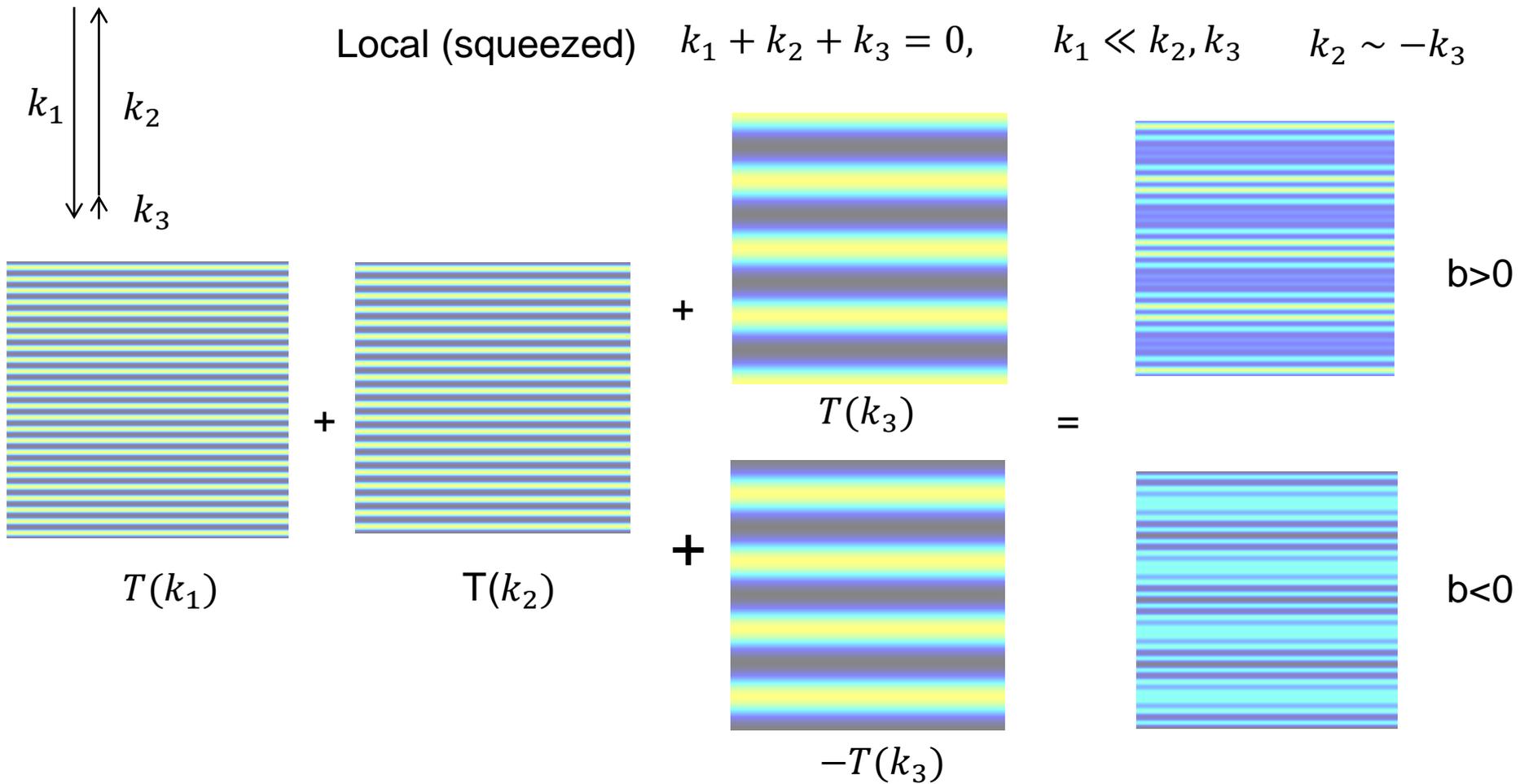
Trispectrum

$$\langle \Theta(l_1)\Theta(l_2)\Theta(l_3)\Theta(l_4) \rangle_C = (2\pi)^{-2} \delta(l_1 + l_2 + l_3 + l_4) T(l_1, l_2, l_3, l_4)$$

$$\langle \Theta(l_1)\Theta(l_2)\Theta(l_3)\Theta(l_4) \rangle_C = \frac{1}{2} \int \frac{d^2 \mathbf{L}}{(2\pi)^2} \delta(l_1 + l_2 + \mathbf{L}) \delta(l_3 + l_4 - \mathbf{L}) \mathbb{T}_{(l_3 l_4)}^{(l_1 l_2)}(L) + \text{perms.}$$



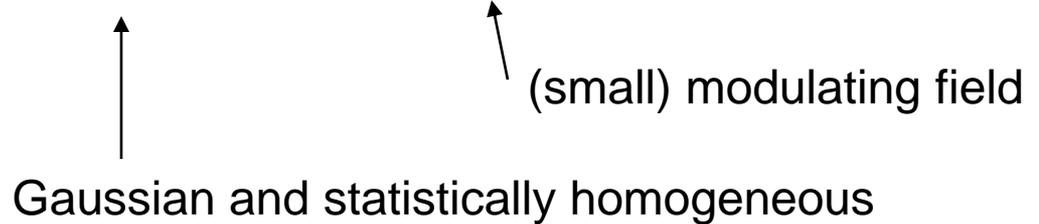
N-spectra...



Modulation of small-scale power by large-scale modes

Local primordial spatial modulation

$$\chi(\mathbf{x}) = \chi_0(\mathbf{x})[1 + \phi(\mathbf{x})]$$



 Gaussian and statistically homogeneous
 (small) modulating field

Gives squeezed non-Gaussianity

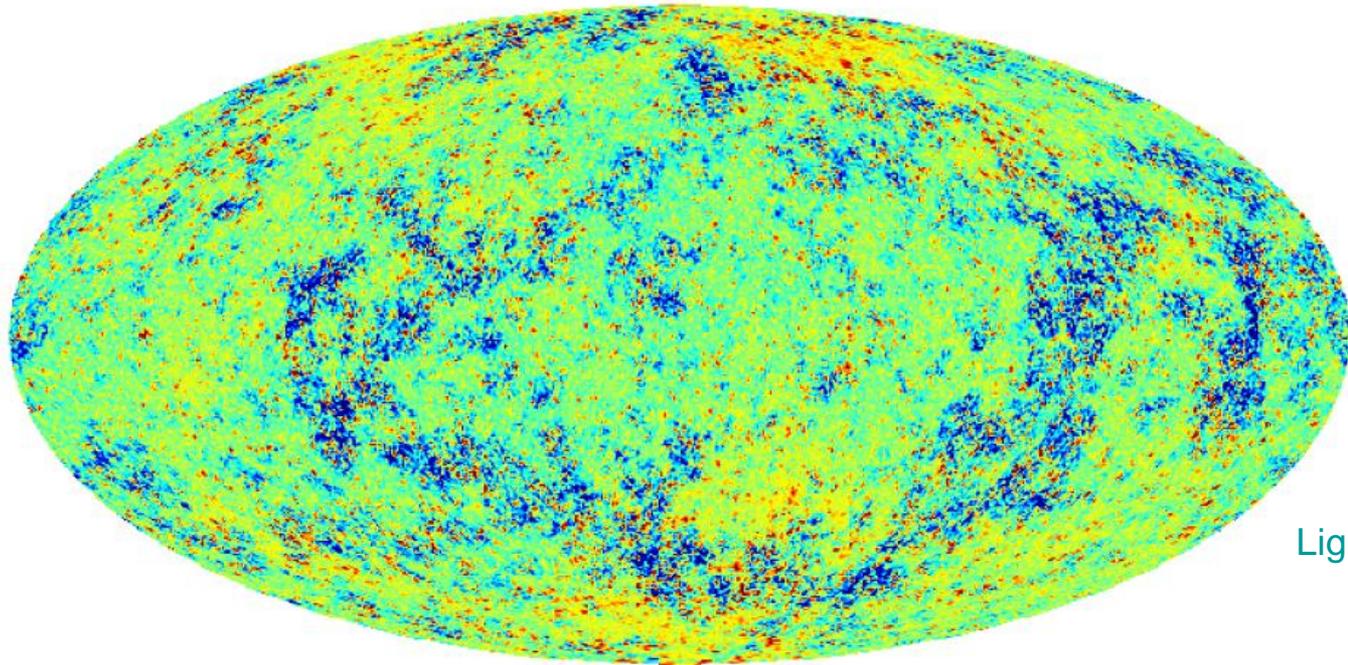
$$\langle \chi\chi\chi \rangle \sim \langle \chi_0\chi_0\chi_0\phi \rangle + \dots \sim P_{\chi_0\chi_0} P_{\chi_0\phi}$$

$$\begin{aligned} \langle \chi\chi\chi\chi \rangle &\sim \langle \chi_0\chi_0\chi_0\chi_0 \rangle + \langle \chi_0\chi_0\chi_0\chi_0\phi\phi \rangle + \dots \\ &\sim (\text{Gaussian}) + P_{\chi_0\chi_0} P_{\chi_0\chi_0} P_{\phi\phi} \end{aligned}$$

Since $P_{\chi_0\phi}^2 \leq P_{\chi_0\chi_0} P_{\phi\phi}$ $P_{\chi_0\chi_0} \langle \chi\chi\chi\chi \rangle_{\text{squeezed}} \geq \langle \chi\chi\chi \rangle_{\text{squeezed}}^2$

In conventional definitions $\tau_{NL} \geq \left(\frac{6f_{NL}}{5}\right)^2$ (also L by L if quasi local)

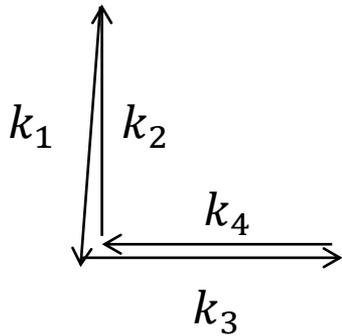
Temperature ($f_{\text{NL}} = 10^4$)



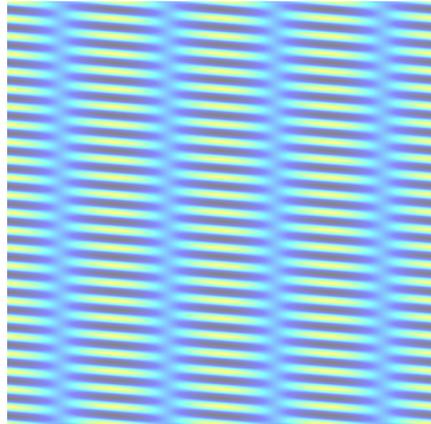
-0.00016 0.00016

Liguori et al 2007

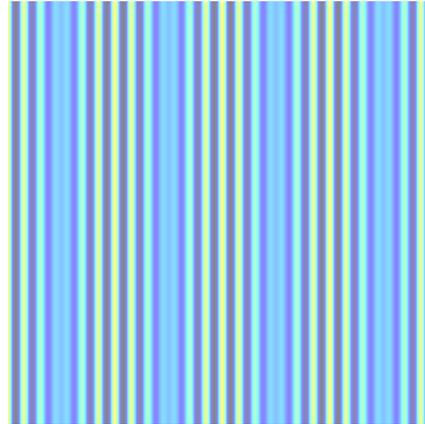
Squeezed trispectrum \sim power spectrum of modulation field



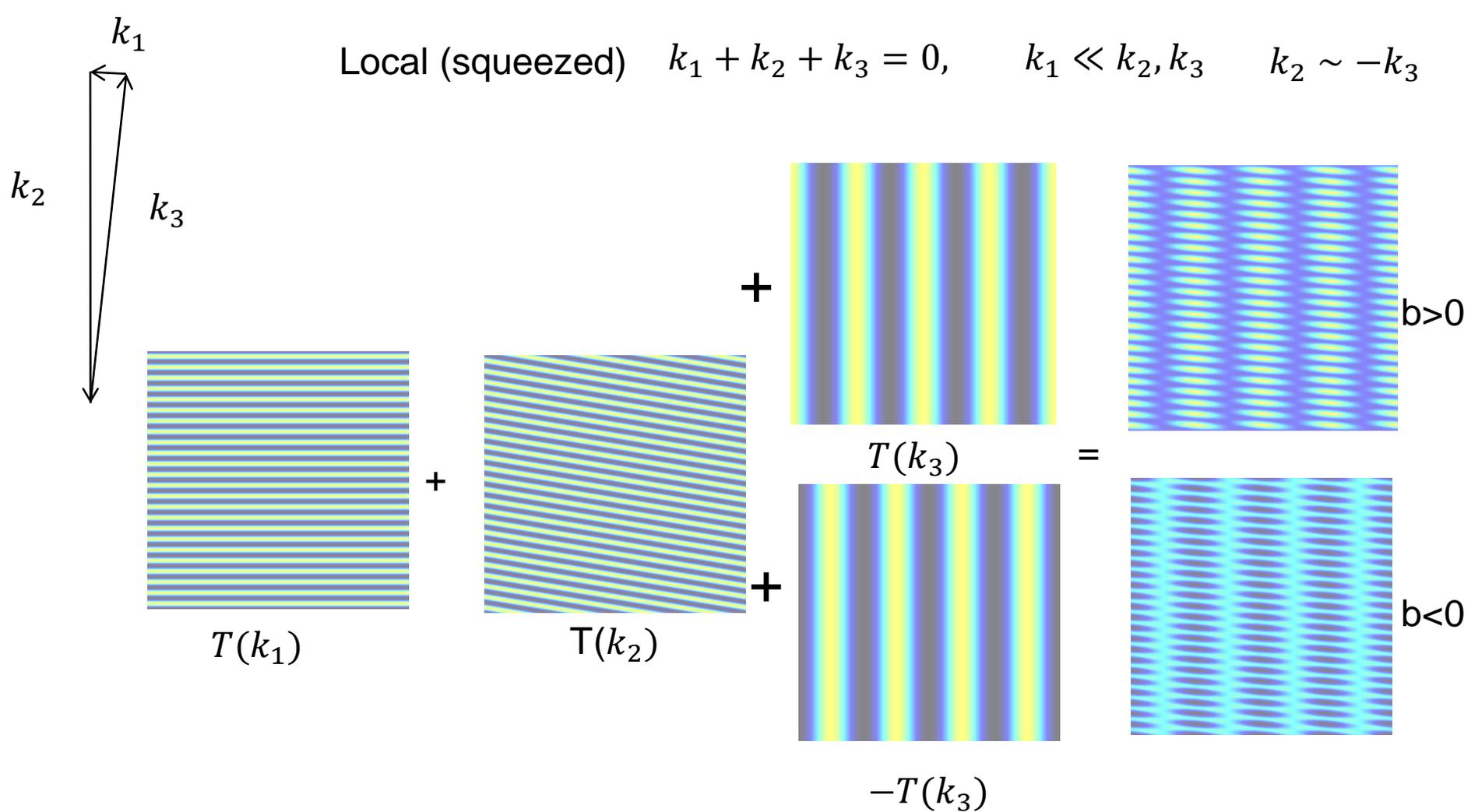
$T(k_1)+T(k_2)$



$T(k_3)+T(k_4)$



Small scale power is modulated by mode with $K = k_1 + k_2 = -(k_3 + k_4)$
- may or may not be correlated to large scale T modes



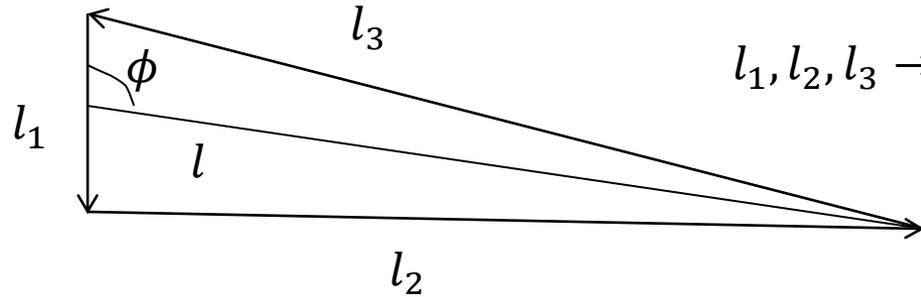
Possible direction-dependent modulation.

Local f_{NL} is isotropic, but e.g. CMB lensing is not:

$$b_{l_1 l_2 l_3} \approx -C_{l_1}^{T\psi} \left[(l_1 \cdot l_2) \tilde{C}_{l_2}^{TT} + (l_1 \cdot l_3) \tilde{C}_{l_3}^{TT} \right]$$

$$\approx l_1^2 C_{l_1}^{T\psi} \left[\frac{(l_1 \cdot l_2)^2}{l_1^2 l_2^2} \frac{d\tilde{C}_{l_2}^{TT}}{d \ln l_2} + \tilde{C}_{l_2}^{TT} \right].$$

Shape decomposition of squeezed triangles



$$l_1, l_2, l_3 \rightarrow l_1, l, \phi$$

$$b_{l_1 l_2 l_3} = \sum_m b_{l_1 l}^m e^{im\phi}$$

Local isotropic modulations: $m = 0$

CMB lensing:

$$m = 0$$

+

$$m = 2$$



Looks like $f_{NL} \sim 9$
 $\sim 2\sigma$ signal

Orthogonal to f_{NL}
 $\sim 4\sigma$ signal

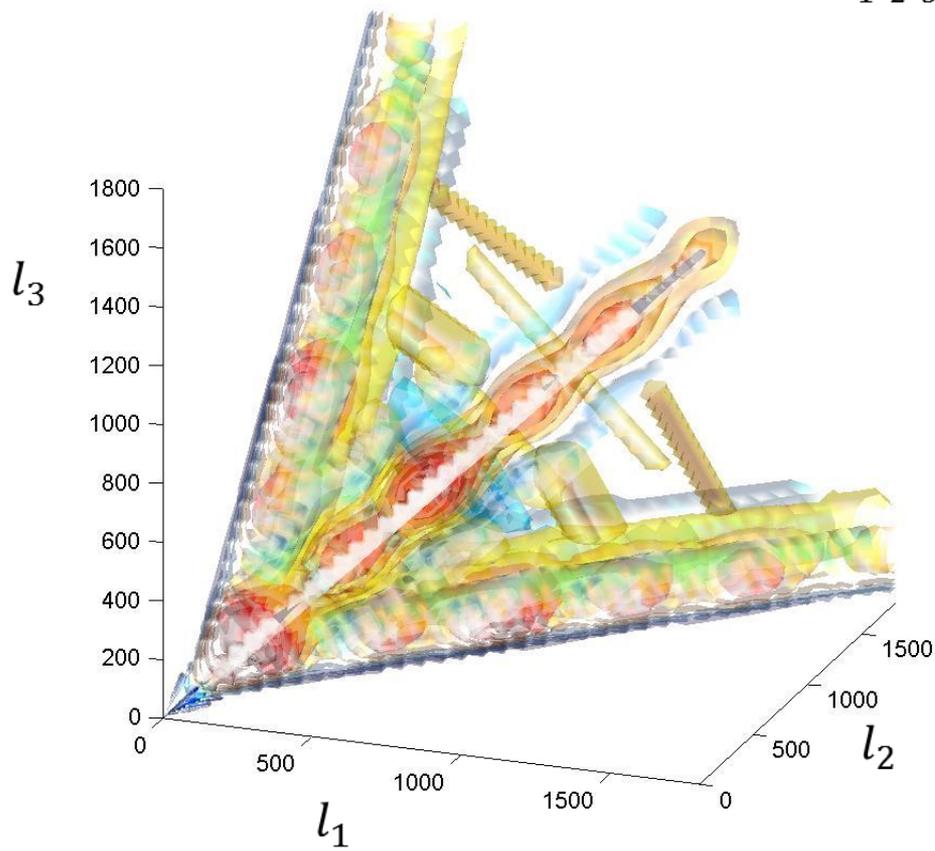
Angular dependence can be used to isolate and subtract different signals

- also l_1 dependence and phase of l dependence can be distinctive

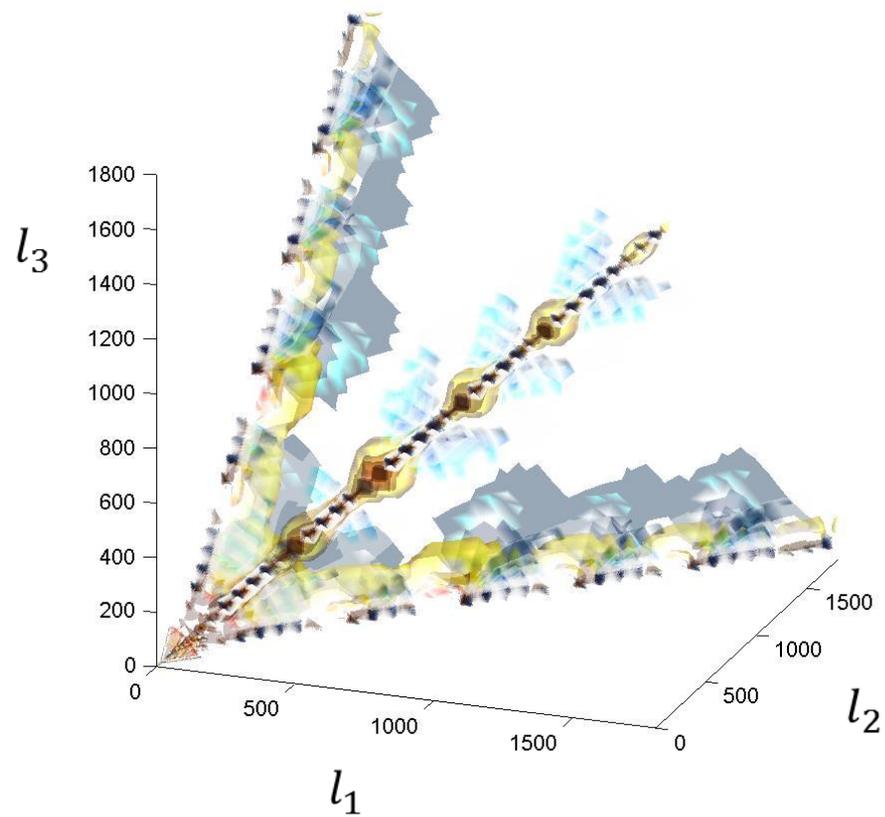
Lensing also modifies primordial signals. For $m = 0$ just like the power spectrum, otherwise:

$$\tilde{b}_{l_1 l}^m \approx \int r dr J_m(lr) \int dl' l' b_{l_1 l'}^m e^{-l'^2 \sigma^2(r)/2} \sum_n I_n[l'^2 C_{gl,2}(r)/2] J_{2n+m}(l'r)$$

$b_{l_1 l_2 l_3}$



Local f_{NL}



CMB lensing

Anisotropy estimators – just reconstruct the ‘modulating’ field

Following Hu et al 2000-2003

Munshi & Heavens 2009

Hanson & Lewis, [0908.0963](#)

First treat modulation field h as fixed. If other fields Gaussian:

$$-\mathcal{L}(\hat{\Theta}|\mathbf{h}) = \frac{1}{2} \hat{\Theta}^\dagger (C^{\hat{\Theta}\hat{\Theta}})^{-1} \hat{\Theta} + \frac{1}{2} \ln \det(C^{\hat{\Theta}\hat{\Theta}})$$

Maximum likelihood:

$$\frac{\delta \mathcal{L}}{\delta \mathbf{h}^\dagger} = -\frac{1}{2} \hat{\Theta}^\dagger (C^{\hat{\Theta}\hat{\Theta}})^{-1} \frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta \mathbf{h}^\dagger} (C^{\hat{\Theta}\hat{\Theta}})^{-1} \hat{\Theta} + \frac{1}{2} \text{Tr} \left[(C^{\hat{\Theta}\hat{\Theta}})^{-1} \frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta \mathbf{h}^\dagger} \right] = 0$$

First iteration solution: Quadratic Maximum Likelihood (QML)



$$\hat{\mathbf{h}} = \mathcal{F}^{-1}[\tilde{\mathbf{h}} - \langle \tilde{\mathbf{h}} \rangle].$$

$$\begin{aligned} \tilde{\mathbf{h}} = \mathcal{H}_0 &= \frac{1}{2} \bar{\Theta}^\dagger \frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta \mathbf{h}^\dagger} \bar{\Theta} & \bar{\Theta} &= (C^{\hat{\Theta}\hat{\Theta}})^{-1}|_0 \hat{\Theta} \\ &= \frac{1}{2} \sum_{lm, l'm'} \left[\frac{\delta C_{lm, l'm'}^{\hat{\Theta}\hat{\Theta}}}{\delta \mathbf{h}^\dagger} \right] \Theta_{lm}^* \Theta_{l'm'}, \end{aligned}$$

CMB lensing

Reconstruct lensing potential, ψ_{lm}

Bispectrum measured by $C_l^{T\psi} = \langle T_{lm}^* \psi_{lm} \rangle$ (Probes ISW, some info on dark energy)

Trispectrum measured by $C_l^{\psi\psi} = \langle \psi_{lm}^* \psi_{lm} \rangle$ (All scales, most of the information)

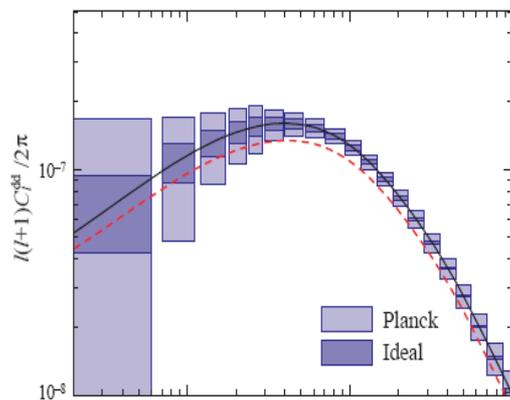
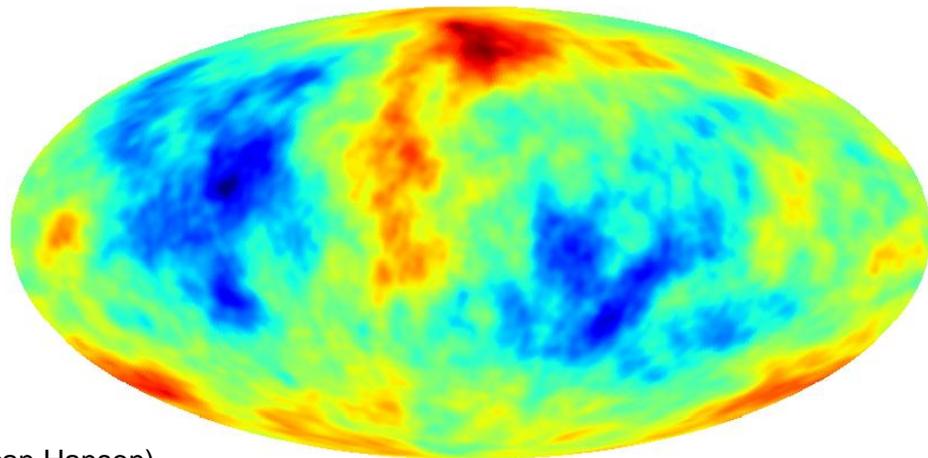
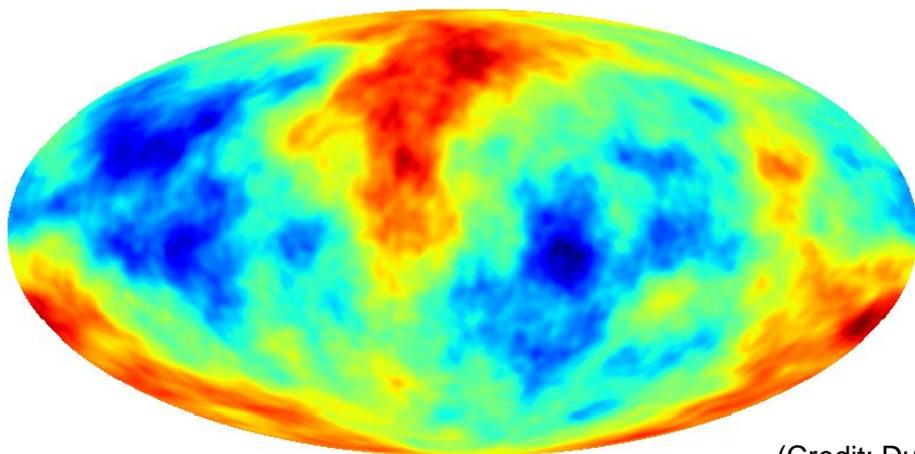


FIG. 3. CMB lensing power spectra for the fiducial $w = -1$ model (solid) and the degenerate $w = -2/3$ model (dashed) of Fig. 1. Boxes represent 1σ errors on band powers assuming the Planck and ideal experiments of Tab. I. Top: deflection power spectra. Bottom: cross correlation of deflection and temperature fields.

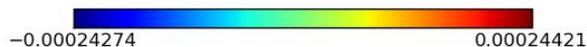
Hu: [astro-ph/0108090](https://arxiv.org/abs/astro-ph/0108090)

True (simulated)

Reconstructed (Planck noise, Wiener filtered)



(Credit: Duncan Hanson)

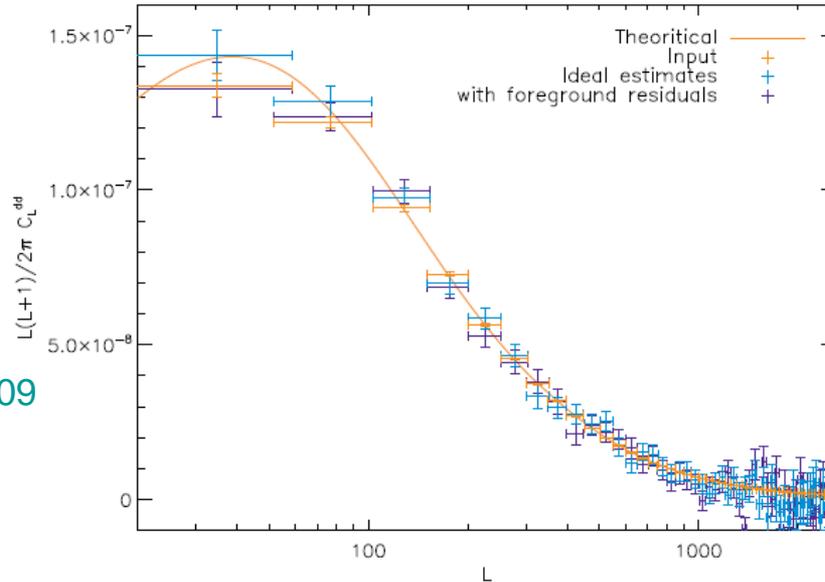


What does a reconstruction of lensing ψ_{lm} and hence estimate of $C_l^{\psi\psi}$ do for us?

Probe $0.5 < z < 6$: depends on geometry and matter power spectrum

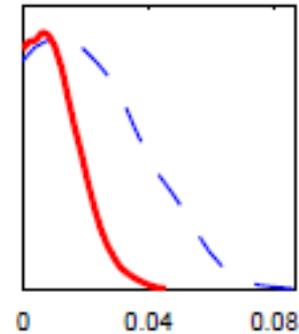
Can break degeneracies in the linear CMB power spectrum

- Better constraints on neutrino mass, dark energy, Ω_K , ...



Perotto et al. 2009
(simulation)

Neutrino mass fraction with and without lensing (Planck only)



Perotto et al. 2006

Local modulation

Reconstruct $\phi_{lm}(\chi)$ – modulation field at distance χ

General bispectra

Similar construction, but a bit more complicated

- Numerically challenging unless separable; or use modes ([Ferguson et al](#))

Anisotropic primordial power spectrum

$$\mathcal{P}_\chi(\mathbf{k}) = \mathcal{P}_\chi(k)[1 + a(k)g(\hat{\mathbf{k}})]$$

e.g.

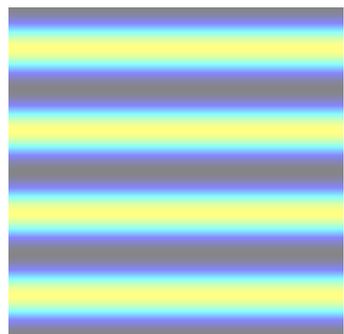
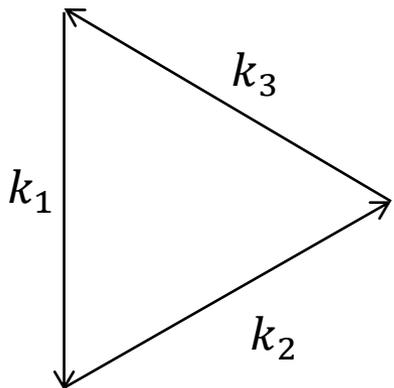
[Ackerman et.al. astro-ph/0701357](#)

[Gumrukcuoglu et al 0707.4179](#)

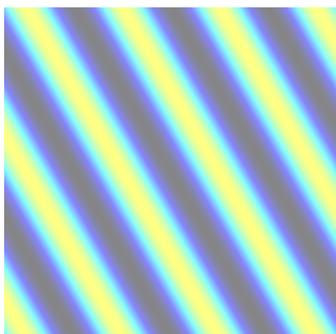
- Would show up in trispectrum, or just reconstruct g
(there is *not* any evidence for primordial $g \neq 0$ in WMAP)

Many other possibilities..

Equilateral $k_1 + k_2 + k_3 = 0, |k_1| = |k_2| = |k_3|$

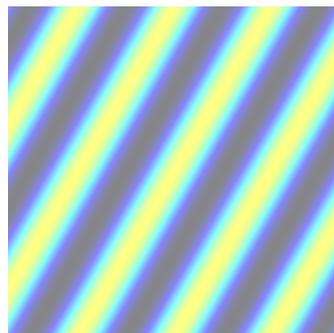


$T(k_1)$



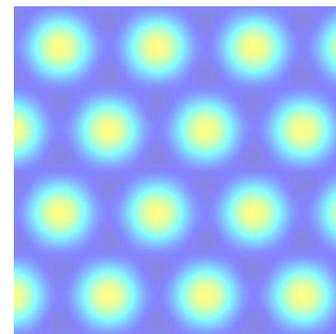
$T(k_2)$

+



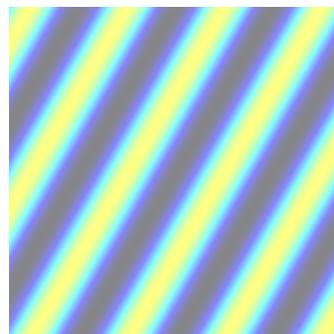
$T(k_3)$

=



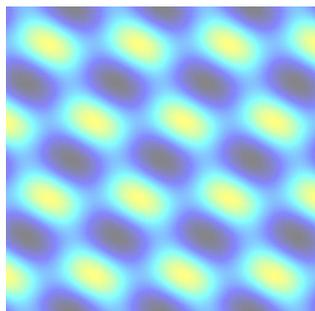
$b > 0$

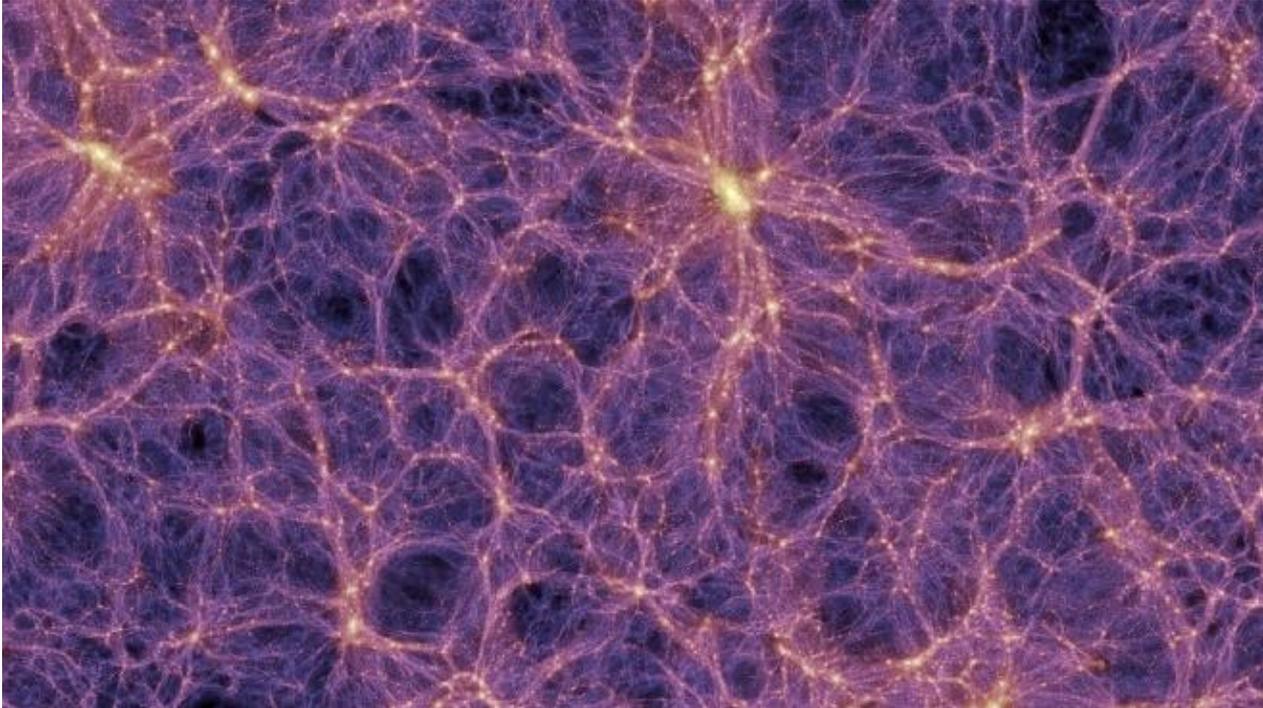
+



$-T(k_3)$

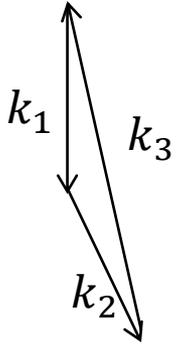
$b < 0$



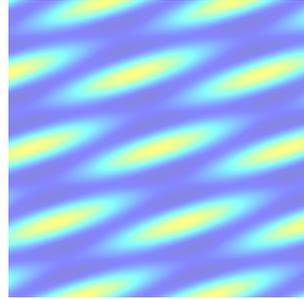


Millennium simulation

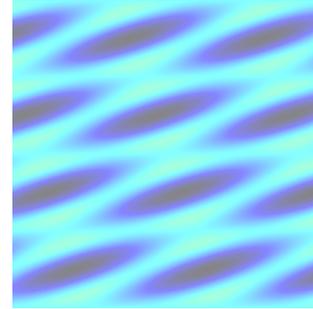
Near-equilateral to flattened:



$b > 0$



$b < 0$



- In general can measure full dependence on shape, but need specific models to have $S/N > 1$

e.g. 'orthogonal' shape changes sign of b between equilateral and flattened

Non-Gaussianities from non-linear effects till recombination

Pitrou, Uzan, Bernardeau (2010) claim local and equilateral components with $f_{NL} \sim 5$

Important for Planck and beyond. Do we believe this?

Total signal is about 2σ , so not easy to check directly against the data, but still important bias if neglected.

Things we might expect:

Isotropic local power modulation via large-scale modulation of Silk scale
(*but shouldn't this give negative f_{NL} ?*)

Large-scale modulation of sound horizon

- squeezed contribution out of phase with primary signal $\propto \frac{dC_l}{d \ln l}$

Equilateral contribution from non-linear growth of density perturbations

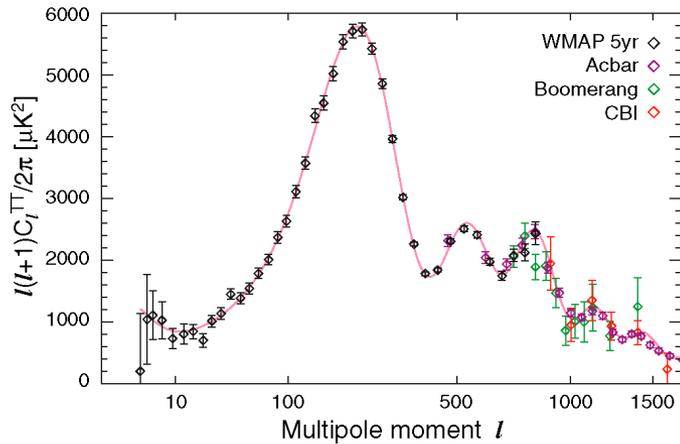
Can anyone give a physical explanation of the squeezed signal?

'Anomalies' in WMAP

Some will never be measured better.. but Planck will give check on WMAP

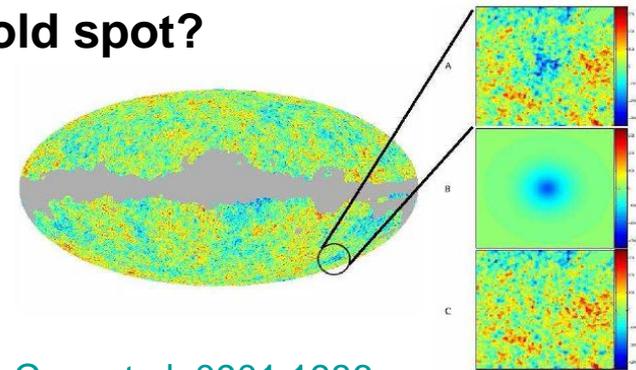
.. and polarization measurements can give good consistency check on models

(e.g. Dvorkin et al at [0711.2321](#))



Low quadrupole?

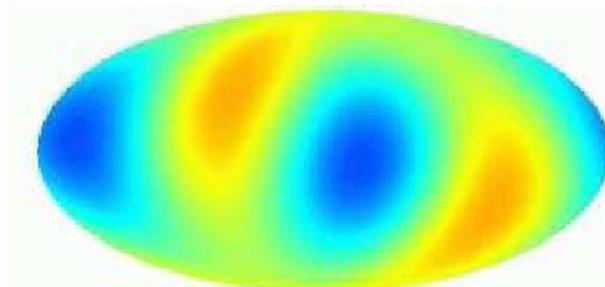
Cold spot?



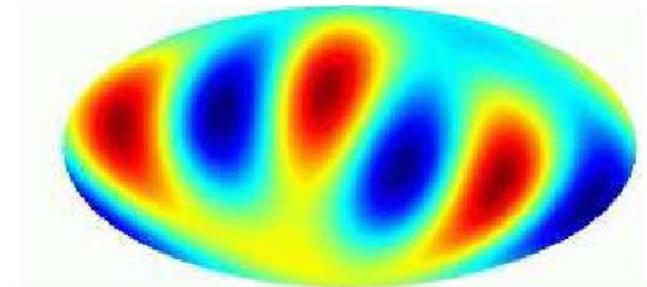
Cruz et al, 0901.1986

Alignments?

Tegmark et al.



Quadrupole



Octopole

etc.

Conclusions

- CMB is still by far the cleanest probe of early-universe physics and cosmological parameters
- Measure some parameters very accurately, but degeneracies and cosmic variance limitations
- Power spectrum and relation to parameters well understood (recombination? reionization?)
- Polarization can cleanly identify non-scalar signals and give powerful consistency checks on results from the temperature alone
- Statistical anisotropy/non-Gaussianities

Many possibilities

- primordial signals cosmic variance limited to around $f_{NL} \sim 2$
- some signals definitely present and easily detectable
 - Lensing; valuable new information, break degeneracies
 - Non-linear effects at recombination (??)
 - Local effects (SZ, point sources, foregrounds...???)