Anisotropy in the CMB

Antony Lewis
Institute of Astronomy &
Kavli Institute for Cosmology, Cambridge
http://cosmologist.info/
(almost) uniform 2.726K blackbody

Dipole (local motion)

\( O(10^{-5}) \) perturbations (+galaxy)

Observations: the microwave sky today

Source: NASA/WMAP Science Team
Can we predict the primordial perturbations?

- Maybe..

Quantum Mechanics
  “waves in a box”

Inflation
  make $>10^{30}$ times bigger

After inflation
  Huge size, amplitude $\sim 10^{-5}$
CMB temperature

End of inflation

gravity + pressure + diffusion

Last scattering surface
14 000 Mpc

$z \approx 1000$

$z = 0$

$\theta$

$14 000 \text{ Mpc}$

$z \approx 1000$
Observed CMB temperature power spectrum

Observations

Constrain theory of early universe + evolution parameters and geometry

Hinshaw et al.
The Vanilla Universe Assumptions

• Translation invariance - statistical homogeneity
  (observers see the same things on average after spatial translation)

• Rotational invariance - statistical isotropy
  (observations at a point the same under sky rotation on average)

• Primordial adiabatic nearly scale-invariant Gaussian fluctuations filling a flat universe

Statistically isotropic CMB with Gaussian fluctuations and smooth power spectrum
WMAP spice - not so vanilla?

Low quadrupole?

Alignments?

Tegmark et al.

Quadrupole

Octopole
Cold spot?

Cruz et al, 0901.1986

Power asymmetry?

Eriksen et al, Hansen et al.

+Non-Gaussianity?... +....?
Gaussian statistical anisotropy

- CMB lensing
- Power asymmetries
- Anisotropic primordial power
- Spatially-modulated primordial power
- Non-Gaussianity
Gaussian anisotropic models

\[ -\mathcal{L}(\hat{\Theta}|h) = \frac{1}{2} \hat{\Theta}^\dagger (C^{\hat{\Theta}\hat{\Theta}})^{-1} \hat{\Theta} + \frac{1}{2} \ln \det(C^{\hat{\Theta}\hat{\Theta}}) \]

Or is it a statistically isotropic non-Gaussian model??
Example: CMB lensing

\[ \alpha = -2 \int_0^{\chi^*} d\chi \frac{f_K(\chi^* - \chi)}{f_K(\chi^*)} \nabla \perp \Psi(\chi \hat{n}; \eta_0 - \chi) \]

\[ \tilde{T}(\hat{n}) = T(\hat{n}') = T(\hat{n} + \alpha) \]
Lensig field is FIXED:

Anisotropic Gaussian temperature distribution

- Different parts of the sky magnified and demagnified
- Re-construct the actual lensing field

Lensig field is RANDOM:

Non-Gaussian statistically isotropic temperature distribution

- Significant connected 4-point function
- Excess variance to anisotropic-looking realizations
- Lensed temperature power spectrum

We see only one sky - both interpretations can be useful

See forthcoming Hanson et al. review for details
Anisotropy estimators

\[- \mathcal{L}(\hat{\Theta} \mid h) = \frac{1}{2} \hat{\Theta}^\dagger (C^{\hat{\Theta}\hat{\Theta}})^{-1} \hat{\Theta} + \frac{1}{2} \ln \det (C^{\hat{\Theta}\hat{\Theta}}) \]

Maximum likelihood:

\[
\frac{\delta \mathcal{L}}{\delta h^\dagger} = -\frac{1}{2} \hat{\Theta}^\dagger (C^{\hat{\Theta}\hat{\Theta}})^{-1} \frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta h^\dagger} (C^{\hat{\Theta}\hat{\Theta}})^{-1} \hat{\Theta} + \frac{1}{2} \text{Tr} \left[ (C^{\hat{\Theta}\hat{\Theta}})^{-1} \frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta h^\dagger} \right] = 0
\]

\[\mathcal{H}\]

\[
\text{Tr}(A) = \langle x^\dagger AC^{-1}x \rangle \quad \Rightarrow \quad \frac{\delta \mathcal{L}}{\delta h^\dagger} = \langle \mathcal{H} \rangle - \mathcal{H} = 0
\]

\[
\langle x^\dagger AC^{-1}x \rangle = \langle \text{Tr}(AC^{-1}xx^\dagger) \rangle = \text{Tr}(AC^{-1}C) = \text{Tr}(A)
\]
Newton-Raphson solution:

\[
\begin{align*}
    h_{i+1} &= h_i - \left[ \frac{\delta}{\delta h^\dagger} (\langle \mathcal{H} \rangle - \mathcal{H})^\dagger \right]^{-1} (\langle \mathcal{H} \rangle_i - \mathcal{H}_i) \\
    \approx \left\langle \frac{\delta}{\delta h^\dagger} (\langle \mathcal{H} \rangle - \mathcal{H})^\dagger \right\rangle &= \left[ \langle \mathcal{H} \mathcal{H}^\dagger \rangle - \langle \mathcal{H} \rangle \langle \mathcal{H} \rangle^\dagger \right]
\end{align*}
\]

\[
\hat{h} = \mathcal{F}^{-1} [\tilde{h} - \langle \tilde{h} \rangle].
\]

First iteration solution: Quadratic Maximum Likelihood (QML)

\[
\tilde{\Theta} = (C^{\hat{\Theta} \hat{\Theta}})^{-1} |_0 \hat{\Theta}
\]

\[
\begin{align*}
    \tilde{h} &= \mathcal{H}_0 = \frac{1}{2} \hat{\Theta}^\dagger \frac{\delta C^{\hat{\Theta} \hat{\Theta}}}{\delta h^\dagger} \hat{\Theta} \\
    &= \frac{1}{2} \sum_{lm, l'm'} \left[ \frac{\delta C^{\hat{\Theta} \hat{\Theta}}_{lm, l'm'}}{\delta h^\dagger} \right] \Theta^*_{lm} \Theta_{l'm'},
\end{align*}
\]
Sky modulation?

Popular modulation model:

$$\Theta_f(\hat{n}) = [1 + f(\hat{n})] \Theta_f^i(\hat{n})$$

QML estimator for $f$:

$$\tilde{h}_{lm}^f = \int d\Omega Y_{lm}^* \left[ \sum_{l_1 m_1} \Theta_{l_1 m_1} Y_{l_1 m_1} \right] \left[ \sum_{l_2 m_2} C_{l_2} \tilde{\Theta}_{l_2 m_2} Y_{l_2 m_2} \right]$$

Approx Fisher:

$$\left[ F_{iso}^{ff} \right]_{lm, l'm'} = \delta_{ll'} \delta_{mm'}$$

$$\times \sum_{l_1, l_2} \frac{(2l_1 + 1)(2l_2 + 1)}{8\pi} \begin{pmatrix} l & l_1 & l_2 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}^2 \frac{(C_{l_1} + C_{l_2})^2}{C_{l_1}^{tot} C_{l_2}^{tot}}$$
Reconstruction recipe

\[ \hat{\Theta} = (C_{\hat{\Theta}}^{-1}|_0 \hat{\Theta} \]

(sets to zero in sky cut)

\[ \hat{\Theta} \]

Inverse variance filter

Make filtered maps

\[ F_1 = \sum_{l_1 m_1} \bar{\Theta}_{l_1 m_1} Y_{l_1 m_1} \]

\[ F_2 = \sum_{l_2 m_2} C_{l_2} \bar{\Theta}_{l_2 m_2} Y_{l_2 m_2} \]

\[ \tilde{h}_{lm}^f = \int d\Omega Y_{lm}^* \quad F_1 F_2 \]

Quadratic estimator

Simulate * many times to calculate \( \langle \tilde{h} \rangle \) (accounts for anisotropic noise/sky cut)

\[ \hat{h} = \mathcal{F}^{-1}[\tilde{h} - \langle \tilde{h} \rangle] \]

\[ \mathcal{F} \]

Approximated or from sims
WMAP power reconstruction
(V band, KQ85 mask, foreground cleaned; reconstruction smoothed to 10 degrees)

$\ell_{\text{max}} = 25$

Cold spot?
\( \ell_{\text{max}} = 64 \)

\( \ell_{\text{max}} = 100 \)

+ peak of QML dipole
Modulation power spectrum $l_{\text{max}}=64$

Dipole power asymmetry?
Dipole amplitude as function of $l_{\text{max}}$

Only $\sim 1\%$ modulation allowed on small scales

Consistent with Hirata 2009 - Very small observed anisotropy in quasar distribution
Is it just the cold spot?  
Or just the low multipoles?  
Or foregrounds?

- No

May be something interesting, but only ~1% significance at most
Primordial power anisotropy

Look for direction-dependence in primordial power spectrum:

$$\langle \chi_0(k) \chi_0^*(k') \rangle = (2\pi)^3 \delta(k - k') P_\chi(k)$$

Assume late-time isotropization.

$$\Theta_{lm} = 4\pi i^l \int \frac{d^3k}{(2\pi)^3} \Delta_l(k) \chi_0(k) Y_{lm}^*(\hat{k})$$

Anisotropic covariance:

$$C_{l_1 m_1 l_2 m_2} = i^{l_1 - l_2} \frac{\pi}{2} \int d^3k P_\chi(k) \Delta_{l_1}(k) \Delta_{l_2}(k) Y_{l_1 m_1}^*(\hat{k}) Y_{l_2 m_2}(\hat{k})$$
Simple case:  

\[ \mathcal{P}_X(k) = \mathcal{P}_X(k)[1 + a(k)g(\hat{k})] \]

\[ C_{l_1m_1l_2m_2} = \delta_{l_1l_2} \delta_{m_1m_2} C_{l_1} + \sum_{lm} i^{l_1-l_2} g_{lm} \int d\Omega_k C_{l_1l_2} Y_{lm} Y_{l_1m_1}^* Y_{l_2m_2} \]

where

\[ C_{l_1l_2} \equiv 4\pi \int d\ln k \mathcal{P}_X(k) a(k) \Delta_{l_1}(k) \Delta_{l_2}(k). \]
• Reconstruct $g(k)$.

QML estimator:

$$\tilde{h}_{l m}^g = \frac{1}{2} \int d\Omega Y_{l m}^* \sum_{l_1 l_2} i^{l_1 - l_2} C_{l_1 l_2}$$

Quadrupole primordial power asymmetry??

![Graph showing $C_{l l}$ for different $a(k)$ values]
Very significant evidence for ~10% quadrupole angular dependence!

\[ a(k) = 1 \]

Dashed: KQ75
Solid: KQ85

Variance from simulations

Very significant evidence for ~10% quadrupole angular dependence!
Direction close to ecliptic!
Could it be systematics?
- beam asymmetries? uncorrected in WMAP maps

Test with 10 asymmetric beam simulations of Wehus et al, 0904.3998
Intriguing, but probably not mostly primordial:

Signal varies significantly between detectors at the same frequency and aligned with ecliptic

- strong evidence for a systematic origin

Wehus simulations give effect of right order of magnitude

- beam asymmetry very important and must be accounted for
- but not consistent with data in all D/A, not complete explanation
Primordial spatial modulation

\[ \chi(x) = \chi_0(x)[1 + \phi(x)] \]

Modulation field

Gaussian and statistically homogeneous

\[ \langle \chi(k)\chi(k') \rangle = (2\pi)^3 \delta(k + k') P_\chi(k) \]
\[ + \int d^3x e^{-i(k+k')\cdot x} \phi(x) [P_\chi(k) + P_\chi(k')] \]
Expand: \[ e^{i\mathbf{k} \cdot \mathbf{x}} = 4\pi \sum_{lm} i^{l} j_{l}(kx) Y_{lm}(\hat{x}) Y_{lm}^{*}(\hat{k}) \]

Anisotropic covariance:

\[ C_{l_1m_1l_2m_2} = \delta_{l_1l_2} \delta_{m_1m_2} C_{l_1} \]

\[ + \int d^{3}x \phi(x) \alpha_{l_1}(x) \beta_{l_2}(x) Y_{l_1m_1}^{*}(\hat{x}) Y_{l_2m_2}(\hat{x}) \]

\[ + \int d^{3}x \phi(x) \alpha_{l_2}(x) \beta_{l_1}(x) Y_{l_1m_1}^{*}(\hat{x}) Y_{l_2m_2}(\hat{x}). \]

\[ \alpha_{l}(r) \equiv 4\pi \int d\ln k j_{l}(kr) \frac{k^{3} \Delta_{l}(k)}{2\pi^{2}} \]

\[ \beta_{l}(r) \equiv 4\pi \int d\ln k j_{l}(kr) \Delta_{l}(k) \mathcal{P}_{\chi}(k) \]
QML estimator for modulation field at distance $r$

$$
\tilde{h}_{l_1 m_1}^\phi (r) = \int d\Omega Y_{l_1 m_1}^* \left[ \sum_{l_1 m_1} \alpha_{l_1} (r) \tilde{\Theta}_{l_1 m_1} Y_{l_1 m_1} \right] \\
\times \left[ \sum_{l_2 m_2} \beta_{l_2} (r) \tilde{\Theta}_{l_2 m_2} Y_{l_2 m_2} \right]
$$
Integrate over $r$, almost equivalent to spatial modulation model

- Adiabatic model cannot explain dipole power asymmetry at $\ell \sim 60$

- Isocurvature modes decay on small scales, a possibility
Bispectrum non-Gaussianity

- Local model: small scale power correlated with large-scale temperature
- Considering large-scale modes to be fixed, expect power anisotropy

Liguori et al 2007
Local primordial non-Gaussianity

\[ \Psi = \Psi_0 + f_{NL} \Psi_0^2 \]

\[ = \Psi_0 (1 + f_{NL} \Psi_0) \]

Just like the spatial modulation model but modulation is the field itself

General bispectrum defined so that

\[ \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle \equiv B_{l_1 l_2 l_3} \left( \begin{array}{ccc} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{array} \right) \]

\[ = b_{l_1 l_2 l_3} \int d\Omega Y_{l_1 m_1} Y_{l_2 m_2} Y_{l_3 m_3} \]
Construct non-Gaussian field from Gaussian one:

\[ T_{l'm'} = T_{lm} + \frac{1}{6} B_{ll_1l_2} \sum_{m_1m_2} (-1)^{m_1} \begin{pmatrix} l & l_1 & l_2 \\ m & -m_1 & m_2 \end{pmatrix} T_{l_1m_1} T_{l_2m_2} \]

(assume $B$ small)

How about reverse? Make Gaussian from non-Gaussian:

Write general quadratic anisotropy estimator:

\[ 6X_{lm} \equiv \sum_{l_1m_1l_2m_2} B_{ll_1l_2} (-1)^{m_1} \begin{pmatrix} l & l_1 & l_2 \\ m & -m_1 & m_2 \end{pmatrix} \bar{\Theta}_{l_1m_1} \bar{\Theta}_{l_2m_2} \]
\[ = \int d\Omega Y^*_l \sum_{l_1l_2} b_{ll_1l_2} \left[ \sum_{m_1} \bar{\Theta}_{l_1m_1} Y_{l_1m_1} \right] \left[ \sum_{m_2} \bar{\Theta}_{l_2m_2} Y_{l_2m_2} \right] \]

Then

\[ \Theta^G_{lm} \equiv \Theta_{lm} - f_{NL} (X_{lm} - \langle X_{lm} \rangle) \]

is isotropic and Gaussian

\[ f_{NL} = 1 \text{ if } B \text{ has right amplitude} \]
Bispectrum estimators are basically the cross-correlation of an anisotropy estimator with the temperature

$$\mathcal{E} = \frac{1}{F_\mathcal{E}} \Theta^\dagger (X - 3\langle X \rangle),$$

In harmonic space

$$\mathcal{E} = \frac{1}{6F_\mathcal{E}} \sum_{l_1m_1} B_{l_1l_2l_3} \left( \begin{array}{ccc} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{array} \right)$$

$$\times \left[ \Theta_{l_1m_1} \Theta_{l_2m_2} \Theta_{l_3m_3} - 3C^{-1}_{l_1m_1l_2m_2} \Theta_{l_3m_3} \right].$$

Planck and the future, 2009+

High sensitivity and resolution
CMB temperature and polarization

14 May 2009
Scope for better estimators:

- Polarization. More signal, very good check of primordial/local origin.
- If non-zero signal, need more complicated iterative estimators
- Subtract effect of beam asymmetries and other systematics
- Account for uncertainties in cosmological parameters
- Use other probes of density/potential fields
- Remove ISW (e.g. Francis & Peacock 0909.2495)
\[ l \sum_{l_{2}l_{3}} B_{E_{2}E_{3}}^{2} / (6C_{ll_{2}l_{3}}C_{E_{2}E_{3}}) \]

With and without ISW

\(~ 20\% \text{ smaller error on } f_{NL} \)
Conclusions

- Can easily constrain a variety of Gaussian anisotropic models using QML estimators

- Marginal evidence for dipole power asymmetry in WMAP

- Strong evidence for anisotropy with primordial anisotropy model
  - varies between detectors, ecliptic alignment
  - may be partly due to beam asymmetries (right order of magnitude)
  - not mostly primordial

- Can improve with Planck, polarization, ISW modelling
Calculate likelihood:

\[-2 \log P(\Theta^G) \sim \Theta^G \cdot C^{-1} \Theta^G + \text{const} \]

So

\[ P(\Theta) = P(\Theta^G) \left| \frac{\partial \Theta^G}{\partial \Theta} \right| \]

The maximum likelihood satisfies \( \partial_{f_{\text{NL}}} \log P(\Theta) = 0 \):

\[
[\Theta - f_{\text{NL}}(X - \langle X \rangle)]^\dagger C^{-1}(X - \langle X \rangle) = \text{Tr} \left[ (I - f_{\text{NL}} dX/d\Theta)^{-1} \partial X/\partial \Theta \right]
\]

The leading Newton-Raphson solution is then

\[
\mathcal{E} = \frac{1}{F_\mathcal{E}} \left\{ (\Theta^\dagger(X - \langle X \rangle) - \text{Tr} [\partial X/\partial \Theta] \right\} \\
= \frac{1}{F_\mathcal{E}} \Theta^\dagger(X - 3\langle X \rangle),
\]

\( F_\mathcal{E} \sim \langle F_\mathcal{E} \rangle = 3 \text{Tr} \left[ C^{-1} \text{cov}(X) \right] \)

- the optimal estimator for weakly non-Gaussian fields
Take QML estimator for spatial modulation field at $r$

$$\tilde{h}_{l_m}^\phi(r) = \int d^3\Omega Y_{l_m}^* \left[ \sum_{l_1 m_1} \alpha_{l_1}(r) \Theta_{l_1 m_1} Y_{l_1 m_1} \right] \times \left[ \sum_{l_2 m_2} \beta_{l_2}(r) \Theta_{l_2 m_2} Y_{l_2 m_2} \right]$$

Local bispectrum: modulating field is the primordial anisotropy itself

Minimum-variance estimator for chi($r$): $\beta(r)\tilde{\Theta}_{l_m}$

Integrate QML estimator weighted by $r$-dependence of expected signal:

$$\bar{h}_{l_m} = \int dr r^2 \tilde{h}_{l_m}^\phi(r) \beta_{l_1}(r)$$

$$= \int d^3\Omega Y_{l_m}^* \sum_{l_1 l_2} b_{l_1 l_2} \left[ \sum_{m_1} \Theta_{l_1 m_1} Y_{l_1 m_1} \right] \left[ \sum_{m_2} \Theta_{l_2 m_2} Y_{l_2 m_2} \right]$$

$$b_{l_1 l_2 l_3} = \frac{3}{5} f_{NL} \int r^2 dr \beta_{l_1}(r) \beta_{l_2}(r) \alpha_{l_3}(r) + 5 \text{ perms.}$$

Correlating with $\tilde{\Theta}_{l_m}$ this is just the usual $t_{NL}$ estimator