# Stats for Engineers Lecture 11

# Acceptance Sampling Summary

**Acceptable quality level**: $p_1$

(consumer happy, want to accept with high probability)

**Unacceptable quality level**: $p_2$

(consumer unhappy, want to reject with high probability)

**Producer's Risk**: reject a batch that has acceptable quality

$$\alpha = P(\text{Reject batch when } p = p_1)$$

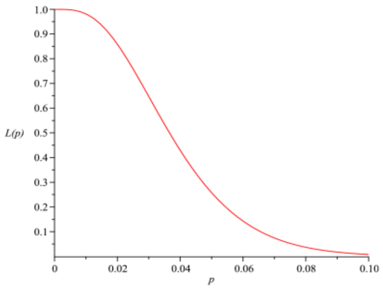
**Consumer's Risk**: accept a batch that has unacceptable quality

$$\beta = P(\text{Accept batch when } p = p_2)$$

**One stage plan**: can use table to find number of samples and criterion

**Two stage plan**: more complicated, but can require fewer samples

Operating characteristic curve
$L(p)$: probability of accepting the batch



**Is acceptance sampling a good way of quality testing?**

**Problems**:

It is too far downstream in the production process; better if you can identify where things are going wrong.

It is 0/1 (i.e. defective/OK) - not efficient use of data; large samples are required.

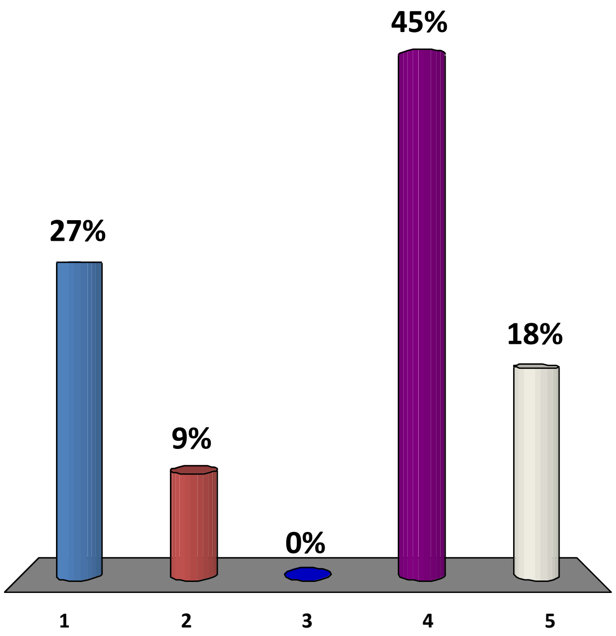
- better to have quality measurements on a continuous scale: earlier warning of deteriorating quality and less need for large sample sizes.

Doesn’t use any information about distribution of defective rates

## **Reliability**: Exponential Distribution revision

Which of the following has an exponential distribution?

1. The time until a new car's engine develops a leak
2. The number of punctures in a car's lifetime
3. The working lifetime of a new hard disk drive
4. 1 and 3 above
5. ✓ None of the above




Exponential distribution gives the time until or between random independent events that happen at constant rate.

(2) is a discrete distribution.

(1) and (3) are times to random events, but failure rate almost certainly increases with time.

# Reliability

Problem: want to know the time till failure of parts

E.g.

- what is the mean time till failure?

- what is the probability that an item fails before a specified time?

If a product lasts for many years, how do you quickly get an idea of the failure time?

***accelerated life*** testing:

**Compressed-time testing**: product is tested under usual conditions but more intensively than usual (e.g. a washing machine used almost continuously)

**Advanced-stress testing:** product is tested under harsher conditions than normal so that failure happens soon (e.g. refrigerator motor run at a higher speed than if operating within a fridge). - requires some assumptions

How do you deal with items which are still working at the end of the test programme?

An example of ***censored data***.

– we don't know all the failure times at the end of the test

**Exponential data** (failure rate $\nu$ independent of time)

Test components up to a time $t_0$

- assuming a rate, can calculate probability of no failures in $t_0$.

- calculate probability of getting any set of failure times (and non failures by $t_0$)

- find maximum-likelihood estimator for the failure rate in terms of failure times

For failure times $t_i$, with $t_i = t_0$ for parts working at $t_0$, and $n_f$ failures

$$\Rightarrow \text{Estimate of failure rate is } \hat{\nu} = \frac{n_f}{\sum_i t_i}$$

*[see notes for derivation]*

***Example:***

50 components are tested for two weeks. 20 of them fail in this time, with an average failure time of 1.2 weeks.

What is the mean time till failure assuming a constant failure rate?

*Answer:*

$$\hat{\nu} = \frac{n_f}{\sum_i t_i}$$

$$n = 50, n_f = 20$$

$$\sum_i t_i = 20 \times 1.2 + 30 \times 2 \quad = 84 \text{ weeks}$$

$$\Rightarrow \hat{\nu} = \frac{n_f}{\sum_i t_i} = \frac{20}{84} = 0.238/\text{week}$$

$\Rightarrow$ mean time till failure is estimated to be $\frac{1}{\hat{\nu}} = \frac{1}{0.238} = 4.2$ weeks

## Reliability function and failure rate

For a pdf $f(x)$ for the time till failure, define:

**Reliability function**

Probability of surviving at least till age $t$. i.e. that failure time is later than $t$

$$R(t) = P(T > t) = \int_t^\infty f(x)dx$$

$$= 1 - F(t)$$

$F(t) = \int_0^t f(t)dt$ is the cumulative distribution function.

**Failure rate**

This is failure rate at time $t$ given that it survived until time $t$: $\phi(t) = \frac{f(t)}{R(t)}$

$$P(\text{fail at } t|\text{OK until } t) = \frac{P(\text{OK until } t \cap \text{fail at } t)}{P(\text{OK until t})} = \frac{f(t)}{R(t)}$$

*Example*: Find the failure rate of the Exponential distribution

*Answer:*

The reliability is $R(t) = \int_t^{\infty} \nu e^{-\nu x} dx = e^{-\nu t}$

Failure rate, $\phi(t) = \frac{f(t)}{R(t)} = \frac{\nu e^{-\nu t}}{e^{-\nu t}} = \nu$ Note: $\nu$ is a constant

The fact that the failure rate is constant is a special "lack of ageing property" of the exponential distribution.

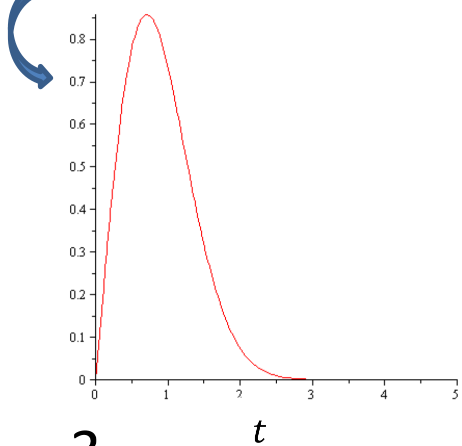
- But often failure rates actually increase with age.

# Reliability function

Which of the following could be a plot of a reliability function?

($R(t)$: probability of surviving at least till age $t$. i.e. that failure time is later than $t$)

1.

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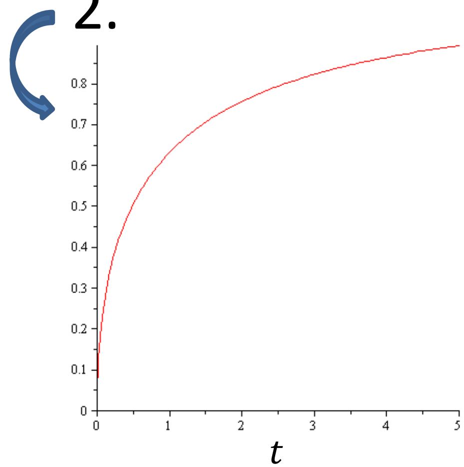


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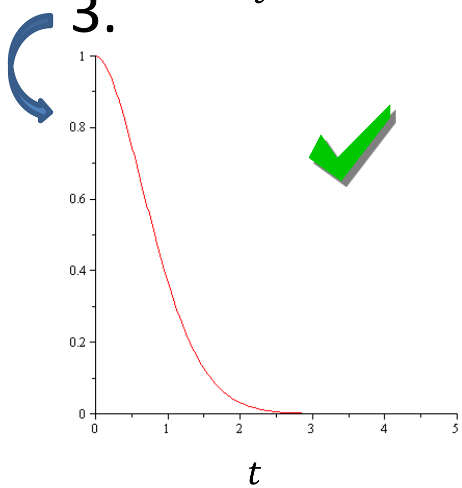


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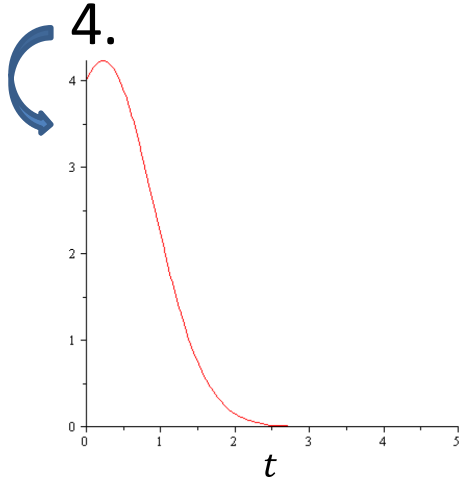
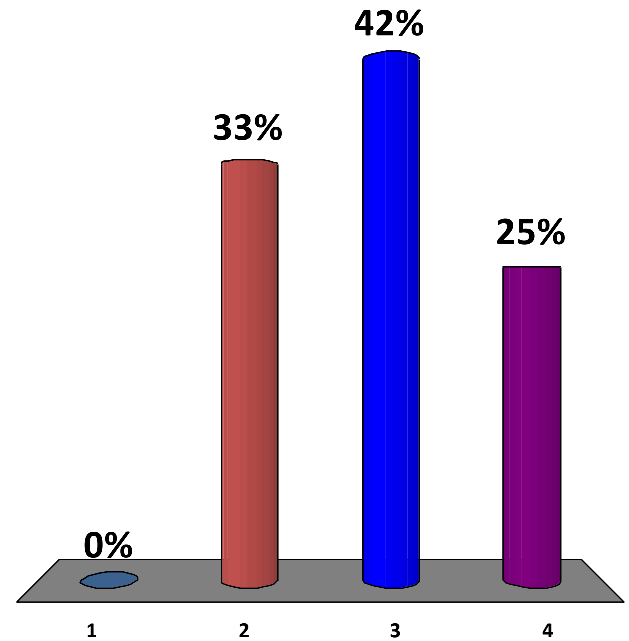


If we *measure* the failure rate $\phi(t)$, how do we find the pdf?

$$\phi(t) = \frac{f(t)}{R(t)} = \frac{\left(\frac{dF}{dt}\right)}{1 - F(t)} \quad = -\frac{d}{dt}\left[\ln\left(1 - F(t)\right)\right]$$

$F(0) = 0$ $\quad \Rightarrow \ln[1 - F(t)] = -\int_0^t \phi(t')dt'$

- can hence find $F(t)$, and hence $f(t), R(t)$

*Example*

Say failure rate $\phi(t)$ measured to be a constant, $\phi(t) = \nu$

$$\Rightarrow \ln[1 - F(t)] = -\int_0^t \nu dt = -\nu t$$

$$\Rightarrow 1 - F(t) = e^{-\nu t} \Rightarrow F(t) = 1 - e^{-\nu t} \qquad \Rightarrow f(t) = \frac{dF(t)}{dt} = \nu e^{-\nu t}$$

- Exponential distribution

## The Weibull distribution

- a way to model failure rates that are not constant

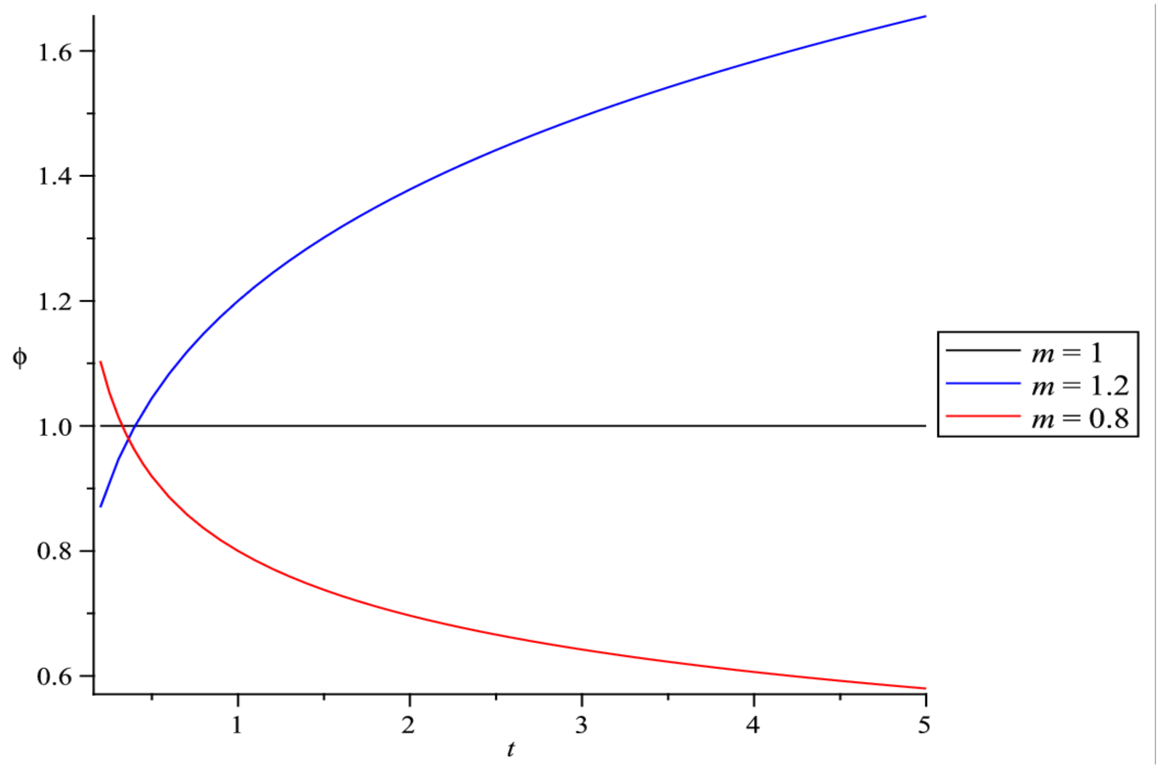
Failure rate: $\phi(t) = m\nu t^{m-1}$

Parameters $m$ (shape parameter) and $\nu$ (scale parameter)

$m = 1$: failure rate constant, Weibull=Exponential

$m > 1$: failure rate increases with time

$m < 1$: failure rate decreases with time




Failure rate: $\phi(t) = m\nu t^{m-1}$

$$\Rightarrow \ln\big(1 - F(t)\big) = -\int_0^t m\nu x^{m-1} dx = -\nu t^m$$

$$\Rightarrow F(t) = 1 - e^{-\nu t^m}$$

Reliability: $R(t) = e^{-\nu t^m}$

Pdf: $f(t) = \frac{dF(t)}{dt} = m\nu t^{m-1} e^{-\nu t^m}$

The End!

# THE UNIVERSITY OF SUSSEX G1042

**BSc/MMath EXAMINATIONS 2011**

MATHEMATICS: STATISTICS FOR ENGINEERS

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**You may attempt as many questions as you wish, but marks will be given for the best FOUR answers only.**

*Time allowed: ONE hour.*

*Each question carries TWENTY FIVE marks. The numbers beside the questions indicate the approximate marks that can be gained from the corresponding parts of the questions.*

*Examination handout: Maths Dept Statistical Tables, Statistical Formulae(Engineering Statistics)*

6. The time till failure of a part, $T$ years, has probability density function:

$$f(t) = kt^{-4} \quad (t > 1)$$

and is zero elsewhere, where $k$ is a constant.

(a) Find the value of $k$. **[5 marks]**

(b) Find the mean time till failure. **[5 marks]**

(c) Find the failure rate. **[5 marks]**

(d) Sketch a graph of the failure rate against time. What does the shape of this graph tell you? **[5 marks]**

*[Note: as from 2011 questions are out of 25 not 20]*

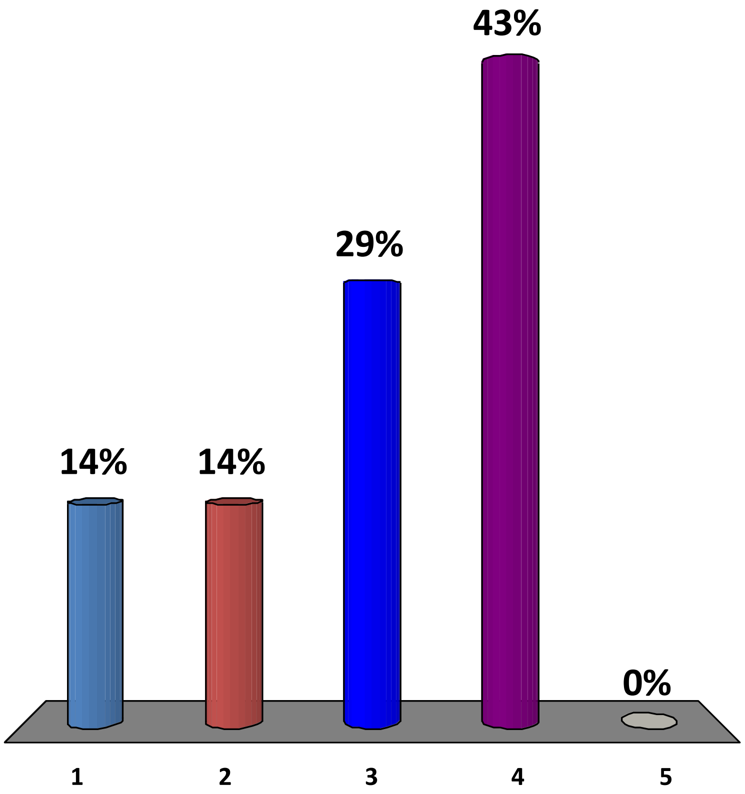
The time till failure of a part, $T$ years, has probability density function:

$$f(t) = kt^{-4} \ (t > 1)$$

and is zero elsewhere, where $k$ is a constant.

(a) Find the value of $k$.

1. 1/4
2. 3 ✓
3. 4
4. -3
5. 1/3



The time till failure of a part, $T$ years, has probability density function:

$$f(t) = kt^{-4} \quad (t > 1)$$

and is zero elsewhere, where $k$ is a constant.

(a) Find the value of $k$.

*Answer*

$$\int_a^b x^n = \left[\frac{x^{n+1}}{n+1}\right]_a^b$$

$$\int_{-\infty}^{\infty} f(t) = 1 \Rightarrow \int_1^{\infty} kt^{-4}\,dt = 1$$

$$\int_1^{\infty} kt^{-4}\,dt = \left[\frac{kt^{-3}}{-3}\right]_1^{\infty} = 0 - \left(-\frac{k}{3}\right) = \frac{k}{3}$$

$$\Rightarrow k = 3$$

The time till failure of a part, $T$ years, has probability density function:

$$f(t) = kt^{-4} \quad (t > 1)$$

and is zero elsewhere, where $k$ is a constant.

(b) Find the mean time till failure. **[5 marks]**

*Answer*

Know $k = 3$

$$\langle T \rangle = \int_{-\infty}^{\infty} t f(t) dt = \int_{1}^{\infty} \frac{k}{t^3} = \left[\frac{-k}{2t^2}\right]_1^{\infty} = 0 - \frac{-k}{2} = \frac{3}{2}$$

The time till failure of a part, $T$ years, has probability density function:

$$f(t) = kt^{-4} \quad (t > 1)$$

and is zero elsewhere, where $k$ is a constant.

(c) Find the failure rate. **[5 marks]**

*Answer*

$$\phi(t) = \frac{f(t)}{R(t)} \qquad R(t) = 1 - F(t)$$

$$F(t) = \int_0^t f(x)dx \quad = \int_1^t \frac{k}{x^4}dx \quad = \left[\frac{k}{-3t^3}\right]_1^t \quad = -\frac{k}{3t^3} - \left(-\frac{k}{3}\right) = 1 - \frac{1}{t^3}$$

$$\Rightarrow R(t) = 1 - F(t) = \frac{1}{t^3}$$

$$\Rightarrow \phi(t) = \frac{f(t)}{R(t)} = \frac{k}{t^4} \times t^3 = \frac{3}{t} \qquad (t > 1)\text{, otherwise } 0$$

The time till failure of a part, $T$ years, has probability density function:

$$f(t) = kt^{-4} \quad (t > 1)$$

and is zero elsewhere, where $k$ is a constant.

(d) Sketch a graph of the failure rate against time. What does the shape of this graph tell you? **[5 marks]**

*Answer*