THE UNIVERSITY OF SUSSEX G1042

BSc/MMath EXAMINATIONS 2007

STATISTICS FOR ENGINEERS

You may attempt as many questions as you wish, but marks will be given for the best FOUR answers only.

Time allowed: ONE hour.

Each question carries TWENTY marks. The numbers beside the questions indicate the approximate marks that can be gained from the corresponding parts of the questions.

Examination handout: Maths Dept Statistical Tables, Statistical Formulae(Engineering Statistics)

1. Give the formula for the conditional probability of event A happening, given that event B has already happened. Prove that if A and B are independent, then $P(A\, and\, B) = P(A) \times P(B)$. [2,2]
   A software company runs a help-line service for customers. Each call involves one, and only one, of three types of problems- applications, hardware incompatibility and installation. 48% of the calls involve applications, 38% are about incompatibility with hardware, and 14% involve installation problems. These three categories can be resolved on-line with probabilities 0.90, 0.15 and 0.80 respectively. Find the probability that

   (a) a call involves a problem that cannot be resolved [8]

   (b) if a call does involve a problem that cannot be resolved, then it is concerned with hardware incompatibility. [8]

2. Light bulbs are manufactured with a defect rate of 5%.

   (a) In a sample of 20, find the probability of 2 or more defectives. State two assumptions you have made in this calculation. [6]

   (b) Find the smallest sample size n for which the probability of having no defectives is less than 0.5. [6]

   (c) In a sample of 100 bulbs, use the Poisson approximation to find the probability of less than 5 defective bulbs. [8]

Turn over/

3. Five samples of diesel fuel had the following flash-points (in ${}^0C$):74,77,76,72,74

   (a) Find the sample mean and variance [6]

   (b) Find the 99% confidence interval for the population mean of flash-point temperatures. [7]

   (c) Find the number of samples required for the 99% confidence interval to have a width of 0.2 ${}^0C$. Briefly comment on your result. [7]

4. From extensive testing of washing machines, it is found that the time T in years before a machine needs a major repair has the probability density function:
   $f(t) = ke^{-t/4} \qquad t \geq 0$
   $f(t) = 0 \qquad$ elsewhere

   (a) Find k [6]

   (b) Find the probability that no major repair is needed in the first 5 years, giving your answer to 3 sig. fig. [6]

   (c) Find the median time until repair. Explain how your answer agrees with your result in (b). [8]

5. A survey was carried out to find the effect of aircraft noise on hearing. The following results were obtained, where x is the number of weeks spent living near an airport, and y is the hearing range:

x	47	56	116	178	19	75	160	31	12	164	43	74
y	15.1	14.1	13.2	12.7	14.6	13.8	11.9	14.8	15.3	12.6	14.7	14.0

   (so $n = 12; \Sigma x = 975; \Sigma x^2 = 117,397; \Sigma y = 166.8; \Sigma y^2 = 2,331.54, \Sigma xy = 12,884.4.$)

   (a) Show that the fitted regression line of y on x is
   $y = 15.3 - 0.0175x.$ [8]

   (b) Briefly describe the physical meaning of the constants in the regression line equation. [4]

   (c) Given that $\hat{\sigma}^2$, the variance of the random errors, is 0.133, find a 95 % confidence interval for the slope of the regression line of y on x. [8]

6. A two-stage sampling plan for a quality control procedure is as follows:
   Select 50 items from a batch, and accept the batch if none are faulty. Reject the batch if two or more items are defective. Otherwise, select another 20 items, and reject the batch if any of these are faulty.

   (a) Find the probability that a batch is rejected under this plan, if the probability p of any particular item being faulty is i) $p = 0.01$ ii) $p = 0.05$. [12]

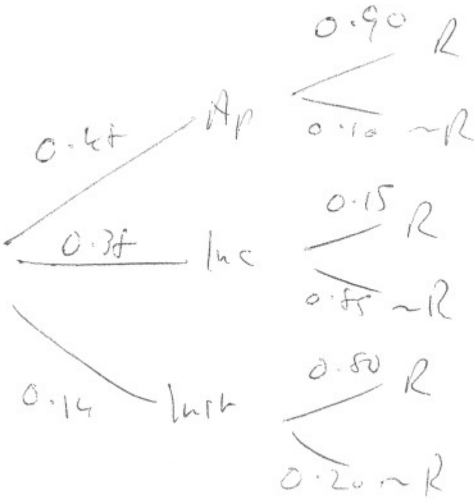
   (b) Show that the expected number of items sampled under this method is
   $50(1 + 20p(1 - p)^{49})$, and that this has a maximum value of 57.4. [4,4]

Stats for Engineers

1 $P(A|B) = P(A \text{ and } B)/P(B)$ | Blw

If ind. $P(A|B) = P(A)$ $\therefore$ $P(A \text{ and } B) = P(A) \times P(B)$

a)



$0.48 \times 0.1 = 0.048$

$0.38 \times 0.85 = 0.323$

$0.14 \times 0.2 = 0.028$

$\overline{0.399}$

b) $P(\text{Inc} | \sim R) = 0.323/0.399 = 0.8095$

2. $D \sim Bin(20, 0.05)$

$P(D \geq 2) = 1 - P(0, 1) = 1 - (0.95^{20} + 20 \times 0.95^{19} \times 0.05)$

$= 0.264$

Random sample
Independent items
Success or Fail.

b) Need $0.95^n < \cancel{0.1} \; 0.5$

$n > \frac{\log 0.5}{\log 0.95} = 13.5$

$\therefore$ Need 14 minimum

c) $Bin(100, 0.05) \approx Po(5)$

$\therefore Prob(0, 1, 2, 3, 4) = e^{-5}\left(1 + 5 + \frac{5^2}{2!} + \frac{5^3}{3!} + \frac{5^4}{4!}\right)$

$= 0.440$

3a $\bar{x} = 373/5 = 74.6$

$$s^2 = \frac{5}{4}\left(\frac{27841}{5} - 74.6^2\right) = 3.8$$

b) $CI = \bar{x} \pm t_4 \frac{s}{\sqrt{n}} = 74.6 \pm 4.604\sqrt{\frac{3.8}{5}} = 74.6 \pm 4.0$

$= 70.6$ to 78.6

for large n

c) Need $2 \times \overset{2.58}{\cancel{t_{?}}} \sqrt{\frac{3.8}{n}} = 0.2$

$$\frac{3.8}{n} = \left(\frac{0.1}{2.58}\right)^2 = 0.0015028$$

$\therefore \quad n = 2520$

4 a) Need $\int_0^\infty k e^{-t/4}\,dt = 1$ $\therefore$ $\left[-4ke^{-t/4}\right]_0^\infty = 1$

$\therefore\ 0 - (-4k) = 1 \qquad k = 1/4$

b) Prob $= \int_5^\infty \frac{1}{4} e^{-t/4}\,dt = \left[-e^{-t/4}\right]_5^\infty = 0 - \left(-e^{-(\frac{5}{4})}\right)$

$= 0.287$

c) Need $\int_0^T \frac{1}{4} e^{t/4}\,dt = \frac{1}{2}$

$\therefore \left[-e^{-t/4}\right]_0^T = \frac{1}{2}$

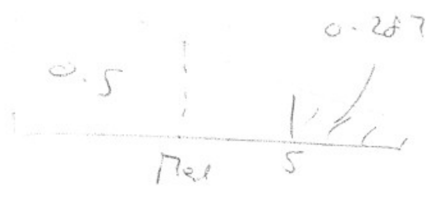
$-e^{-T/4} - (-1) = \frac{1}{2}$

$e^{-\frac{T}{4}} = +\frac{1}{2}$

$T/4 = -\ln\frac{1}{2}$

$T = 2.773$ years.

This is less than 5, as expected



5 a) $y = a + bx$ where $b = \dfrac{S_{xy}}{S_{xx}}$, $a = \bar{y} - b\bar{x}$

$$b = \left(12{,}884.4 - \frac{975 \times 1668}{12}\right) \Big/ \left(117{,}397 - \frac{975^2}{12}\right)$$

$$= -\frac{668.1}{38178.25} = -0.0175 \quad , \quad a = \frac{1668}{12} + 0.0175 \times \frac{975}{12}$$

$$= 15.3$$

$$\therefore \quad y = 15.3 - 0.0175x$$

~~That minimises, the top~~

b) 15.3 is hearing range after 0 weeks

-0.0175 is loss of hearing range for each week.

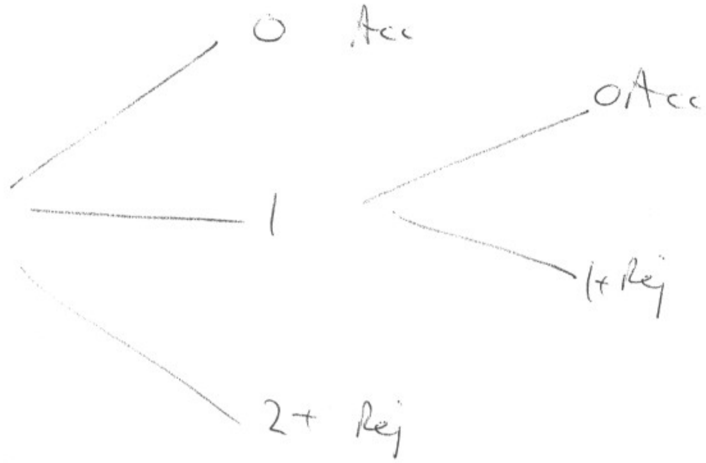
c) CI for b is $-0.0175 \pm \overset{2.228}{t_{10}} \sqrt{\dfrac{0.133}{38178.25}}$

$$= -0.0175 \pm 0.00416$$

$$= -0.0217 \text{ to } -0.0133$$

50 20

6



a) $\text{Prob}(\text{Acc}) = (1-p)^{50} + 50p(1-p)^{49} \times (1-p)^{20}$

$p = 0.01$, prob Acc $= 0.855$ $\therefore P(\text{Rej}) = 0.145$

$p = 0.05$, prob Acc $= 0.150$ $\therefore P(\text{Rej}) = 0.850$

b) Expected no $= \cancel{50\times(1-p)}^{50} + \cancel{70\times 50p(1-p)}^{49}$

$$= \cancel{(1-p)}^{49}(50\cancel{-50p+3500p})$$

$$= \cancel{50(1-p)}^{49}(\cancel{1+69}p)$$

$$= 50 + 20\times 50p(1-p)^{49}$$

$$E = 50 + 1000p(1-p)^{49}$$

$\frac{dE}{dp} = 1000(1-p)^{49} - 1000p\,49(1-p)^{48} = 0$ for max/min

$\therefore 1-p-49p = 0 \quad p = \frac{1}{50} = 0.02$

$E = 50 + 1000\times\frac{1}{50}\times 0.98^{49} = 57.4$