THE UNIVERSITY OF SUSSEX G1042

BSc/MMath EXAMINATIONS 2010

MATHEMATICS: STATISTICS FOR ENGINEERS

Wednesday, 23rd June 2010 **5.00 pm–6.00 pm**

You may attempt as many questions as you wish, but marks will be given for the best FOUR answers only.

Time allowed: ONE hour.

Each question carries TWENTY marks. The numbers beside the questions indicate the approximate marks that can be gained from the corresponding parts of the questions.

Examination handout: Maths Dept Statistical Tables (Normal), Maths Dept Statistical Tables (Acceptance Sampling), Statistical Formulae (Engineering Statistics)

1. (a) It is estimated that the probability of a rocket exploding during lift-off is 0.02, and that the chance of its guidance system failing is 0.05, where these events are independent. Find the probabilities that

   (i) the rocket will not explode during lift-off, [2]

   (ii) it will either explode or have its guidance fail, [3]

   (iii) it will neither explode nor have guidance failure. [3]

   (b) An assembly plant receives voltage regulators from three suppliers - 60% from A, 30% from B and 10% from C. Of those from source A, 95% perform to specification, from B 80% and from C 65%.

   (i) Find the total probability that a randomly selected regulator performs according to specification. [6]

   (ii) A regulator is found to fail; find the probability that it is from supplier C. [6]

Turn over/

2. At a service till, customers arrive at an average rate of 2.5 per minute.

   (a) Describe the statistical model for the number of people arriving each minute. [2]

   (b) Find the probability that at most 3 will arrive in any given minute. [6]

   (c) Find the probability that at least 3 will arrive during an interval of two minutes. [6]

   (d) Use the Normal approximation to find the probability that at least 20 will arrive in an interval of six minutes. [6]

   (All probabilities should be given correct to 4 decimal places.)

3. Some paint is sold in 5-litre tins. It is found that one tin of paint covers a mean area of 318 m^2, with a standard deviation of 26 m^2. Assuming that the area covered is normally distributed, find

   (a) the probability that one tin will cover at least 380 m^2, [4]

   (b) the probability that 40 such tins will cover an area of at least $12,500$ m^2. [8]

   (c) Find a 95% confidence interval for the area covered by 20 tins of paint. [8]

4. The mileage (in thousands of miles) that car owners get from a certain type of tyre is a random variable x having the probability density
   $f(x) = 0$ for $x < 0$
   $f(x) = \dfrac{e^{-x/20}}{20}$ for $x \geq 0$.

   (a) Find the mean mileage achieved with this type of tyre. [6]

   (b) Find the probability that a tyre fails before 10,000 miles have been driven on it. [4]

   (c) Find the probability that it lasts for at least 30,000 miles. [4]

   (d) Find the median lifetime of the tyre. [6]

5. The following data shows how the length of repair time y (in hours) for five jet engines varies with their length of usage x (in hundreds of hours):

Usage time (x)	1	2	3	4	5
Repair time (y)	30	40	70	80	100

(a) Find the equation of the line of best fit of y on x. [8]

(b) Briefly give a physical interpretation of each of the constants in the equation. [6]

(c) Use this to predict the repair times after (i) 60 hours and (ii) 360 hours. Which of these is likely to be the more accurate prediction? [6]

6. (a) In the context of Acceptance Sampling, give a brief explanation of Acceptable Quality Level, Unacceptable Quality Level, Producer Risk and Consumer Risk. Sketch a typical graph of an Operating Characteristic, and indicate on it where the four items above will lie. [10]

(b) In a particular production process, both Producer Risk and Consumer Risk are to be 0.1, the AQL is 0.01 and the UQL is 0.08.

(i) Use the Acceptance Sampling Tables to find the sample size, and the conditions under which a batch should be accepted. [4]

(ii) In this case, what is the probability that a batch is accepted when the proportion of defectives is 0.02? [6]

~~VERSION 1 – 2/2/10~~

Stats for Engineers 2010 FINAL VERSION

1(a)(i) $1 - 0.02 = 0.98$

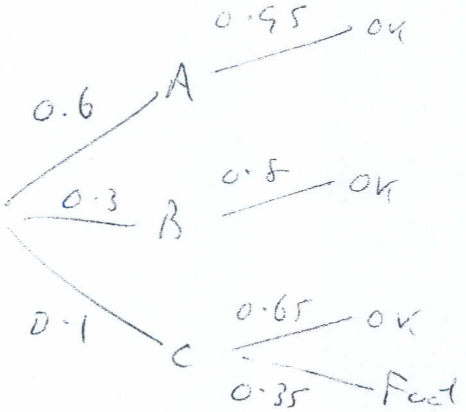
(ii) 'explode' or 'not-explode and guidance fail'

$= 0.02 + 0.98 \times 0.05 = 0.069$

(iii)

$0.98 \times 0.95 = 0.9310$

b)



Overall prob OK

$= 0.6 \times 0.95 + 0.3 \times 0.8 + 0.1 \times 0.65$

$= 0.57 + 0.24 + 0.065$

$= 0.875$

P(Firm C, given it has failed) $= \dfrac{0.035}{1 - 0.875}$

$= \dfrac{0.035}{0.125} = \dfrac{7}{25} = 0.28$

2 $\quad N \sim Po(2.5)$

a) $P = P(0) + P(1) + P(2) + P(3)$

$$= e^{-2.5} + 2.5e^{-2.5} + \frac{2.5^2}{2}e^{-2.5} + \frac{2.5^3}{6}e^{-2.5}$$

$$= e^{-2.5}(1 + 2.5 + 3.125 + 2.604) = 0.758$$

b) $P(\text{at least } 3) = 1 - P(0 \text{ or } 1 \text{ or } 2)$

In 2 minutes, $\lambda = 5$

$\therefore P(\text{at least } 3) = 1 - \left(e^{-5} + 5e^{-5} + \frac{5^2}{2}e^{-5}\right)$

$$= 1 - e^{-5}(1 + 5 + 12\tfrac{1}{2}) = 1 - 0.125 = 0.875$$

c) In 6 minutes, $\lambda = 6 \times 2.5 = 15$

$Po(15) \sim N(15, 15)$

$\therefore P(x \geq 20) = P(x > 19.5)$

$$= P\left(z > \frac{19.5 - 15}{\sqrt{15}}\right) = P(z > 1.1619)$$

$$= 1 - 0.8774$$

$$= 0.1226$$

3 a) $P(c > 380) = P\left(z > \frac{380-318}{26}\right) = P(z > 2.385)$

$= 1 - 0.9914 = 0.0086$

b) For 40 tins, coverage $\sim N(40 \times 318, 40 \times 26^2)$

$= N(12720, 164.4^2)$

$\therefore P(c > 12500) = P\left(z > \frac{12500 - 12720}{164.4}\right)$

$= P(z > -1.338) = 0.9096$

c) $CI = 20 \times 318 \pm 1.96\sqrt{20} \times 26$

$= 6360 \pm 228 = 6132$ to 6588

4 a) Mean $= \int_0^\infty \frac{x e^{-x/20}}{20}\,dx$

$= \left[-x e^{-\frac{x}{20}}\right]_0^\infty - \int \frac{e^{-\frac{x}{20}}}{20} \times -20\,dx$

$= 0 + \int e^{-\frac{x}{20}}\,dx = \left[-20 e^{-\frac{x}{20}}\right]_0^\infty = 0 + 20$
$= 20$

b) $P = \int_0^{10} \frac{e^{-x/20}}{20}\,dx = \left[-e^{-x/20}\right]_0^{10}$

$= -e^{-\frac{1}{2}} + 1 = 0.393$

c) $P = \int_{30}^\infty \frac{e^{-\frac{x}{20}}}{20} = \left[-e^{-\frac{x}{20}}\right]_{30}^\infty$

$= 0 + e^{-\frac{30}{20}} = 0.223$

d) Need $\int_0^M \frac{e^{-\frac{x}{20}}}{20}\,dx = \frac{1}{2}$

$\therefore\ -e^{-\frac{M}{20}} + 1 = \frac{1}{2}$

$e^{-\frac{M}{20}} = \frac{1}{2} \qquad -\frac{M}{20} = \ln\frac{1}{2}$

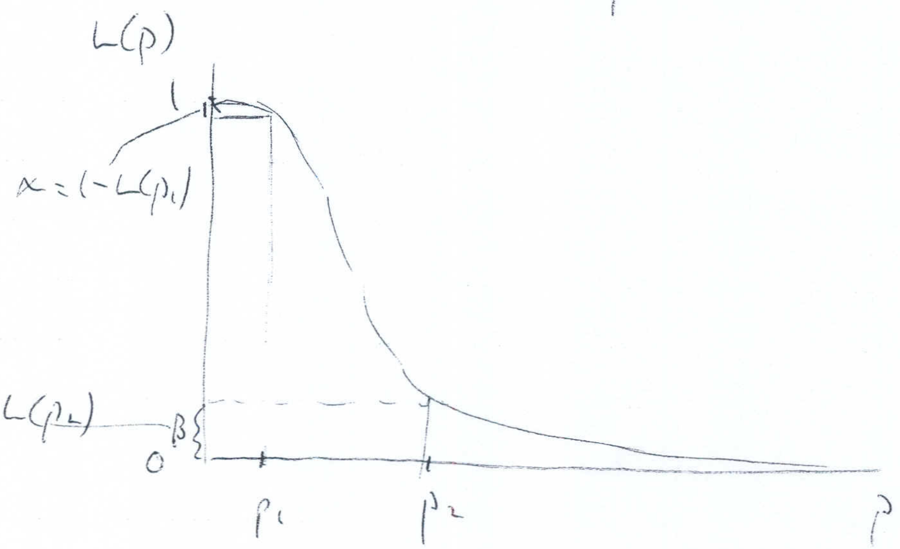
$\therefore\ M = 13.863$

ie 13,863 miles.

6 a) If p = proportion of defectives

AQL $= p_1$ where consumer happy if $p < p_1$
UQL $= p_2$ where consumer unhappy if $p > p_2$

Producer Risk α = prob batch is rejected if $p = p_1$
Consumer Risk β = prob batch is accepted if $p = p_2$



b) i) Sample 48 items, accept up to 1 defect

ii) $\text{Prob}(1) = 48 \times 0.02 \times 0.98^{47} = 0.3714$
$\text{Prob}(0) = 0.98^{48} = 0.3792$

$\therefore$ Prob accept $= 0.7506$