1. In a particular area, the weather each day is classified as wet or dry. If a day is dry, there is a probability of 0.7 that the next day will also be dry. If a day is wet, the probability that the next day will be wet is 0.6.

   If Sunday is wet, find the probability that

   (a) Monday, Tuesday and Wednesday are all wet, **[5 marks]**

   (b) Wednesday is wet, **[7 marks]**

   (c) if Wednesday is wet, then Monday was dry. **[8 marks]**

2. The lengths of a batch of widgets are distributed Normally. 6 % are longer than 5 cm, and 11 % are shorter than 4 cm.

   (a) Show that the mean length of widgets is 4.441 cm and that the standard deviation of the lengths of the widgets is 0.359 cm. **[8 marks]**

   (b) For a randomly selected widget, find the probability that its length exceeds 5.2 cm.

   [4 marks]

   (c) In a sample of 100 widgets, find the probability that the mean length exceeds 4.5 cm. **[8 marks]**

3. Records show that the probability that a car will have a flat tyre while going through a certain tunnel is 0.00004.

   (a) If 100,000 cars go through the tunnel each year, write down the probability distribution of x, the number of cars with flat tyres in the tunnel each year. Write down the mean and standard deviation of x, and calculate the probability that no cars have a flat tyre in the tunnel in a particular year. **[8 marks]**

   (b) Use the Poisson approximation to the binomial to estimate the probability that no cars have a flat tyre in a particular year, and compare your answer with that in a).

   [3 marks]

   (c) Use the Poisson approximation to the binomial to estimate the probability that $x \geq 5$ in a particular year. **[9 marks]**

4. The stress X Newtons in an engineering component has the probability density function:
   $f(x) = kx^3(10 - x)$ for $0 \leq x \leq 10$
   $f(x) = 0$ elsewhere,
   where k is a constant.

   (a) Show that $k = 0.0002$ **[6 marks]**

   (b) Find the mean and standard deviation of the stress in the component. **[10 marks]**

   (c) The component breaks if $X > 9$. Find the proportion of components that fail.

   [4 marks]

5. An investigation of Hooke's Law gave the following results for x, the tensile force in kilo-Newtons applied to a steel bar, and y, the resulting elongation of the bar, in micrometres:

x	1	2	3	4	5	6
y	10	33	40	63	76	85

   (a) Find the least squares estimate of the regression line of y on x. **[8 marks]**

   (b) Predict the elongation when a) $x = 4.7$ and b) when $x = 8.2$. Comment on the reliability of each of these estimates. By considering the behaviour of y when x is small, show that this linear model fails for small x. **[8 marks]**

   (c) Suppose that the units of measurement in the experiment are changed, to millimetres and Newtons. Write down the equation of the regression line of Y(mm) on X(N).

   [4 marks]

6. (a) In acceptance sampling, the following terms are used: Acceptable Quality Level, Unacceptable Quality Level, Producer Risk and Consumer Risk. Briefly explain each of these terms. **[8 marks]**

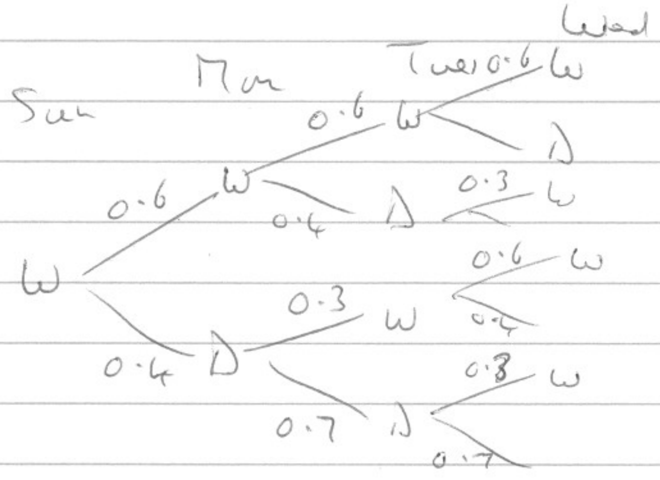
   (b) In a certain production process, both risks are to be 0.05, the AQL is 0.01 and the UQL is 0.08. Use the Acceptance Sampling Tables to find the sample size, and conditions under which a batch should be accepted. **[4 marks]**

   (c) In this case, what is the probability that a batch is accepted when the proportion of defectives is 0.04? **[6 marks]**

   (d) Briefly discuss the advantages and disadvantages of using a two-stage plan instead of a single-stage plan for sampling. **[2 marks]**

Stats for Engineers 2008

1 $P(A|B) = P(A \text{ and } B)/P(B)$.



a) $0.6 \times 0.6 \times 0.6 = 0.216$

b) $0.6^3 + 0.6 \times 0.4 \times 0.3 + 0.4 \times 0.3 \times 0.6 + 0.4 \times 0.7 \times 0.3$

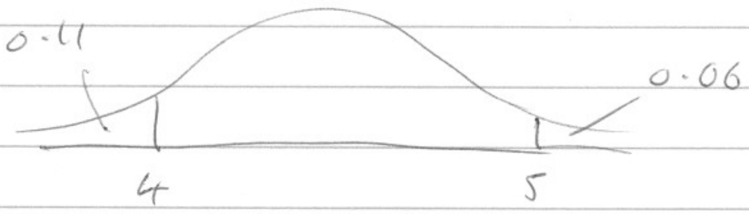
$= 0.216 + 0.072 \times 2 + 0.084$

$= 0.444$

c) $\dfrac{P(\text{Wed wet and Mon dry})}{P(\text{Wed wet})} = \dfrac{0.6 \times 0.4 \times 0.3 + 0.4 \times 0.7 \times 0.3}{0.444}$

$= \dfrac{0.072 + 0.084}{0.444} = \dfrac{156}{444} = \dfrac{52}{148} = \dfrac{13}{37}$

2 a)



$$5 = \mu + 1.555\sigma$$
$$4 = \mu - 1.227\sigma$$

$1 = 2.782\sigma$

$\sigma = 0.359$

$\therefore \mu = 4.441$

b) $P(\text{wob}) = 1 - \Phi\left(\frac{5.2 - 4.441}{0.359}\right) = 1 - \Phi(2.111)$

$= 1 - 0.9826 = 0.0174$

c) $\bar{X} \sim N\left(4.441, \frac{0.359^2}{100}\right)$

$\therefore P(\text{wob}) = 1 - \Phi\left(\frac{4.5 - 4.441}{0.0359}\right) = 1 - \Phi(1.64)$

$= 1 - 0.9495$

$= 0.0505.$

3 a) $X \sim Bin(100{,}000, 0.00004)$

Mean $= 4$, variance $= 3.9998$ sd $= 1.99996$.

$P(0 \text{ fails}) = 0.99996^{100{,}000} = 0.0183$

b) $X \sim Po(4)$ $P(0) = e^{-4} = 0.0183$ – very similar to above.

$P(x > 5) = 1 - (P(0) + P(1) + P(2) + P(3) + P(4))$

$= 1 - e^{-4}\left(\frac{4^0}{0!} + \frac{4^1}{1!} + \frac{4^2}{2!} + \frac{4^3}{3!} + \frac{4^4}{4!}\right)$

$= 1 - e^{-4}\left(1 + 4 + 8 + 10\tfrac{2}{3} + 10\tfrac{2}{3}\right)$

$= 1 - e^{-4} \times 34\tfrac{1}{3} = 1 - 0.6288$

$= 0.3712$

4 a) $\int_0^{10} kx^3(10-x)\,dx = 1$

$\therefore\ k\left[\frac{10x^4}{4} - \frac{x^5}{5}\right]_0^{10} = 1$

$10^5 k\left(\frac{1}{4} - \frac{1}{5}\right) = 1$ ie $\frac{10^5 k}{20} = 1$ $k = \frac{20}{10^5}$

$= 0.0002$

b) $\mu = \int_0^{10} kx^4(10-x)\,dx = k\left[\frac{10x^5}{5} - \frac{x^6}{6}\right]_0^{10}$

$= 0.0002 \times 10^6\left(\frac{1}{5} - \frac{1}{6}\right) = \frac{20}{3} = 6.6\dot{6}$ N

$\sigma^2 = \int_0^{10} kx^5(10-x)\,dx - 6.6\dot{6}^2 = k\left[\frac{10x^6}{6} - \frac{x^7}{7}\right]_0^{10} - 6.6\dot{6}^2$

$= k \times 10^7\left(\frac{1}{6} - \frac{1}{7}\right) - 6.6\dot{6}^2 = 47.619 - 44.\dot{4}$

$= 3.1746$

$\therefore\ \sigma = 1.782$ N

c) Prob (failure) $= \int_9^{10} kx^3(10-x)\,dx = k\left[\frac{10x^4}{4} - \frac{x^5}{5}\right]_9^{10}$

$= \frac{k \times 10^5}{20} - k9^4\left(\frac{10}{4} - \frac{9}{5}\right) = 1 - 0.9185$

$= 0.0815.$

5 a) $\Sigma x = 21$ $\Sigma y = 311$ $\Sigma x^2 = 91$

$\Sigma xy = 1352$

Then $y = a + bx$ has $b = \dfrac{1352 - \frac{21 \times 311}{6}}{91 - \frac{21^2}{6}}$

$= 15.057$

$a = \bar{y} - b\bar{x} = 51\frac{5}{6} - \qquad = -0.866\dot{6}$

$\therefore \quad y = -0.86\dot{6} + 15.057x$

b) $4.7 \rightarrow 69.9$ – reliable, as within the existing data range

$8.2 \rightarrow 122.6$ – not reliable, as extrapolation

For small x, y becomes –ve, which is clearly impossible. This shows the model needs modifying for small x.

c) $b = 15.057 \ \mu m/kN = \dfrac{15.057 \times 10^{-3} \ mm}{1000 \ N}$

$= 15.057 \times 10^{-6} \ mm/N$

$a = -0.86\dot{6} \ \mu m = -0.00086\dot{6} \ mm$

$\therefore \quad Y = -0.00086\dot{6} + 15.057 \times 10^{-6} X$

6 a) AQL – the max proportion of defective items acceptable by consumer

UQL – the min proportion of defective items unacceptable to consumer

PR – prob batch is rejected even if $p = AQL$

CR – prob batch is accepted even if $p = UQL$.

b) Sample size $= 77$ – accept if 2 or less are defective

c) $P(0, 1 \text{ or } 2) = P(0) + P(1) + P(2) = 0.96^{77} + 77 \times 0.96^{76} \times 0.04 + \frac{77 \times 76}{2} \times 0.96^{75} \times 0.04^{2}$

$= 0.96^{75}\left(0.96^{2} + 77 \times 0.96 \times 0.04 + \frac{77 \times 76}{2} \times 0.04^{2}\right)$

$= 0.4007$

d) 2-stage – more complicated to organise

– if designed properly, on average it requires smaller sample size, so cheaper.