STATISTICS FOR ENGINEERS

Exercise Sheet 3

Hand in solutions to the two starred questions.

1. The lengths of a batch of steel rods are approximately Normally distributed with mean 3.1m and standard deviation 0.15m. Estimate the proportions of the batch which are (i) longer than 3.42m, (ii) shorter than 3m, (iii) between 3m and 3.2m in length. Also, find x (in metres) such that approximately 5% of the rods are longer than x.

2*. A particular mix of concrete reaches a specified level of hardness after X hours, where $X \sim N(25, 3.8^2)$. A building is constructed in which concrete is laid in three stages. As soon as the concrete is found to be hard in one stage, the concrete for the next stage is laid.
(i) Calculate the probability that Stage 3 commences at least 45 hours after Stage 1 commences
(ii) A contractor erects several buildings in the above manner and is penalised in 8% of the cases because the concrete took too long to set. What is the penalty time?

3. I'm interested in testing whether it works better for students to work on question sheets on their own and hand them in before a workshop – so they get help on questions they had difficultly with - compared to the usual Sussex system of having workshops before handing in work. I decide to measure performance by calculating students' average exam scores assuming the variance remains the same. There are 100 students in total, and I plan to split them into two randomly chosen groups of 50, each group having different workshop arrangements. Using the central limit theorem:

(i) If the standard deviation of the students' exam marks is 15, what is the standard deviation of the average of 50 students' marks? (assuming they are independent)
(ii) What is the distribution of the difference in the mean mark of the two groups of 50 students if they had the same workshop arrangements?
(iii) When the two groups have different arrangements, how large would the difference in average mark have to be to have only 5% chance that the difference was that size or larger due to random fluctuations?

4*. In a digital communications channel, the probability that a bit is received in error is 5×10^{-6} and errors occur at random. A message contains 10^7 bits.
(i) What is the distribution of the number of bits in error in the message? What is the Normal approximation to this distribution? Use this Normal approximation in the rest of this question.
(ii) Find the approximate probability that between 45 and 55 (inclusive) bits are in error.
(iii) The message uses an error-correcting code, so that the content can be understood correctly as long as there are less than 13 errors in each consecutive chunk of 10^6 bits. Estimate the probability the message cannot be understood correctly.

5. State, with reasons, which of the Binomial, Poisson, Normal or another distribution is likely to best model the following variates, and mention any likely reasons for failure of the model:

(i) The mean diameter of combustion chambers in each batch of 1000 mass-produced cars.
(ii) The number of faulty items in a random sample of 10 items from a production line.
(iii) The weekly amount of beer consumed by each adult male.
(iv) The time from now until earth is hit by an unidentified asteroid killing more than 1000 people
(v) The percentage vote of the Liberal Democrats reported by different opinion polls on the same day
(vi) The number of light bulbs examined until a defective one is found.

6. The Chaotic Bus Company has buses running to the university on average 4 times an hour, but the timings are completely random and independent so they are well modelled as a Poisson process. Assume there is only one bus stop where everyone gets on.

1. When I arrive at the bus stop in the morning, what is the average time I have to wait?
2. What is the average time from my arrival at the bus stop since the previous bus?
3. What is the average time between the bus I catch and the previous bus?
4. Passengers arrive at the bus stop regularly once every minute. At the end of the day a random sample of these passengers are questioned. What is the average total number of passengers that passengers report seeing on their bus? What is the average number of passengers reported by the bus drivers?