STATISTICS FOR ENGINEERS

Exercise Sheet 2

Hand in solutions to the two starred questions.

1. 10% of nails in a large batch are just too long for the required purpose. What is probability of getting less than two bad nails if you randomly pick 10?

2. A network consists of 800 components (links and nodes), each component having a probability of 0.001 of failure. Different components fail independently. The network has some redundancy, so it has a chance of functioning correctly as long as not more than 2 components fail. In fact, the network will function correctly with probability 1.0 if no component fails, with probability 0.5 if 1 component fails and with probability 0.2 if 2 components fail.
(i) Using the Poisson approximation, find the probabilities that 0, 1, 2 or more than 2 components fail.
(ii) Find the approximate probability that the network functions correctly.

3*. Chocolates are drawn randomly from a production process and placed in boxes which hold 30. At intervals a box is inspected and if more than 3 chocolates are damaged the process is adjusted. On average, 5% of the chocolates are damaged, independently of other chocolates. Calculate the probability that a randomly selected box contains more than 2 damaged chocolates (i) using the exact distribution, and (ii) using a Poisson approximation.

4. A help line receives calls at independent random times, with an average rate of 20 per hour. The phone switching system needs to be upgraded, resulting in a down time of 2 minutes when all calls will fail to get through. Find the probability that no calls arise during the down time. How long can the down time afford to be if this probability is to exceed 95%?

5. Integration revision: calculate the following integrals, and sketch the corresponding functions and the areas corresponding to the integrals:

$$\int x^3 dx \qquad \int_1^3 \frac{dx}{x^2} \qquad \int_0^\infty e^{-2x} dx \qquad \int_0^1 x e^{x/2}\, dx$$

6*. X, the proportion of cars coming off a production line which need re-working, has pdf:

$$f(x) = \begin{cases} k(0.1 - x) & 0 < x < 0.1 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(i) Use the condition $\int_{-\infty}^{\infty} f(x)dx = 1$ to find the value of k.

(ii) What is the probability that X lies between 0.02 and 0.06?

(iii) Find the mean and variance of X.

7. A climate change skeptic is 70% sure that there is nothing to worry about. He thinks that effective action on climate change will have a long-term cost of around 2% of GDP, and that if he is wrong and no action is taken then the long-term cost will on average be 20% of GDP. If the skeptic is only interested in maximizing expected GDP, should he support effective action?

8. The time, T seconds, between messages arriving at a telecommunications centre has probability density function (pdf):

$$f(t) = \begin{cases} 5e^{-5t} & t > 0 \\ 0 & t \le 0 \end{cases}$$

Calculate the mean, median and standard deviation of T. [Note: the median of T is the value m satisfying $P(T \le m) = P(T \ge m) = \frac{1}{2}$.] Find the probability that the interval between messages exceeds 1 second.