STATISTICS FOR ENGINEERS

Exercise Sheet 1

Hand in solutions to the two starred questions.

1. It has been found that 15% of spacing washers are thicker that their advertised tolerance. An inspector keeps selecting washers at random until she finds a washer that is too thick. What is the probability that the inspector will have to inspect (i) exactly 10 washers, and (ii) 10 or more washers?

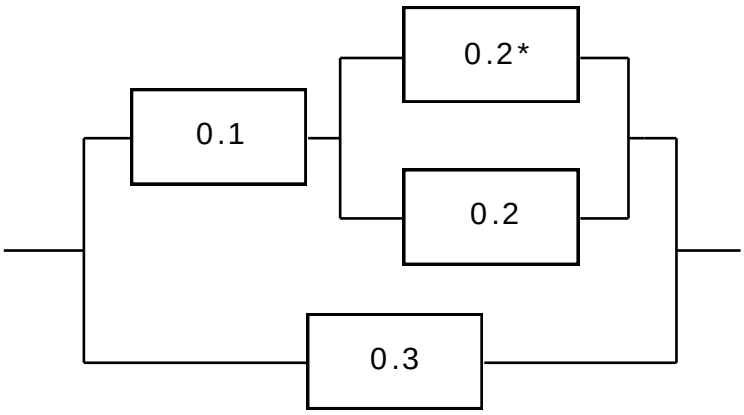
2. A test for a specific cancer is not 100% reliable: it correctly identifies someone with cancer with 95% reliability, and correctly gives a negative result for 90% of people without cancer (so 10% false positive). In the population as a whole 1% of people have the cancer. If someone is selected at random from the population and tests positive, what is the probability that they have the cancer?

3*. In a communications systems, a string of 0's and 1's are transmitted from a sender to a receiver. Noise in the system means that some of the bits are incorrectly received. In fact, there is a probability of 0.1 that a 0 is received as a 1, and a probability of 0.05 that a 1 is received as a 0. The accuracy of reception of different bits is independent and 60% of the bits sent are 1's.

(i) What proportion of bits are accurately received?
(ii) If a 1 is received, what is the probability that a 1 was sent?
(iii) If a 0 is received, what is the probability that a 0 was sent?

In an attempt to improve the accuracy of transmission, each bit is repeated 3 times i.e. 0 is sent as the triple 000, and 1 is sent as the triple 111. On receiving a triple, it is interpreted as a 0 if it contains at least two 0's and as a 1 otherwise. What proportion of triples are accurately interpreted?

4*. The diagram shows a system with both series and parallel subsystems. (For a parallel subsystem to function, at least one element must function; for a series subsystem to function, all elements must function.) The blocks in the diagram represent components and the numbers indicate the probabilities that the components fail within five years. It can be assumed that components fail independently of one another.



(i) Find the probability that the system does not fail within five years.
(ii) Find the conditional probability that the system does not fail within five years given that component * does fail in this period.
(iii) After five years, it is found that the system is still working. What is the conditional probability that component * has failed?

5. John studies engineering, reads the New Scientist and had a chemistry set as a child. Let M be the statement "John doesn't like playing computer games or doesn't speak Spanish (or both)". Rank the following in order of increasing probability:

a. John enjoys playing computer games and speaks Spanish
b. John speaks a foreign language
c. John will win the lottery this week or M is false (or both)
d. John speaks Spanish

6. Five cards are drawn at random (without replacement) from a full pack of 52, what is the probability

a) they are the 2, 3, 4 , 5 and 6 of hearts
b) there are no hearts
c) there is one or more sevens
d) there are exactly two spades

7. (*for enthusiasts)* I place three pieces of paper on the table, and and put a £10 note behind one of them so you can't see it. We play the following game in two steps: 1) you pick one of the pieces of paper, but don't touch it. 2). I take away one of the other pieces of paper, chosen randomly with the constraints that it is not the one that you picked and it is not the one with the £10 note behind it. What is the probability that the remaining piece of paper (that you did not pick) then has the note behind it?
We play the game again, but after step 1) a gust of wind blows away a random piece of paper, which happens not to be the one you picked and also not to have the note behind it. What is the probability that the other remaining piece of paper then has the £10 note behind it?