6. Acceptance Sampling

In this section we look at one particular method of quality checking called acceptance sampling. It is just a simple recipe that is followed, and may not be the best thing to do.

Situation: large batches of items are produced. We must sample a small proportion of each batch to check that the proportion of defective items is sufficiently low.

One-stage sampling plans

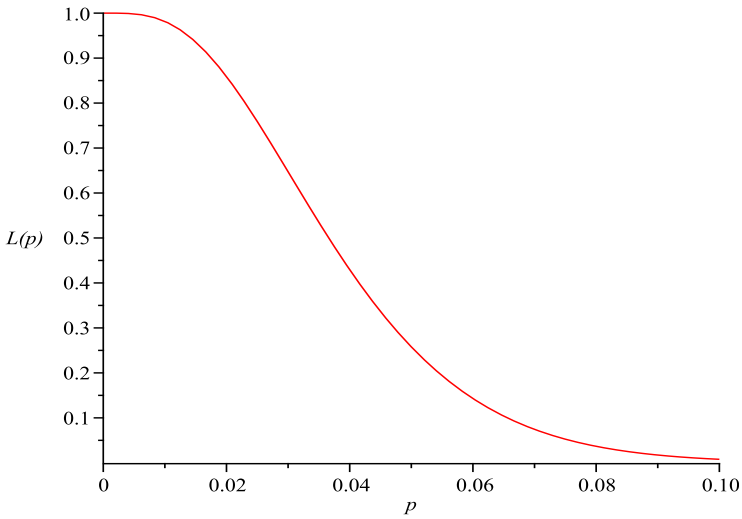
Sample n items, $X=$ number of defective items in the sample. The batch is rejected if $X > c$ and accepted if $X \le c$.

What values should we choose for n and c? Let $p =$ proportion of defective items in the batch (typically small). Then $X \sim B(n, p)$ if the population the samples are drawn from is large.

Operating characteristic (OC): probability of accepting the batch

$$L(p) = P(X \le c) = \sum_{k=0}^{c} \mathrm{P}(X = k) = \sum_{k=0}^{c} \binom{n}{k} p^k (1-p)^{n-k}$$

Plot of a typical OC curve ($n = 100$,$c = 3$):



The Producer and Consumer of the items have to agree some unacceptable defective fraction that should be rejected with high probability, and some good low defective fraction that should be accepted with high probability. So the Producer and Consumer of items have to agree what constitutes:

- Acceptable quality level: p_1 (consumer happy, want to accept with high probability)

- Unacceptable quality level: p_2 (consumer unhappy, want to reject with high probability)

Ideal sampling scheme: always accept batch if $p \leq p_1$ and always reject if $p \geq p_2$, i.e. $L(p \leq p_1) = 1$ and $L(p \geq p_2) = 0$. However the only way to guarantee this would be to inspect the whole batch, which is usually not desirable (esp. if testing requires destruction of the item!). We therefore want to use a sampling scheme, optimized so that the risk of one of these undesirable outcomes is minimized:

- $\alpha = P(\text{Reject batch when } p = p_1) = 1 - L(p_1)$: the *Producer's Risk.*
- $\beta = P(\text{Accept batch when } p = p_2) = L(p_2)$: the *Consumer's Risk.*

Of course the Producer really cares about rejecting the batch when $p \leq p_1$, but taking $p = p_1$ is conservative as the probability is always lower for $p < p_1$. Similarly for the Consumer's risk.

Once the Producer and Consumer have agreed the values of p_1, p_2, α and β, values of n and c can be calculated. See the tables (*Acceptance Sampling*) for the cases $\alpha = \beta = 0.1$ and $\alpha = \beta = 0.05$.

Example: In planning an acceptance sampling scheme, the Producer and Consumer have agreed that the acceptable quality level is 2% defectives and the unacceptable level is 6%; each is prepared to take a 10% risk. What sample size is required and under what circumstances should the batch be rejected?

Answer: $\alpha = \beta = 0.1$, $p_1 = 0.02$ and $p_2 = 0.06$. From the tables, $n = 153$ and $c = 5$. So should sample 153 items and reject if the number of defective items is greater than 5.

Example: It has been decided to sample 100 items at random from each large batch and to reject the batch if more than 2 defectives are found. The acceptable quality level is 1% and the unacceptable quality level is 5%. Find the Producer's and Consumer's risks.
Answer:

$n = 100$, $c = 2$, $p_1 = 0.01$, $p_2 = 0.05$.

For the Producer's Risk $X \sim B(100, 0.01)$

$$\begin{aligned}
\alpha &= P(\text{Reject batch when } p = 0.01) = 1 - L(0.01) \\
&= 1 - P(X = 0) - P(X = 1) - P(X = 2) \\
&= 1 - \binom{100}{0} 0.01^0 \times 0.99^{100} - \binom{100}{1} 0.01 \times 0.99^{99} - \binom{100}{2} 0.01^2 \times 0.99^{98} \\
&= 1 - 0.3660 - 0.3697 - 0.1849 = 0.079.
\end{aligned}$$

For the Consumer's Risk $X \sim B(100, 0.05)$

$$\begin{aligned}
\beta &= P(\text{Accept batch when } p = 0.05) = L(0.05) \\
&= P(X=0) + P(X=1) + P(X=2) \\
&= \binom{100}{0} 0.05^0 \times 0.95^{100} - \binom{100}{1} 0.05^1 \times 0.95^{99} - \binom{100}{2} 0.05^2 \times 0.95^{98} \\
&= 0.118
\end{aligned}$$

Two-stage sampling plan

- Sample n_1 items, X_1 = number of defectives in the sample.
- Accept batch if $X_1 \leq c_1$, reject if $X_1 > c_2$ (where $c_2 > c_1$)
- if $c_1 < X_1 \leq c_2$, sample a further n_2 items; let X_2 = number of defectives in 2nd sample;
- accept batch if $X_2 \leq c_3$, otherwise reject batch.

Although more complicated, by suitable choice of n_1, n_2, c_1,c_2 and c_3, you may be able to find a plan with similar $L(p)$ to a single stage design but smaller average sample size.

Quality

Acceptance sample is a rather limited method of ensuring good quality:

- It is too far downstream in the production process; we want a method which identifies where things are going wrong.
- It is 0/1 (i.e. defective/OK) and so does not make efficient use of data; we have seen that large samples are required. It is better to have quality measurements on a continuous scale; there will be an earlier warning of deteriorating quality and less need for large sample sizes.