THE UNIVERSITY OF SUSSEX

MSc PHYSICS AND ASTRONOMY EXAMINATION MAY 2014

COSMOLOGY

Credit will given for the best **TWO** *answers only.*
Total time allowed: **ONE and a HALF** *hours.*

Each question carries 20 marks. The approximate allocation of marks is shown in brackets by the questions.

A list of **physical constants** *is provided.*

Turn over/

1. (i) Explain briefly how *redshift* (z) can be measured by spectroscopic observations of a distant astronomical source. Other than cosmological expansion, give one other physical effect that complicates the interpretation of observed redshifts. [2]

(ii) If a galaxy cluster of physical diameter one megaparsec is observed to subtend an angle on the sky today of one arcminute, what is the (physical) angular diameter distance of the galaxy in megaparsecs? [1]

(iii) Consider a standard flat homogeneous universe containing radiation, matter and a cosmological constant. For typical values of the densities today consistent with observations, on a log plot sketch how the various energy densities vary in the redshift range $5000 > z > 0$. [3]

(iv) Now consider a flat homogeneous universe containing just pressureless matter and a cosmological constant. Explain what is meant by Ω_m and Ω_Λ (defined today), and give a relationship for the Hubble parameter $H(z)$ in terms of Ω_m, Ω_Λ, the redshift, and the Hubble parameter today H_0. [2]

(v) In a flat universe the physical angular diameter distance to a source at redshift z can be calculated using $d_A(z) = \chi(z)/(1+z)$ where $\chi(z)$ is the comoving radial distance travelled by light from the source. For a flat universe containing matter and a cosmological constant derive an explicit expression for d_A in terms of an integral that depends on Ω_Λ (you may find it easier to write it as an integral over scale factor $a = 1/(1+z)$ rather than redshift). Show that for the special case $\Omega_\Lambda = 0$

$$d_A = \frac{2H_0^{-1}}{1+z}\left[1-(1+z)^{-1/2}\right]$$

[5]

(vi) By considering a first order series expansion, show that in a flat universe with $\Omega_\Lambda \ll \Omega_m$ the angular diameter distance $d_A(z;\Omega_\Lambda)$ is related to that in a matter only universe $[d_A(z;\Omega_\Lambda=0)]$ by

$$d_A(z;\Omega_\Lambda) \approx d_A(z;\Omega_\Lambda=0) + A\Omega_\Lambda\left(6(1+z)^{-1} + (1+z)^{-9/2} - 7(1+z)^{-3/2}\right).$$

Give the constant A explicitly in terms of H_0.

Does a small cosmological constant increase or decrease the angular diameter distance to the last scattering surface (for fixed observed H_0)? [7]

2. (i) State the assumptions about the symmetries of the Universe that give rise to the Friedmann equation. Comment on how this can be compatible with structures observed in the Universe, e.g. the voids, clusters and filaments defined by the observed galaxy distribution. [2]

(ii) Assume the expansion rate is governed by the Friedmann equation

$$H^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$$

where G is Newton's constant, a is the scale factor of the Universe, overdot denotes a time derivative, and K is a constant.

The fluid equation in natural units is

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + P\right) = 0$$

where P is the pressure.

Assuming a spatially-flat Universe ($K = 0$) solve these equations to find the expansion rate of the Universe $H(t)$ when the universe is radiation dominated. Recall that the equation of state for radiation is $P = \frac{1}{3}\rho$. [5]

(iii) Approximating the universe as radiation dominated until matter-radiation equality at $z \approx 3500$, find the maximum comoving distance (in Megaparsecs) light could travel between the start of the hot big bang at $t = 0$ and redshift $z = 3500$. Assume today the radiation density parameter is $\Omega_R = 8 \times 10^{-5}$ and $H_0 = 70\text{kms}^{-1}\text{Mpc}^{-1}$, and neglect changes in the degrees of freedom at early times [also take $c = 3 \times 10^8\text{ms}^{-1}$] [5]

(iv) Detailed observations of large scale structure indicate that at matter-radiation equality there were correlations on comoving scales significantly larger than 200Mpc. Describe why this gives a horizon problem, and how the horizon problem motivates the theory that a period of inflation occurred in the very early Universe. [3]

(v) Show that if $P < -\rho/3$ for some epoch in the early universe the horizon problem can be solved. You can use the fact that $\rho(a) = \rho_0 a^{-3(1+w)}$ if $P = w\rho$ and you can assume w is a constant. [5]

3. The effective degrees of freedom for particles in thermal equilibrium at the same temperature T is

$$g_* = \sum_{i,bosons} g_i + \frac{7}{8} \sum_{i,fermions} g_i \,.$$

where g_i are the degeneracy factors. The entropy density is given in terms of the temperature by $s = \frac{2\pi^2}{45} g_* T^3$.

(i) Briefly describe what is meant by *thermal equilibrium*. In an expanding homogeneous universe, qualitatively what determines whether a massive particle continues to have a thermal distribution as the universe expands? [3]

(ii) Give one piece of observational evidence that the early universe was in thermal equilibrium. [1]

(iii) Show that for massless particles in thermal equilibrium, with any distribution function $f(p)$ that depends on the particle energy E only as a function of E/T (i.e. $f(p) = f(E) = g(E/T)$ for some function g), the particle number density n scales as $n \propto T^3$. Why is the energy density $\propto T^4$? [3]

(iv) Muons μ^- are essentially identical to electrons, except that they are heavier ($m_\mu = 106\text{MeV}$); other than that, they also have the same charge and spin as the electron, and there is an antimuon μ^+ analogous to the positron. When T is just above 106MeV, what particles besides the muons are contained in the thermal radiation that fills the universe? What is the total effective g_*? [3]

(v) As T falls below 106MeV the muons annihilate, but everything remains in thermal equilibrium. Letting a denote the scale factor, use conservation of entropy to show that

$$\frac{(aT)_{\text{after}}}{(aT)_{\text{before}}} = \left(\frac{57}{43}\right)^{1/3} .$$

[5]

(vi) It has been suggested that there may be additional particles that we don't yet know about. Show that a single additional massless scalar (spin-zero) boson that was originally in thermal equilibrium but decoupled at T just above 106MeV would contribute to the radiation energy about $0.39\times$ as much as instead having one new additional flavour of massless neutrino that decoupled when $1\text{MeV} < T < 106\text{MeV}$. [5]

End of Paper