Cosmology: Problem Sheet 0

This sheet is not assessed and is for practice; answers will not be marked but worked answers will be provided.

I. HUBBLE PARAMETER

Galaxies typically have some random peculiar velocity relative to the overall Hubble expansion. Assuming galaxy distance and redshift can be measured exactly, and the typical (root mean square) peculiar velocity is $600\,\mathrm{km\,s^{-1}}$, how far away would a galaxy have to be before it could be used to determine H_0 to within 10% accuracy? (consider the two cases where the true Hubble constant is $H_0 = 100\,\mathrm{km\,s^{-1}Mpc^{-1}}$ and $H_0 = 70\,\mathrm{km\,s^{-1}Mpc^{-1}}$).

II. NEWTONIAN FRIEDMANN EQUATIONS WITH COSMOLOGICAL CONSTANT

Consider a generalization of Newton's force law to

$$\frac{F}{m} = -\frac{GM}{r^2} + \frac{\Lambda r}{3}$$

where Λ is a constant. Show that a cosmology with this force law and pressureless matter with density ρ satisfies the Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{K}{a^2} + \frac{\Lambda}{3}.$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G\rho}{3} + \frac{\Lambda}{3}$$

III. BIG RIP

There are good reasons to think that the pressure $P \geq -\rho$, where ρ is the density. However consider the alternative, when the universe contains dark energy with a constant $w_{de} \equiv P_{de}/\rho_{de} < -1$. If a homogenous spatially-flat universe with scale factor a contains only this form of dark energy and pressureless matter with density ρ_m:

1. Show that the energy density of the dark energy increases with time. If the scale factor is $a_0 = 1$ today show that in the future

   $$\Omega_{de}(a) = \left(1 + \frac{\Omega_{m,0}a^{3w}}{\Omega_{de,0}}\right)^{-1}$$

   where $\Omega_{de,0}$ and $\Omega_{m,0}$ and the dark energy and matter densities as a fraction of the critical density today. If $\Omega_{de,0} = 0.75$ and $w = -2$, at roughly what scale factor is 99.9% of the energy density in dark energy?

2. If the dark energy dominates the matter density at time t_{de} [so that $\rho_m(t_{de}) \ll \rho_{de}(t_{de})$], show that the universe has a "big rip" (scale factor $a \to \infty$) in a finite time Δt, and find Δt in terms of w_{de} and the Hubble parameter $H_{t_{de}}$ at time t_{de}.

3. What would happen to the observed wavelength of CMB photons as the big rip is approached?