Impact of post-Born lensing on the CMB

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Lensing of the CMB is affected by post-Born lensing, producing corrections to the convergence power spectrum and introducing field rotation. We show numerically that the lensing convergence power spectrum is affected at the $\lesssim 0.2\%$ level on accessible scales, and that this correction and the field rotation are negligible for observations with arcminute beam and noise levels $\gtrsim 1\,\mu\mathrm{K\,arcmin}$. The field rotation generates $\sim 2.5\%$ of the total lensing B-mode polarization amplitude (0.2% in power on small scales), but has a blue spectrum on large scales, making it highly subdominant to the convergence B modes on scales where they are a source of confusion for the signal from primordial gravitational waves. Since the post-Born signal is non-linear, it also generates a bispectrum with the convergence. We show that the post-Born contributions to the bispectrum substantially change the shape predicted from large-scale structure non-linearities alone, and hence must be included to estimate the expected total signal and impact of bispectrum biases on CMB lensing reconstruction quadratic estimators and other observables. The field-rotation power spectrum only becomes potentially detectable for noise levels $\ll 1\,\mu\mathrm{K\,arcmin}$, but its bispectrum with the convergence may be observable at $\sim 3\sigma$ with Stage IV observations. Rotation-induced and convergence-induced B modes are slightly correlated by the bispectrum, and the bispectrum also produces additional contributions to the lensed BB power spectrum.

arXiv:1605.05662

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http://cosmologist.info/

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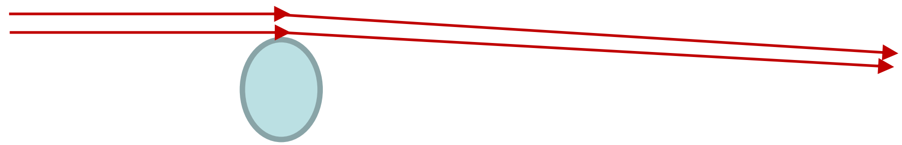
Ray-deflection: first lens changes location of second lensing event

$$\Psi(\mathbf{x}_0 + \delta\mathbf{x}) \approx \Psi(\mathbf{x}_0) + \Psi_{,a}(\mathbf{x}_0)\delta x_a + \frac{1}{2}\Psi_{,ab}(\mathbf{x}_0)\delta x_a \delta x_b + \mathcal{O}(\Psi^4)$$

Linear approximation



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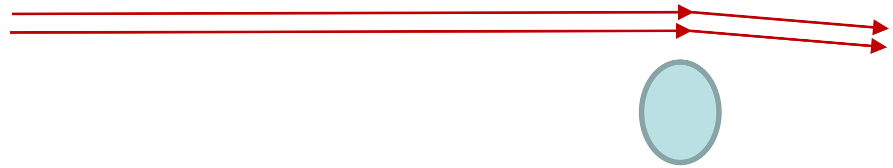

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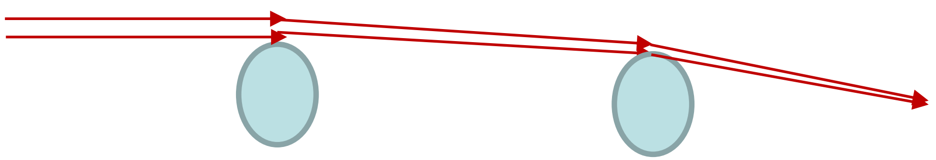
Post-Born lensing

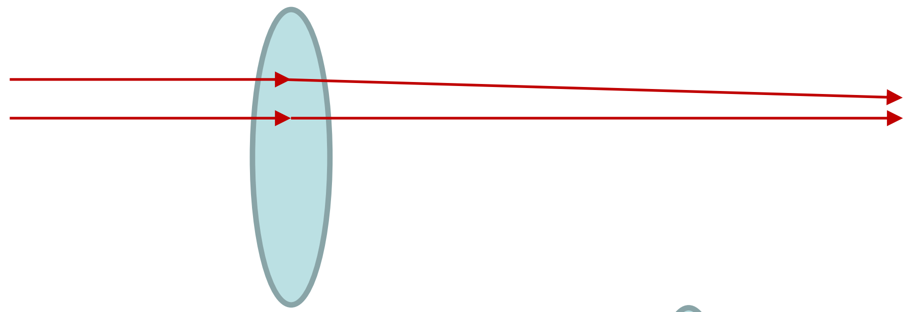
e.g. more net lensing



Lens-Lens coupling: Beam size (and shape) affected by first lensing event

$$\psi_{ab}(\boldsymbol{\theta},\chi) = 2\int_0^{\chi} d\chi'\, \chi'^2\, W(\chi',\chi)\, \Psi_{,ac}(\mathbf{x}')\, [\delta^c_b - \psi^c_b(\boldsymbol{\theta},\chi')]$$

Linear approximation



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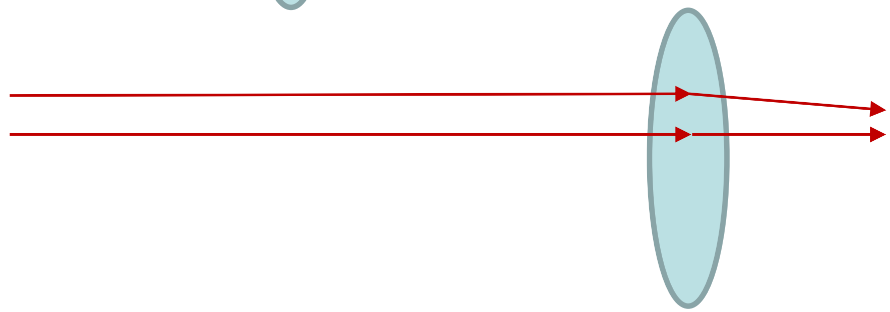

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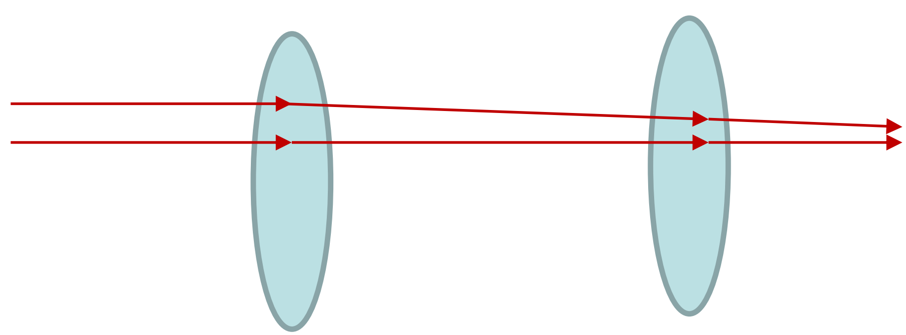
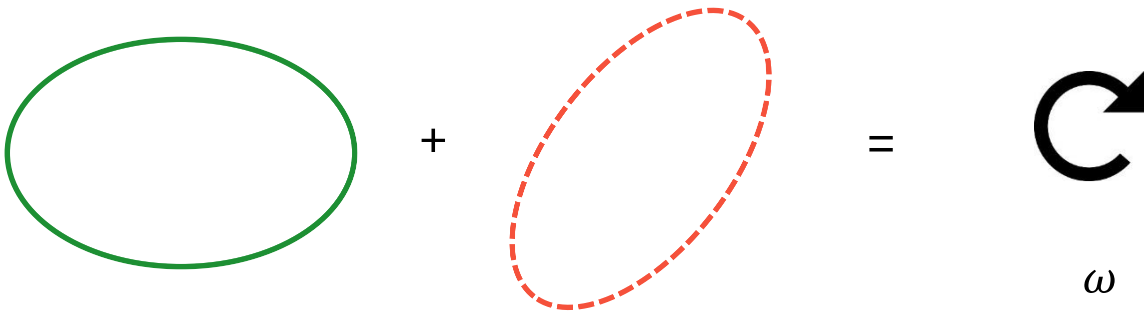
Post-Born lensing

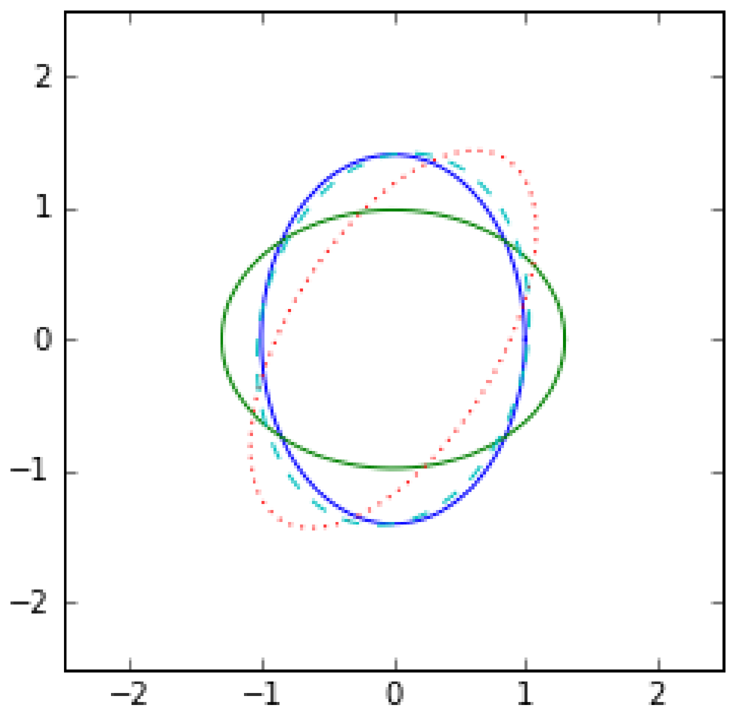


e.g. less net lensing

Lens-Lens with two non-aligned shears ⇒ rotation



ω $+ \kappa + \gamma$

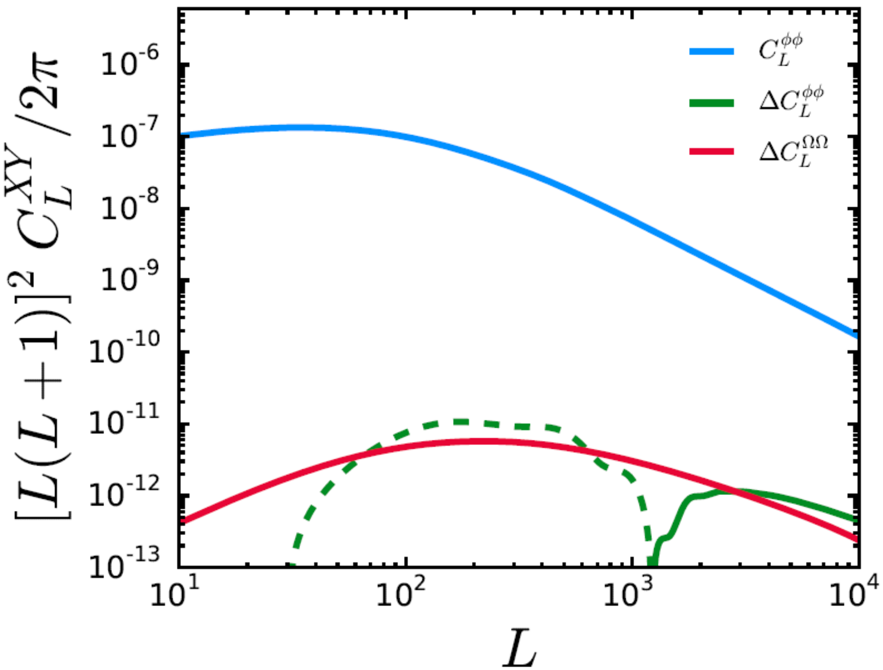


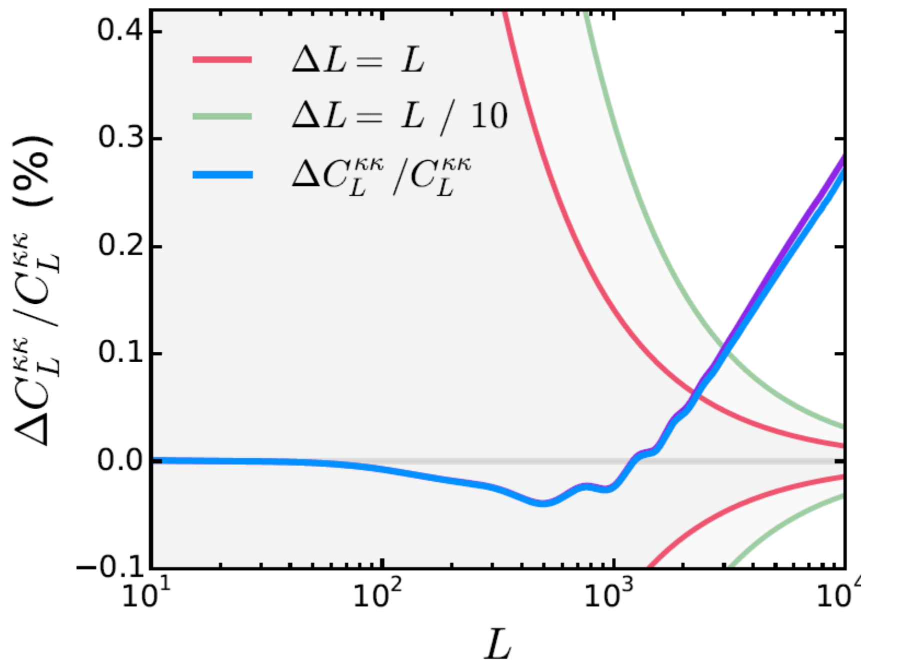
Effect on lensing convergence and rotation power spectra

$$\mathcal{A}^{ab} = \frac{\partial \theta^a_{\mathcal{S}}}{\partial \theta^b} = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2-\omega \\ -\gamma_2+\omega & 1-\kappa+\gamma_1 \end{pmatrix}$$

$$\alpha_a = \nabla_a \phi + \epsilon_{ab} \nabla^b \Omega$$

$$\kappa = -\frac{1}{2}\nabla^a \alpha_a = -\frac{1}{2}\boldsymbol{\nabla}^2\phi, \quad \text{and} \quad \omega = -\frac{1}{2}\epsilon^{ab}\nabla_a\alpha_b = -\frac{1}{2}\boldsymbol{\nabla}^2\Omega$$





- Negligible change to convergence spectrum

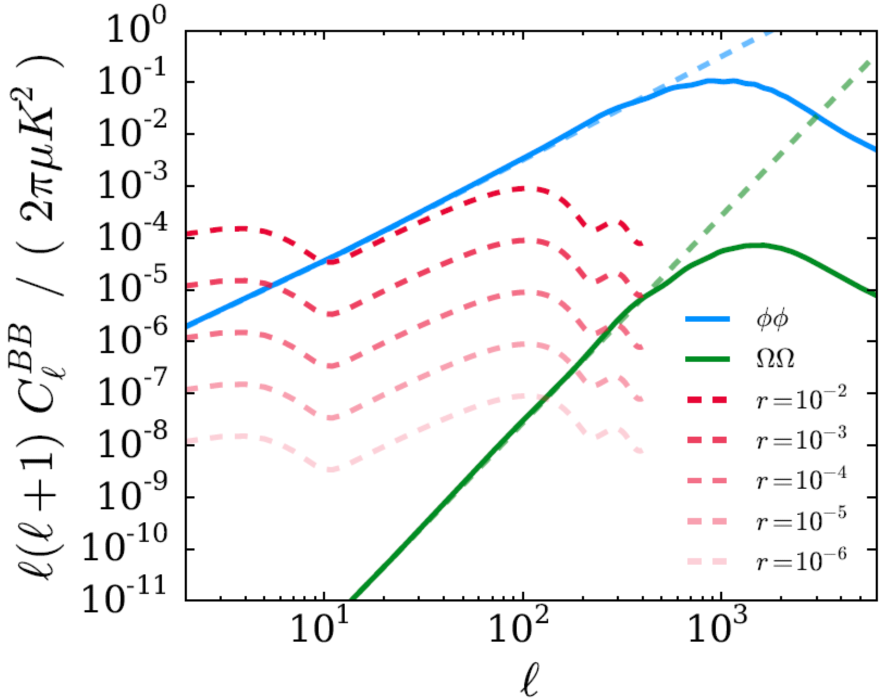
- Non-zero rotation spectrum

Impact on CMB polarization

$$\tilde{P}_{ab}(\boldsymbol{\theta}) = P_{ab}(\boldsymbol{\theta}+\boldsymbol{\alpha}) \approx P_{ab}(\boldsymbol{\theta}) + \underbrace{\boldsymbol{\alpha}_a \nabla^a P_{ab}(\boldsymbol{\theta})}$$

$$\alpha_a = \nabla_a \phi + \epsilon_{ab} \nabla^b \Omega$$

For rotation: $= \epsilon_{cd} \nabla^d (\Omega \nabla^c P_{ab})$



~2.5% of B mode amplitude from rotation

Small on large scales:

$$\tilde{C}_\ell^{BB}(\text{convergence}) \approx \frac{1}{\pi} \int d\ln \ell' C_{\ell'}^{\kappa\kappa} \ell'^2 C_{\ell'}^{EE} \qquad \tilde{C}_\ell^{BB}(\text{rotation}) \approx \frac{\ell^2}{2\pi} \int d\ln \ell' C_{\ell'}^{\omega\omega} C_{\ell'}^{EE}$$

$$\tilde{C}_\ell^{BB}(\text{convergence}) \approx 2.0 \times 10^{-6} \mu\text{K}^2, \qquad \tilde{C}_\ell^{BB}(\text{rotation}) \approx 1.7 \times 10^{-11} \left(\frac{\ell}{100}\right)^2 \mu\text{K}^2$$

How Gaussian is the lensing potential field?

Non-Gaussianity potentially important:

- Useful extra signal? (Namikawa 2016 + Jia Liu talk)
- Biases on lensing quadratic estimators (Boehm et al 2016)
- Corrections to the lensed CMB power spectra (Marozzi et al 2016)

Expected to be quite small:

Large distance to CMB $\Rightarrow$ many independent lenses

$\Rightarrow$ Gaussianization by central limit theorem

But how non-Gaussian, and what shape?...

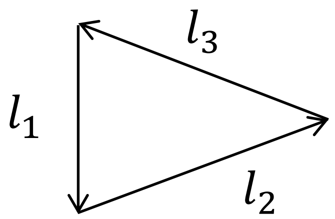
Flat sky approximation: $\kappa(x) = \frac{1}{2\pi}\int d^2l\,\kappa(l)e^{ix\cdot l}$

Gaussian + statistical isotropy

$$\langle\kappa(l_1)\kappa(l_2)\rangle = \delta(l_1 + l_2)C_l^{\kappa}$$

- power spectrum encodes all the information
- modes with different wavenumber are independent

Bispectrum

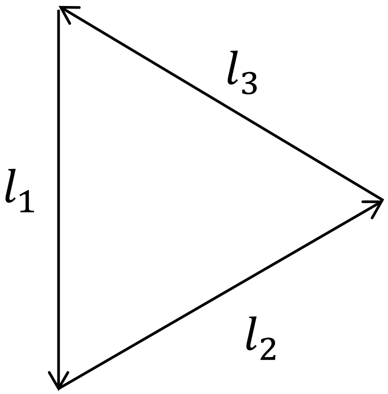


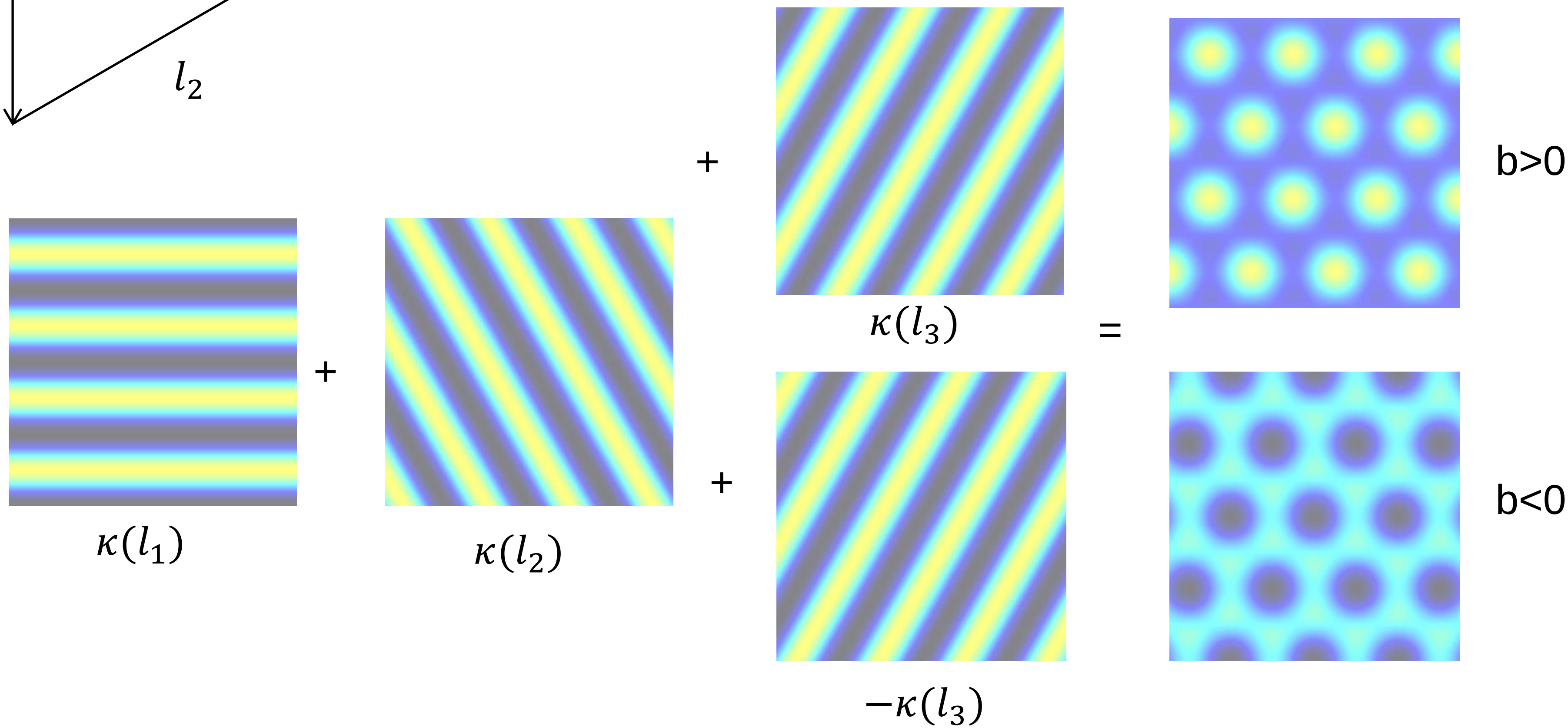
$$\boldsymbol{l_1 + l_2 + l_3 = 0}$$

Flat sky approximation: $\langle\kappa(l_1)\kappa(l_2)\kappa(l_3)\rangle = \frac{1}{2\pi}\delta(l_1 + l_2 + l_3)b_{l_1l_2l_3}$

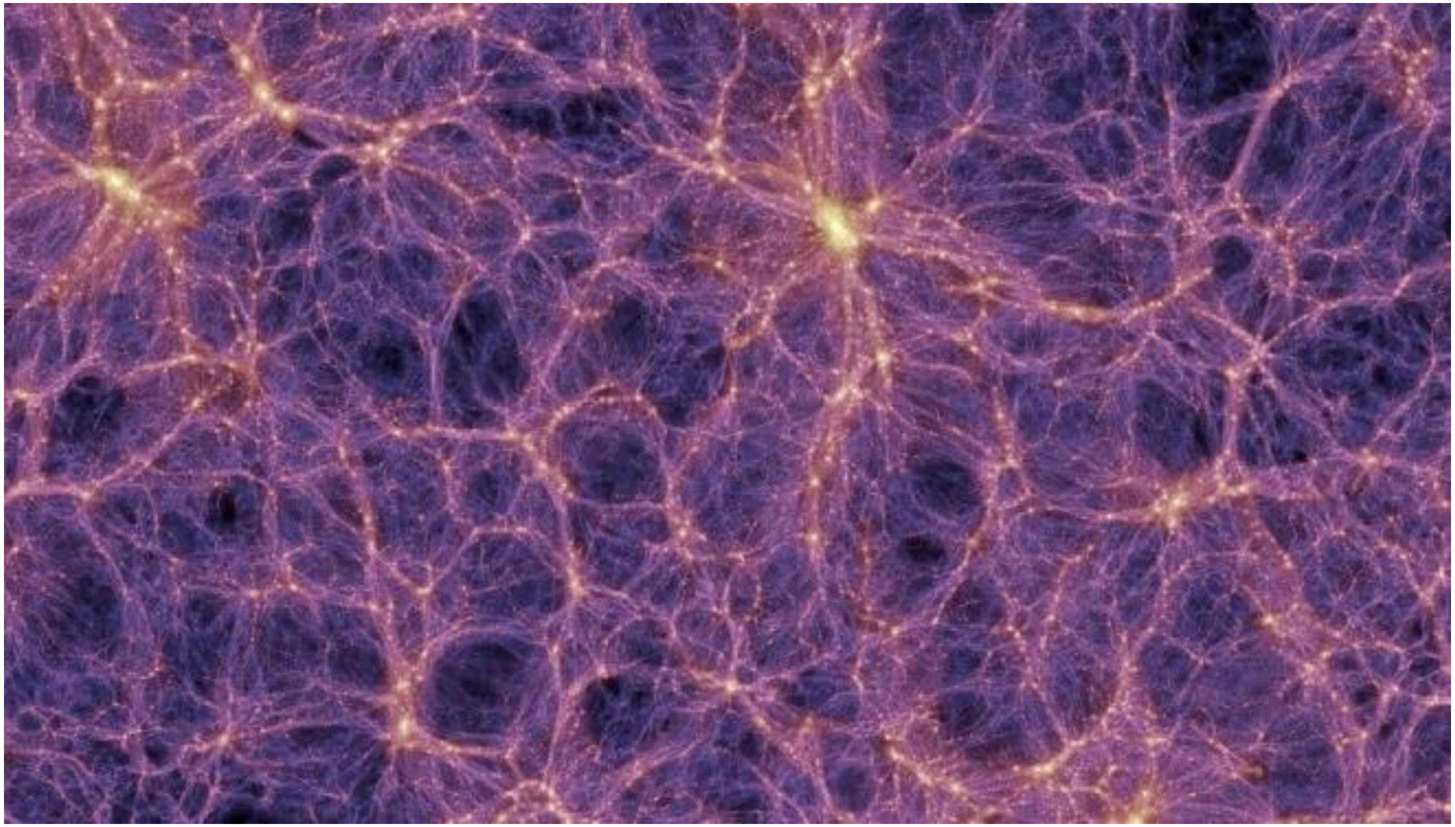
If you know $\kappa(l_1), \kappa(l_2)$, sign of $b_{l_1l_2l_3}$ tells you which sign of $\kappa(l_3)$ is more likely

Equilateral $l_1 + l_2 + l_3 = 0, |l_1| = |l_2| = |l_3|$





AL: The Real Shape of Non-Gaussianities, arXiv:1107.5431



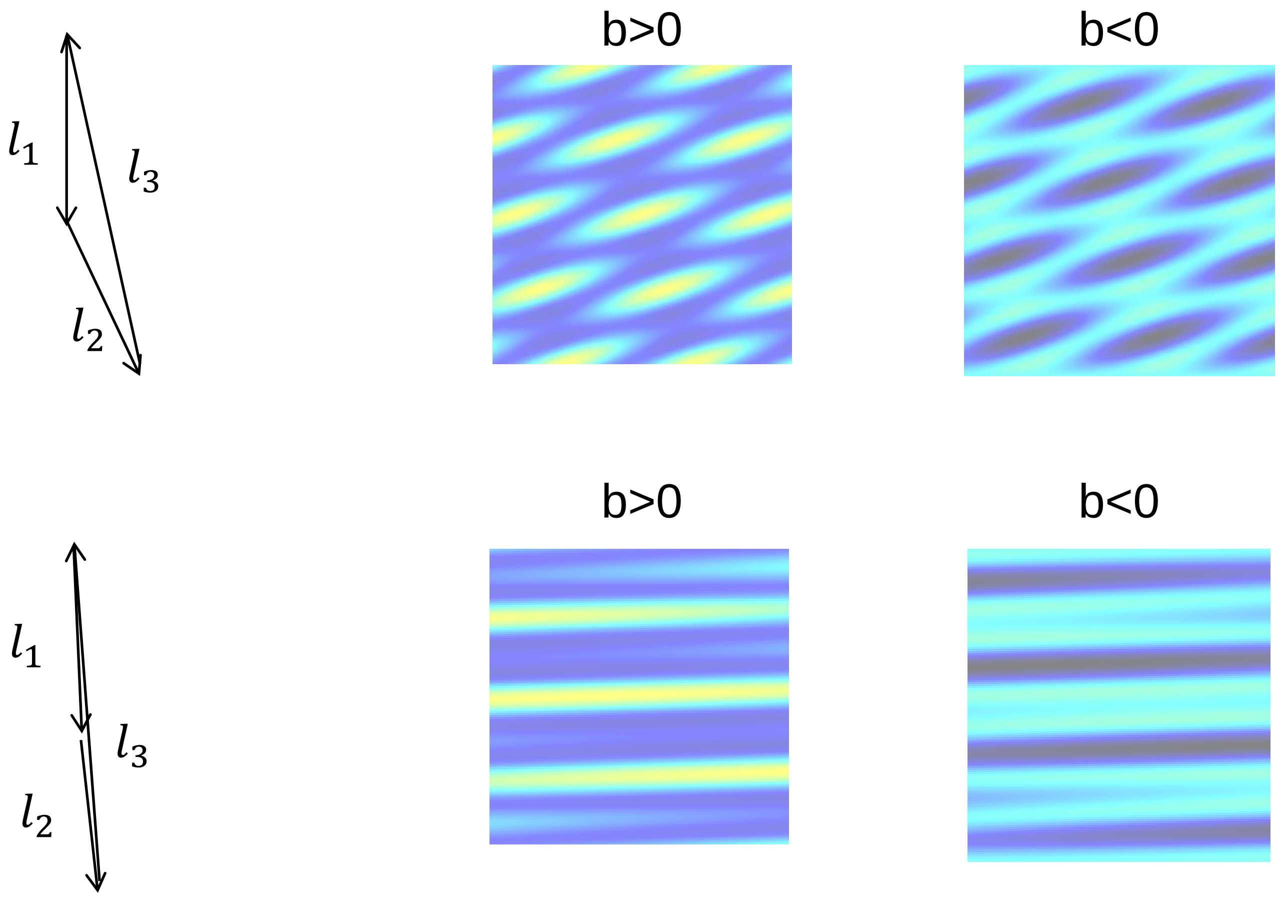
In 2D projection (e.g. lensing)

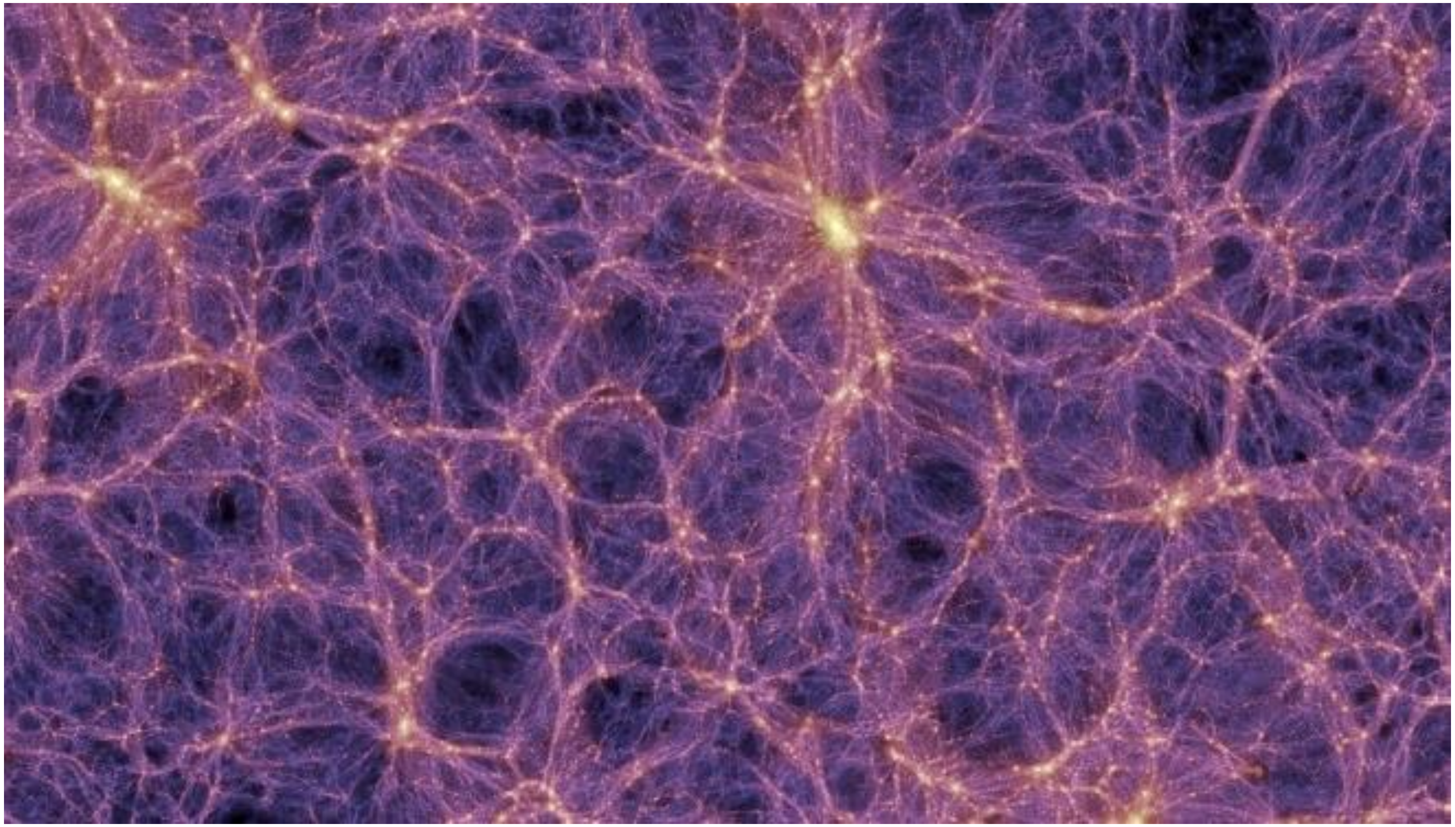
Clusters

(+sight-aligned filaments)

Positive equilateral bispectrum

Near-equilateral to flattened/folded:





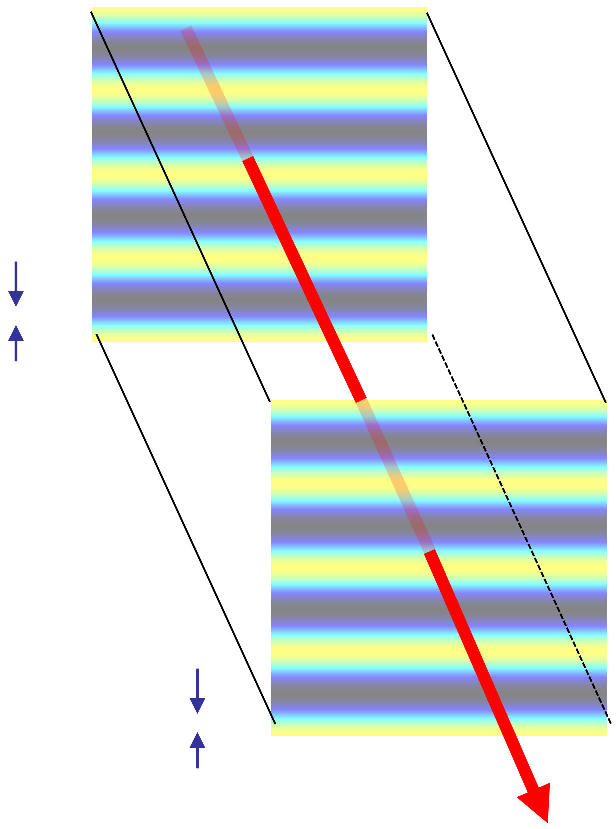
In 2D projection (e.g. lensing)

Filaments

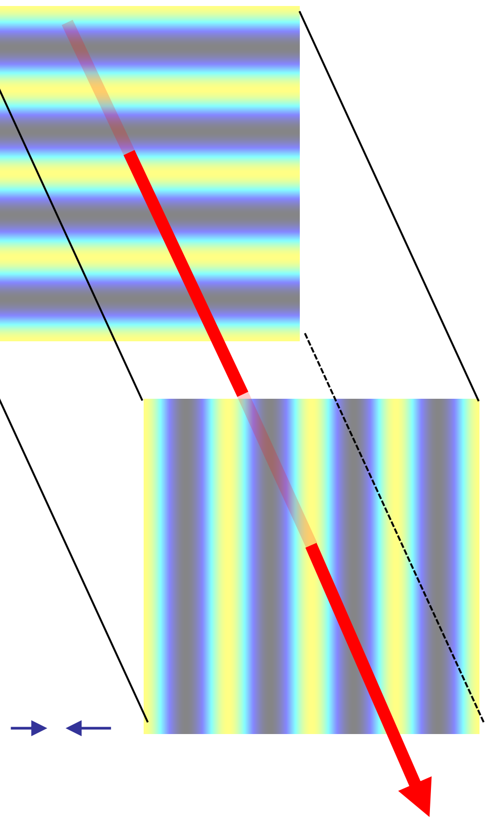
(+sight-aligned sheets)

Positive flattened bispectrum

LSS has positive bispectrum, hence κ bispectrum from LSS also positive.
What about post-Born?



Big negative lens-lens effect

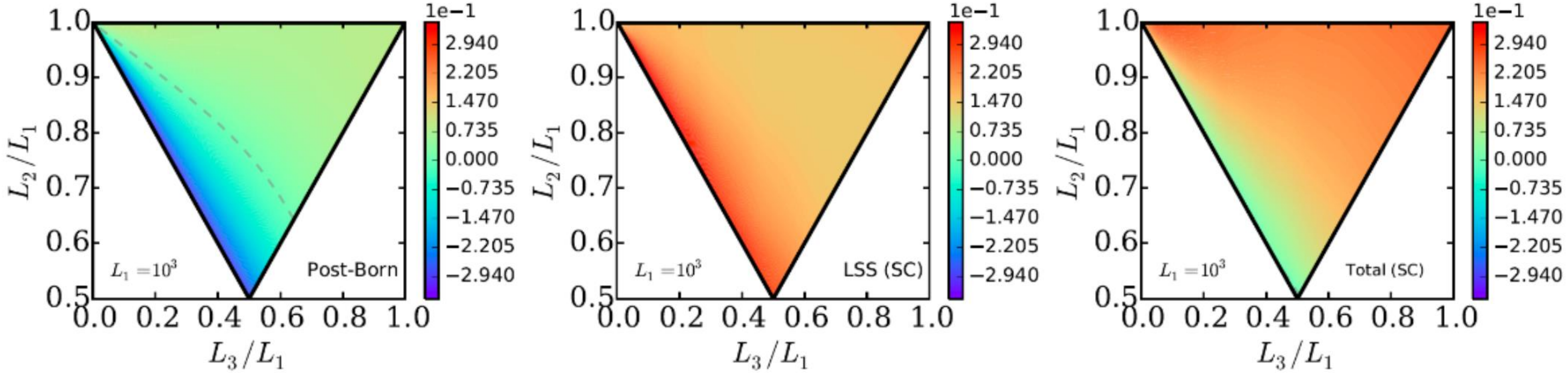


Zero lens-lens effect

Negative flattened bispectrum

Convergence Bispectrum

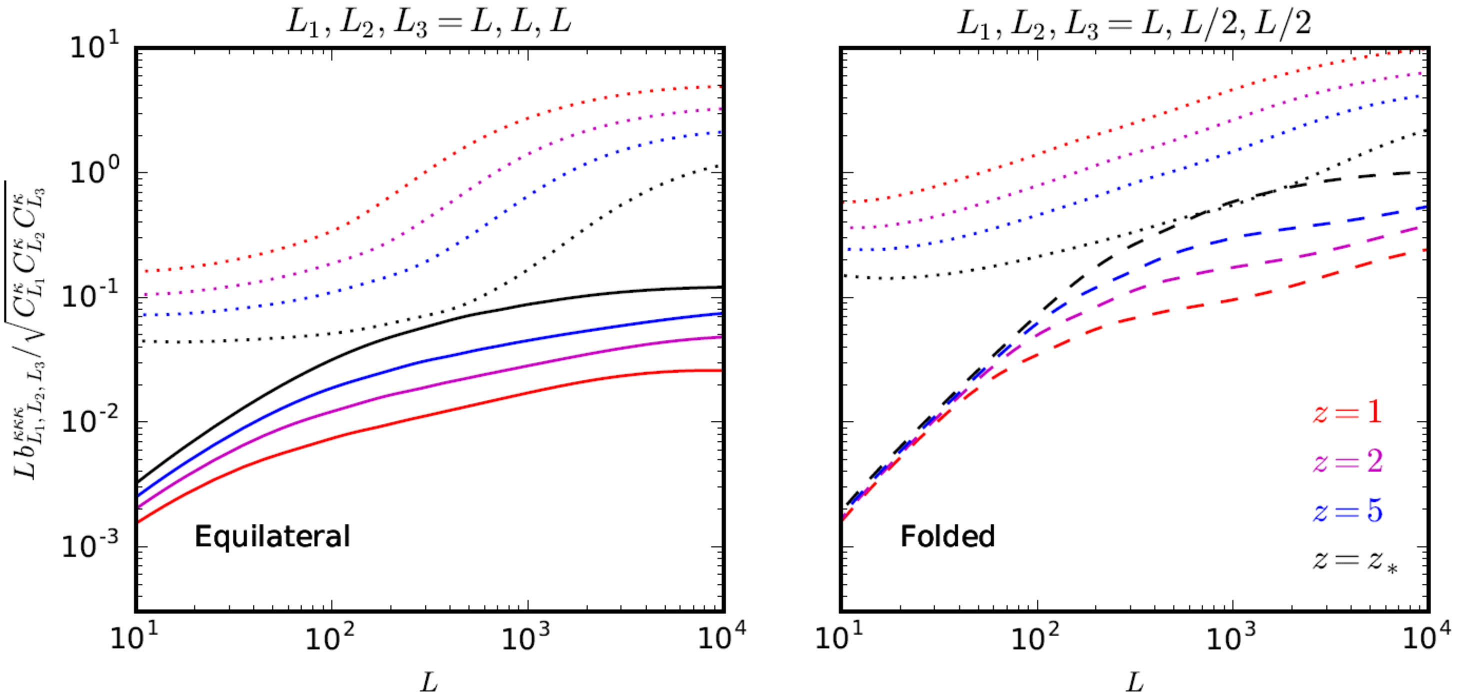
Post-born + LSS = Total



$$(L_2 L_3)^{1/2}\, b^{\kappa\kappa\kappa}_{L_1 L_2 L_3} / (C^{\kappa\kappa}_{L_1} C^{\kappa\kappa}_{L_2} C^{\kappa\kappa}_{L_3})^{1/2}$$

Unexpectedly small folded Gaussianity of the CMB lensing convergence!

This cancellation is a fluke, LSS dominates at lower redshifts



Naïve S/N for post-Born and total bispectrum

Small but may be important for S4

	noise [μK arcmin]	beam [arcmin]	$\ell_{\rm max}$	$f_{\rm sky}$	$\Delta\kappa\kappa$ S/N	$\omega\omega$ S/N	$\kappa\kappa\kappa$ S/N	$\kappa\kappa\omega$ S/N
Planck	33	5	2000	0.7	0.0	0.0	0.8	0.1
Simons Array	12	3.5	4000	0.65	0.0	0.0	3.4	0.4
SPT 3G	4.5	1.1	4000	0.06	0.0	0.0	2.3	0.4
S4	1	3	4000	0.4	0.2	0.7	25	3.1
S5	0.25	1	4000	0.5	0.8	2.7	99	8.8

Negligible

Can measure ω from its bispectrum with κ?

But note bispectrum may be much more significant for cross-correlations (with higher S/N tracer)

Discussion question: What is the optimal estimator for the κ bispectrum from CMB?

Conclusions

Post-Born impact on power spectra negligible for near future

- O(0.2%) on power spectra

- Blue-spectrum rotation B-mode power negligible for $r > \sim 10^{-5}$

As important as LSS for the CMB convergence bispectrum

- Nearly cancels for aligned-mode bispectra (completely changes LSS-only shape: mostly equilateral left)

- Potentially detectable with near-future data (25σ with S4)

May be able to detect rotation via $\kappa\kappa\omega$ bispectrum