Anisotropy in the CMB

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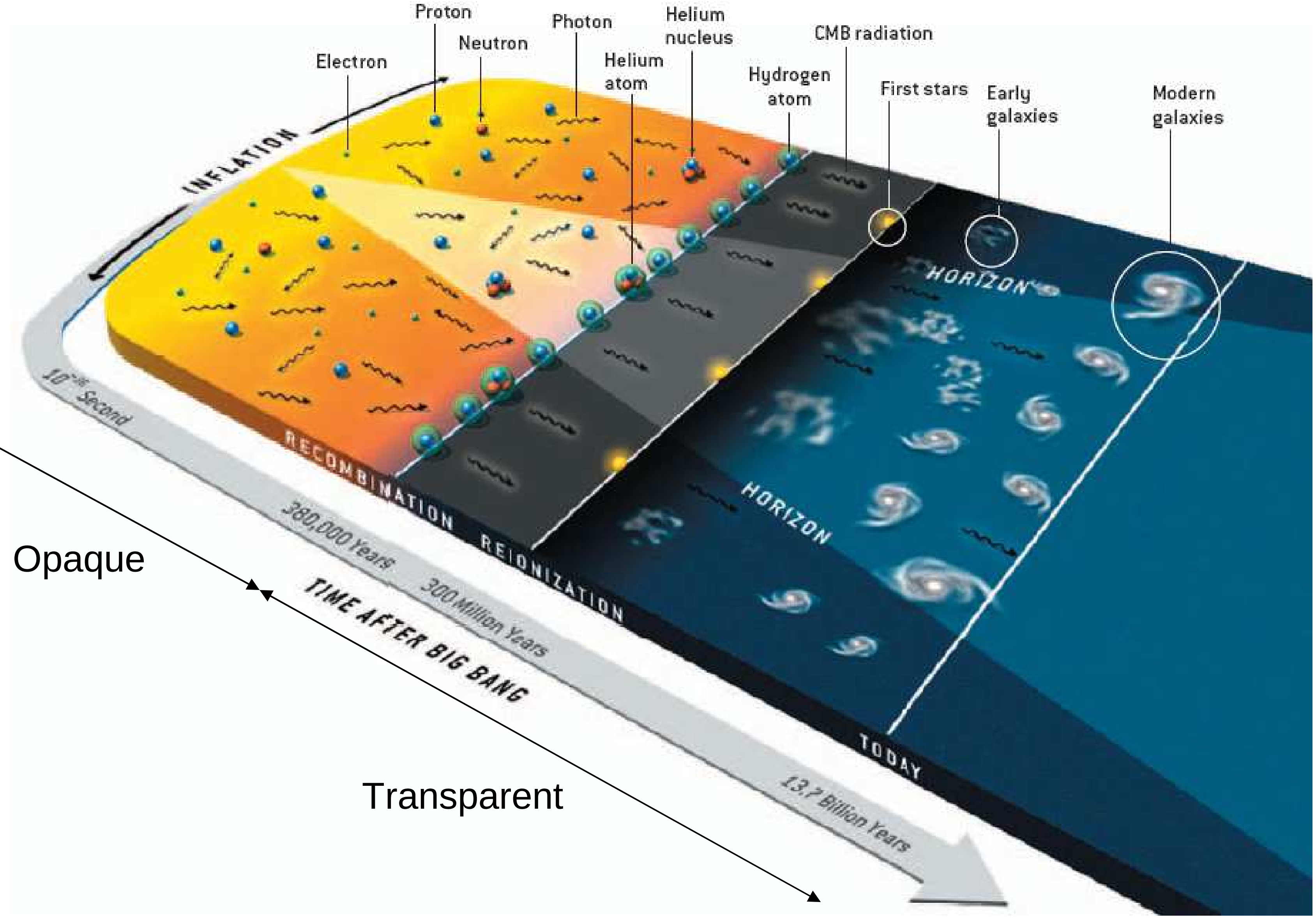
Hanson & Lewis: 0908.0963

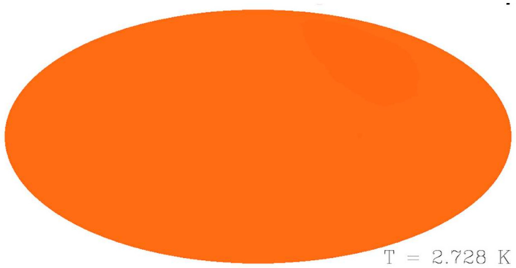
Evolution of the universe



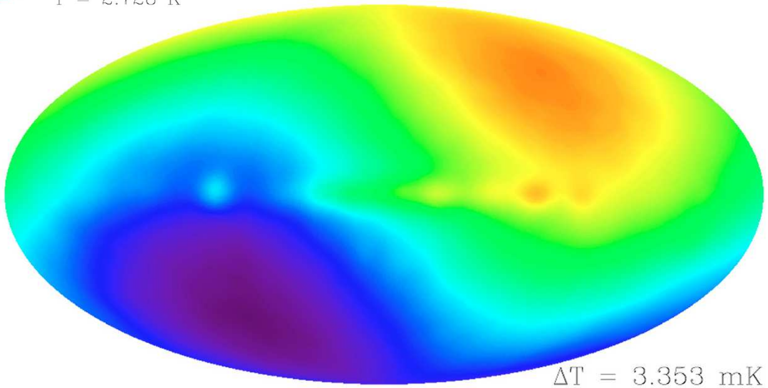
Opaque

Transparent

Hu & White, Sci. Am., 290 44 (2004)

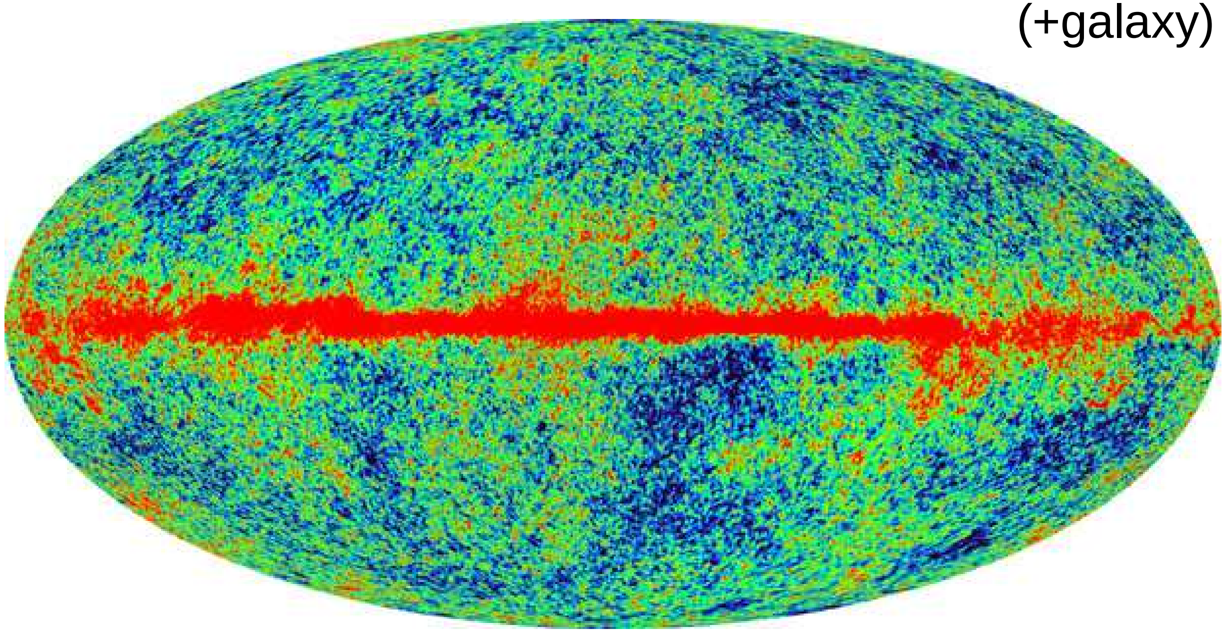


(almost) uniform 2.726K blackbody



Dipole (local motion)

$O(10^{-5})$ perturbations (+galaxy)

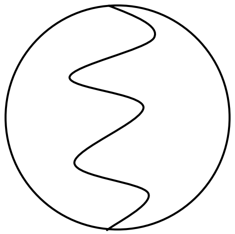


Observations: the microwave sky today

Source: NASA/WMAP Science Team

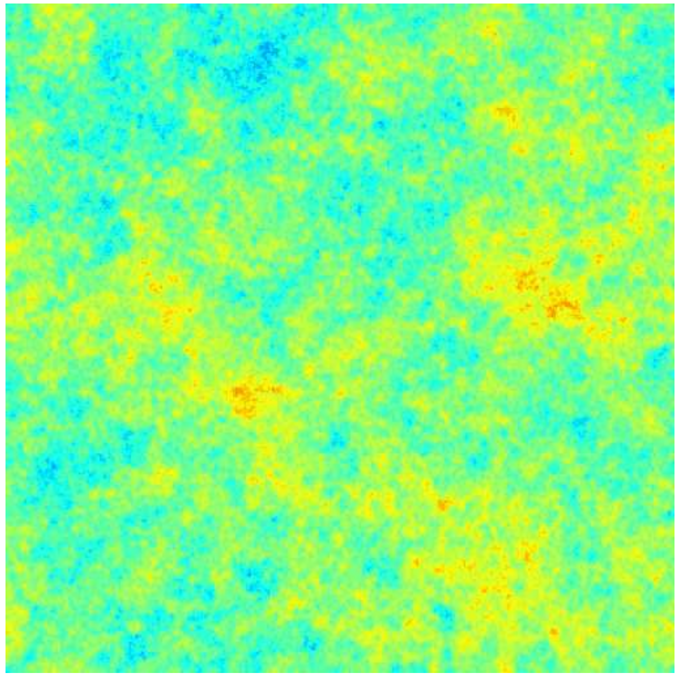
Can we predict the primordial perturbations?

- Maybe..



Quantum Mechanics
"waves in a box"

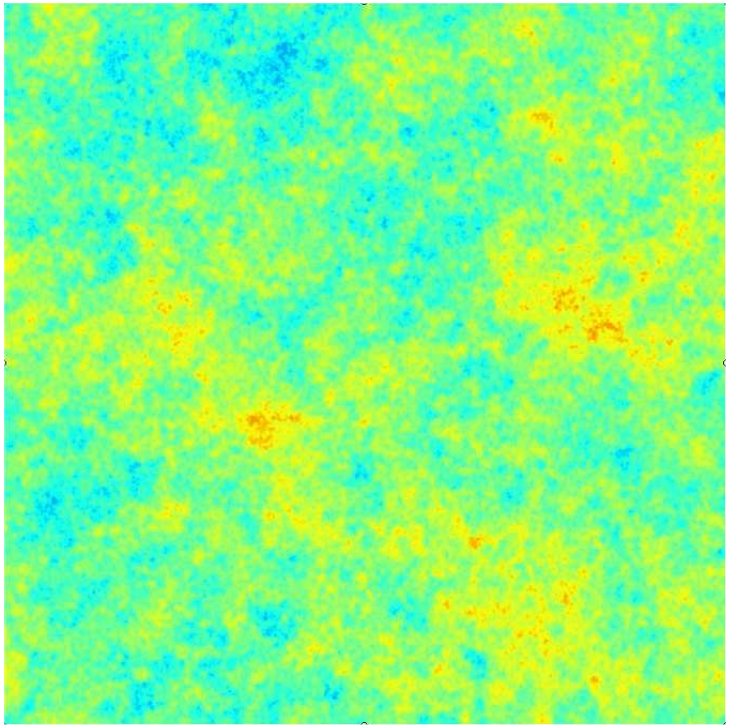
Inflation
make >10^{30} times bigger



After inflation
Huge size, amplitude ~ 10^{-5}

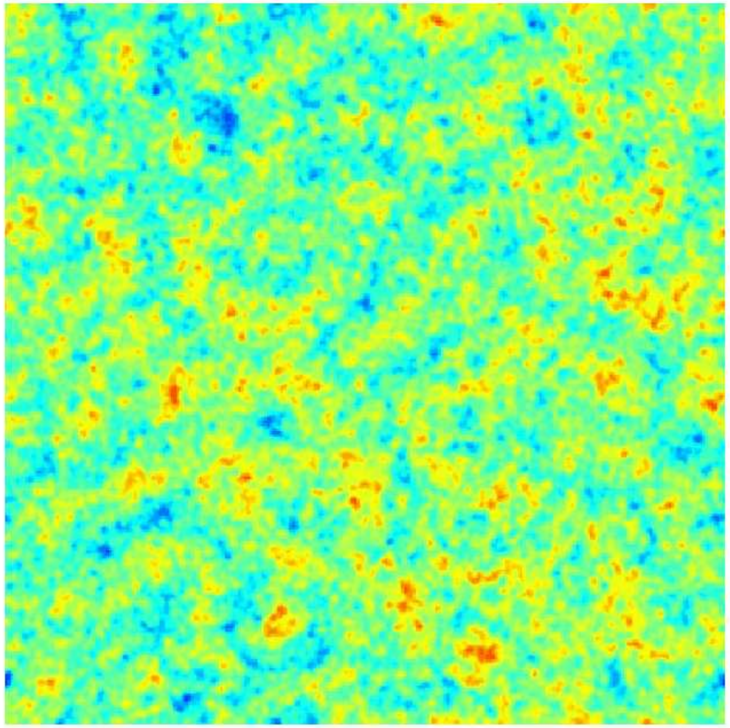
CMB temperature

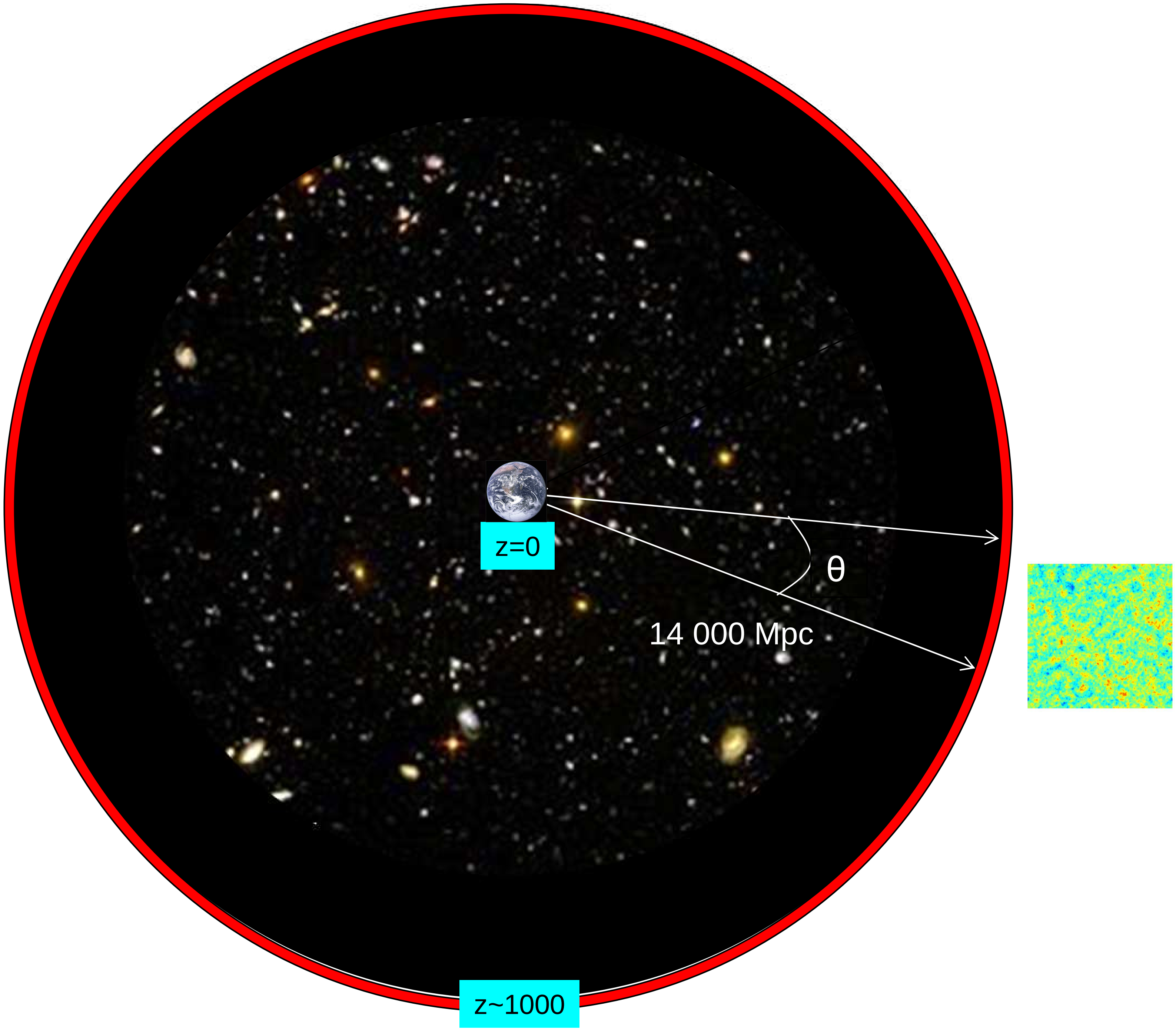
End of inflation



gravity+
pressure+
diffusion

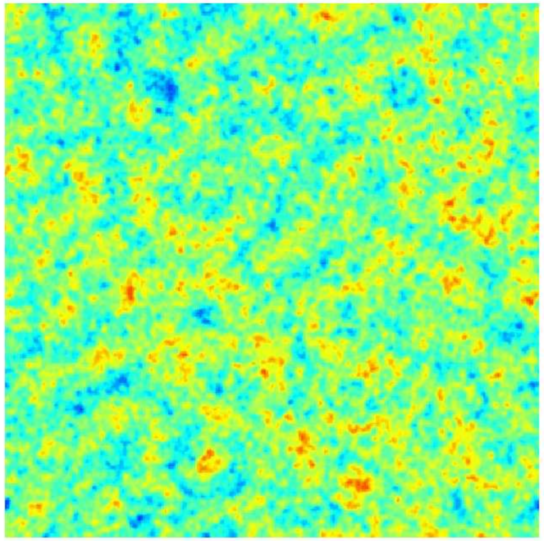
Last scattering surface

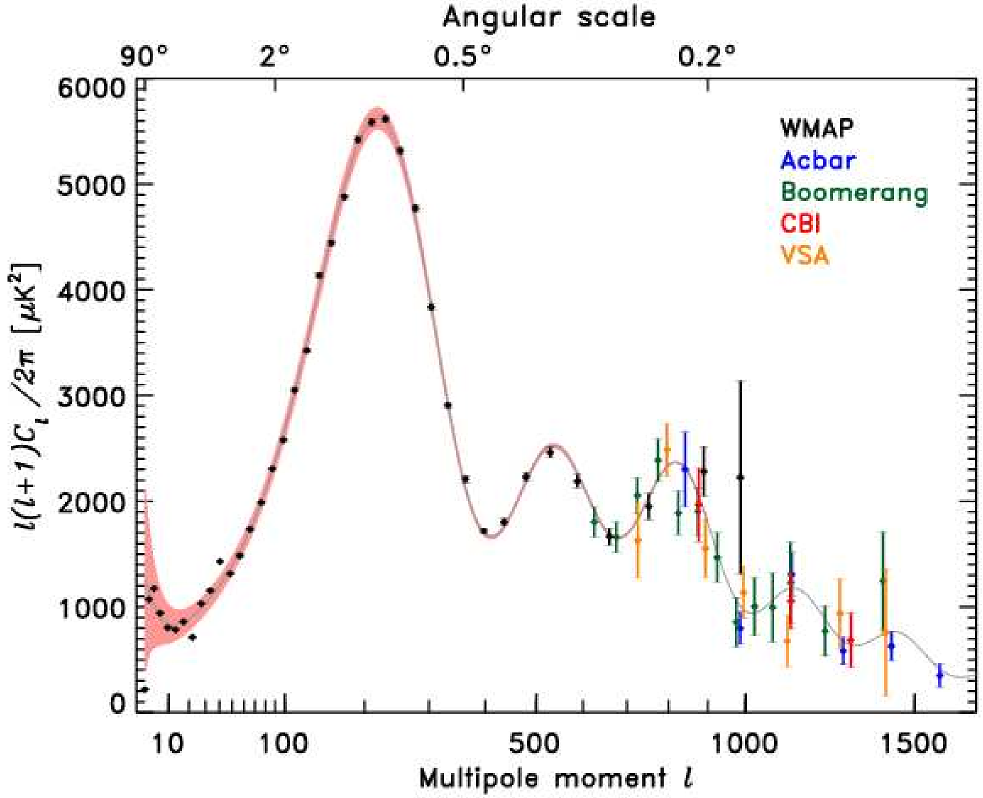




z=0
θ
14 000 Mpc
z~1000

Observed CMB temperature power spectrum





Hinshaw et al.

Observations

Constrain theory of early universe
+ evolution parameters and geometry

The Vanilla Universe Assumptions

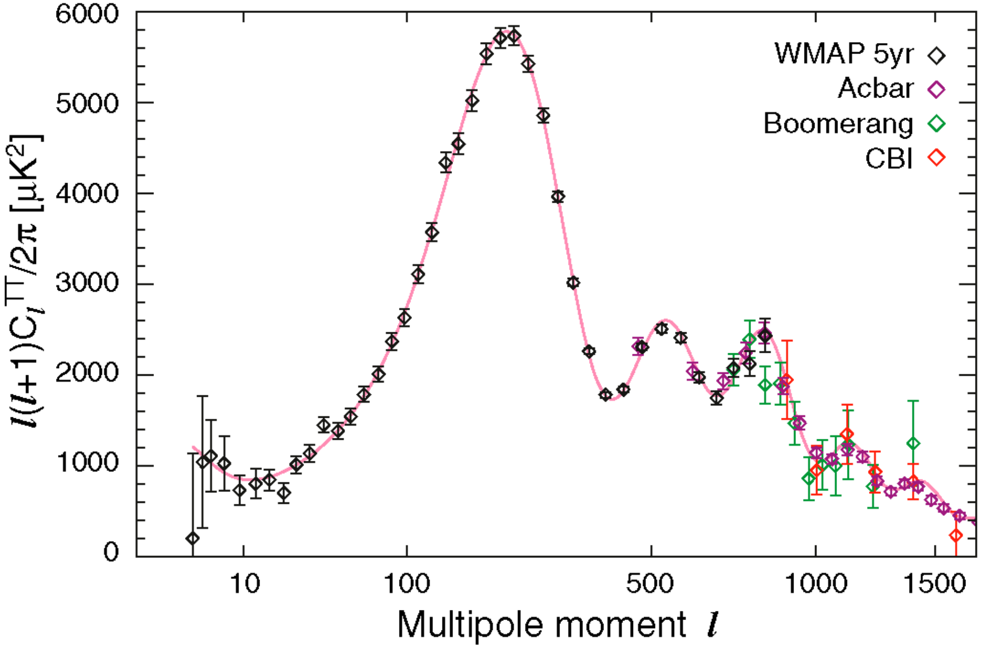
- Translation invariance - statistical homogeneity
  (observers see the same things on average after spatial translation)

- Rotational invariance - statistical isotropy
  (observations at a point the same under sky rotation on average)

- Primordial adiabatic nearly scale-invariant Gaussian fluctuations filling a flat universe

Statistically isotropic CMB with Gaussian fluctuations and smooth power spectrum

WMAP spice - not so vanilla?

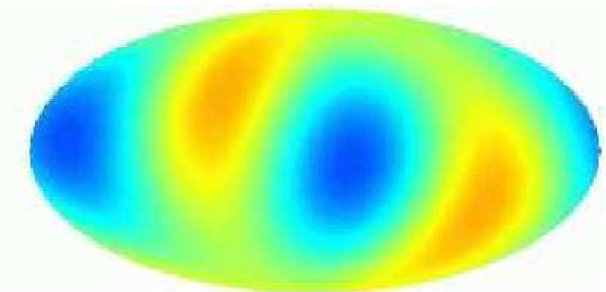


Low quadrupole?

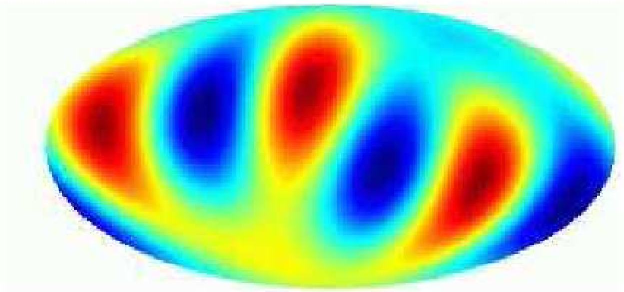
WMAP team

Alignments?

Tegmark et al.

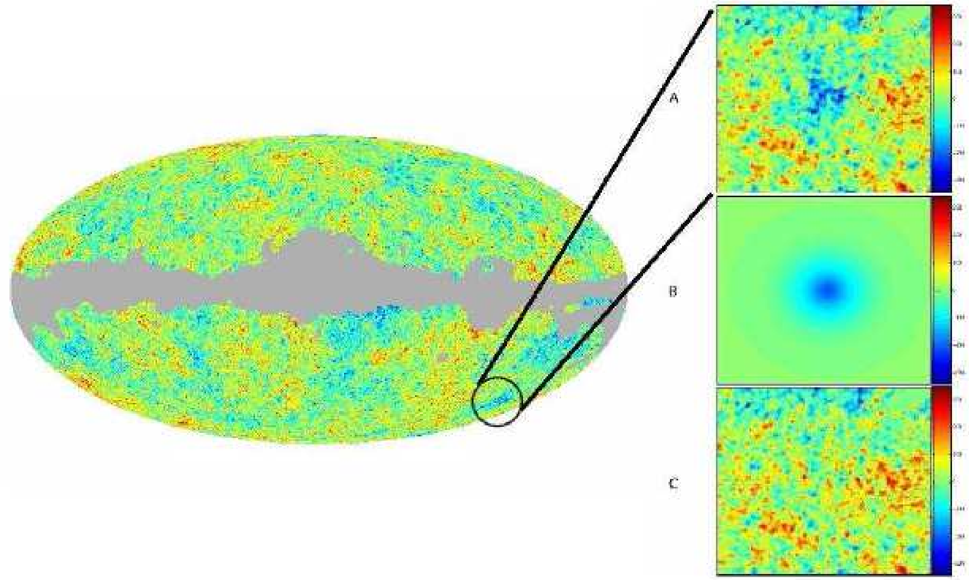


Quadrupole

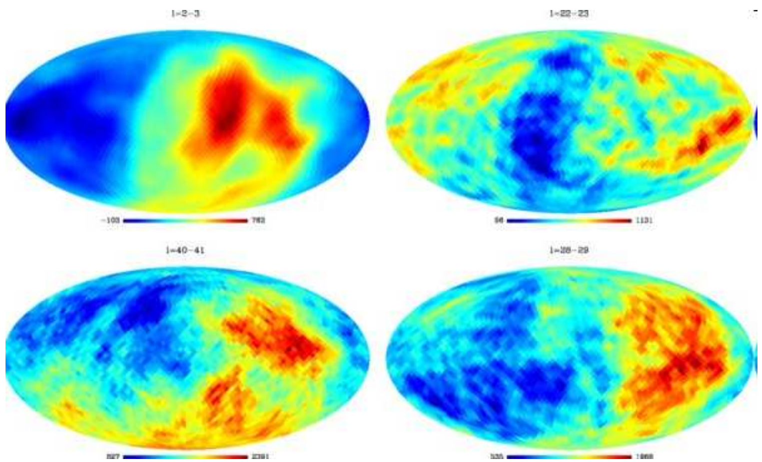


Octopole

Cold spot?



Cruz et al, 0901.1986



Power asymmetry?

Eriksen et al, Hansen et al.

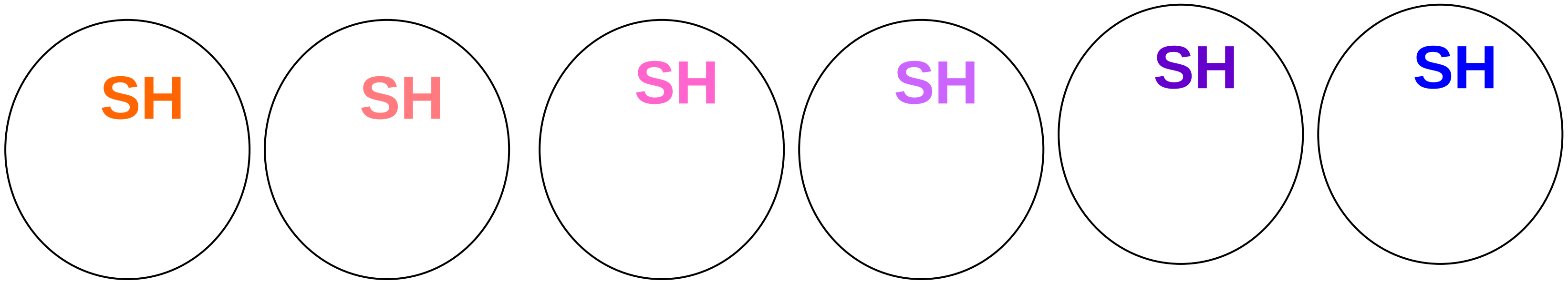
+Non-Gaussianity?… +….?

Gaussian statistical anisotropy

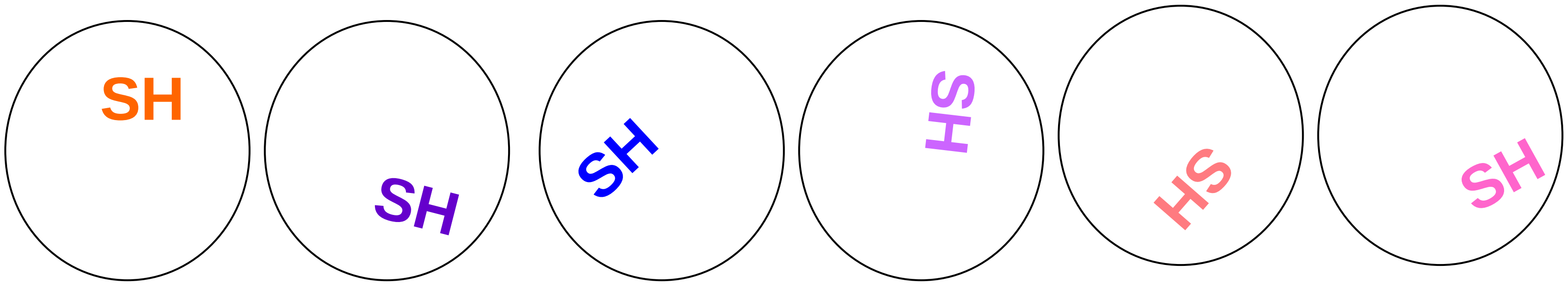
- CMB lensing
- Power asymmetries
- Anisotropic primordial power
- Spatially-modulated primordial power
- Non-Gaussianity

Gaussian anisotropic models

$$-\mathcal{L}(\hat{\Theta}|\mathrm{h}) = \frac{1}{2}\hat{\Theta}^{\dagger}(C^{\hat{\Theta}\hat{\Theta}})^{-1}\hat{\Theta} + \frac{1}{2}\ln\det(C^{\hat{\Theta}\hat{\Theta}})$$

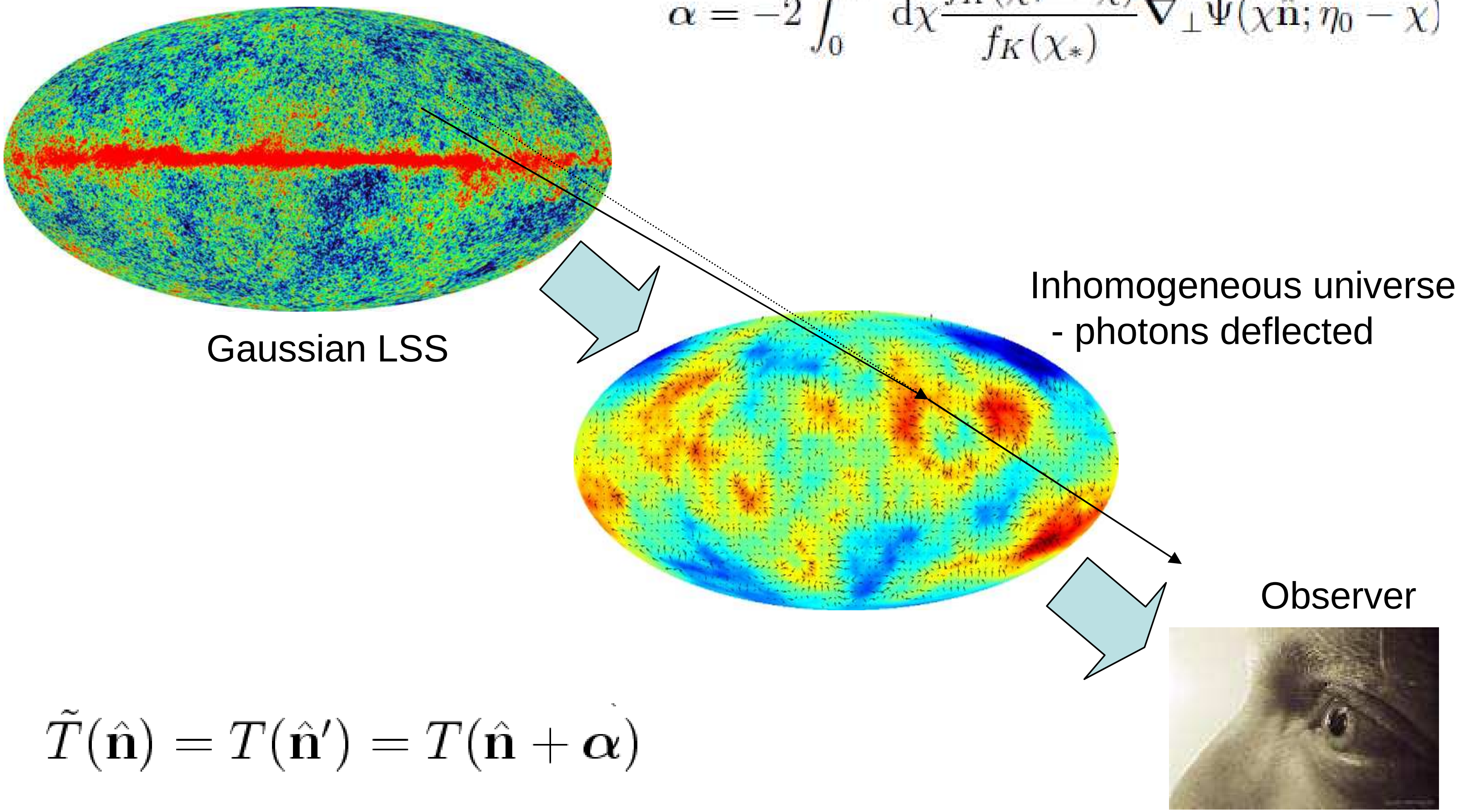


Or is it a statistically isotropic non-Gaussian model??



Example: CMB lensing

$$\boldsymbol{\alpha} = -2\int_0^{\chi_*} \mathrm{d}\chi \frac{f_K(\chi_* - \chi)}{f_K(\chi_*)} \boldsymbol{\nabla}_\perp \Psi(\chi\hat{\mathbf{n}};\eta_0 - \chi)$$



$$\tilde{T}(\hat{\mathbf{n}}) = T(\hat{\mathbf{n}}') = T(\hat{\mathbf{n}} + \boldsymbol{\alpha})$$

Lensing field is FIXED:

Anisotropic Gaussian temperature distribution

- Different parts of the sky magnified and demagnified
- Re-construct the actual lensing field

Lensing field is RANDOM:

Non-Gaussian statistically isotropic temperature distribution

- Significant connected 4-point function
- Excess variance to anisotropic-looking realizations
- Lensed temperature power spectrum

We see only one sky - both interpretations can be useful

See forthcoming Hanson et al. review for details

Anisotropy estimators

$$-\mathcal{L}(\hat{\Theta}|\mathrm{h}) = \frac{1}{2}\hat{\Theta}^\dagger (C^{\hat{\Theta}\hat{\Theta}})^{-1}\hat{\Theta} + \frac{1}{2}\ln\det(C^{\hat{\Theta}\hat{\Theta}})$$

Maximum likelihood:

$$\frac{\delta\mathcal{L}}{\delta\mathrm{h}^\dagger} = \underbrace{-\frac{1}{2}\hat{\Theta}^\dagger (C^{\hat{\Theta}\hat{\Theta}})^{-1}\frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta\mathrm{h}^\dagger}(C^{\hat{\Theta}\hat{\Theta}})^{-1}\hat{\Theta}}_{\mathcal{H}} + \frac{1}{2}\mathrm{Tr}\left[(C^{\hat{\Theta}\hat{\Theta}})^{-1}\frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta\mathrm{h}^\dagger}\right] = 0$$

$$\mathrm{Tr}(A) = \langle \mathrm{x}^\dagger A C^{-1}\mathrm{x}\rangle \Rightarrow \frac{\delta\mathcal{L}}{\delta\mathrm{h}^\dagger} = \langle\mathcal{H}\rangle - \mathcal{H} = 0$$

$$\langle \mathbf{x}^\dagger A C^{-1}\mathbf{x}\rangle = \langle \mathrm{Tr}(AC^{-1}\mathbf{x}\mathbf{x}^\dagger)\rangle = \mathrm{Tr}(AC^{-1}C) = \mathrm{Tr}(A)$$

Newton-Raphson solution:

$$\mathbf{h}_{i+1} = \mathbf{h}_i - \left[\left. \frac{\delta}{\delta \mathbf{h}^\dagger} (\langle \mathcal{H} \rangle - \mathcal{H})^\dagger \right|_i^\dagger \right]^{-1} (\langle \mathcal{H} \rangle_i - \mathcal{H}_i)$$

$$\sim \left\langle \frac{\delta}{\delta \mathbf{h}^\dagger} (\langle \mathcal{H} \rangle - \mathcal{H})^\dagger \right\rangle = \left[\left\langle \mathcal{H}\mathcal{H}^\dagger \right\rangle - \langle \mathcal{H} \rangle \langle \mathcal{H} \rangle^\dagger \right]$$

$$= \mathcal{F}$$

First iteration solution: Quadratic Maximum Likelihood (QML) $\hat{\mathbf{h}} = \mathcal{F}^{-1}[\tilde{\mathbf{h}} - \langle \tilde{\mathbf{h}} \rangle]$.

$$\bar{\Theta} = (C^{\hat{\Theta}\hat{\Theta}})^{-1}|_0 \hat{\Theta}$$

$$\tilde{\mathbf{h}} = \mathcal{H}_0 = \frac{1}{2} \bar{\Theta}^\dagger \frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta \mathbf{h}^\dagger} \bar{\Theta}$$

$$= \frac{1}{2} \sum_{lm, l'm'} \left[\frac{\delta C^{\hat{\Theta}\hat{\Theta}}_{lm,\, l'm'}}{\delta \mathbf{h}^\dagger} \right] \Theta^*_{lm} \Theta_{l'm'},$$

Sky modulation?

Popular modulation model: $\Theta_f(\hat{\mathbf{n}}) = [1 + f(\hat{\mathbf{n}})]\Theta^i_f(\hat{\mathbf{n}})$

QML estimator for f:

$$\tilde{h}^f_{lm} = \int d\Omega Y^*_{lm} \left[\sum_{l_1 m_1}^{l_{\max}} \bar{\Theta}_{l_1 m_1} Y_{l_1 m_1}\right] \left[\sum_{l_2 m_2}^{l_{\max}} C_{l_2} \bar{\Theta}_{l_2 m_2} Y_{l_2 m_2}\right]$$

Approx Fisher:

$$\left[\mathcal{F}^{ff}_{\text{iso}}\right]_{lm,l'm'} = \delta_{ll'}\delta_{mm'} \times \sum_{l_1,l_2} \frac{(2l_1+1)(2l_2+1)}{8\pi} \begin{pmatrix} l & l_1 & l_2 \\ 0 & 0 & 0 \end{pmatrix}^2 \frac{(C_{l_1}+C_{l_2})^2}{C^{\text{tot}}_{l_1} C^{\text{tot}}_{l_2}}$$

Reconstruction recipe

$$\hat{\Theta}$$

$\downarrow$ Inverse variance filter

$$\bar{\Theta} = (C^{\hat{\Theta}\hat{\Theta}})^{-1}|_0\hat{\Theta}$$ (sets to zero in sky cut)

Make filtered maps

*

$$\mathbf{F_1} = \left[\sum_{l_1 m_1}^{l_{\max}} \bar{\Theta}_{l_1 m_1} Y_{l_1 m_1}\right] \qquad \mathbf{F_2} = \left[\sum_{l_2 m_2}^{l_{\max}} C_{l_2} \bar{\Theta}_{l_2 m_2} Y_{l_2 m_2}\right]$$

$$\tilde{h}^f_{lm} = \int d\Omega Y^*_{lm} \mathbf{F_1}\,\mathbf{F_2}$$ Quadratic estimator

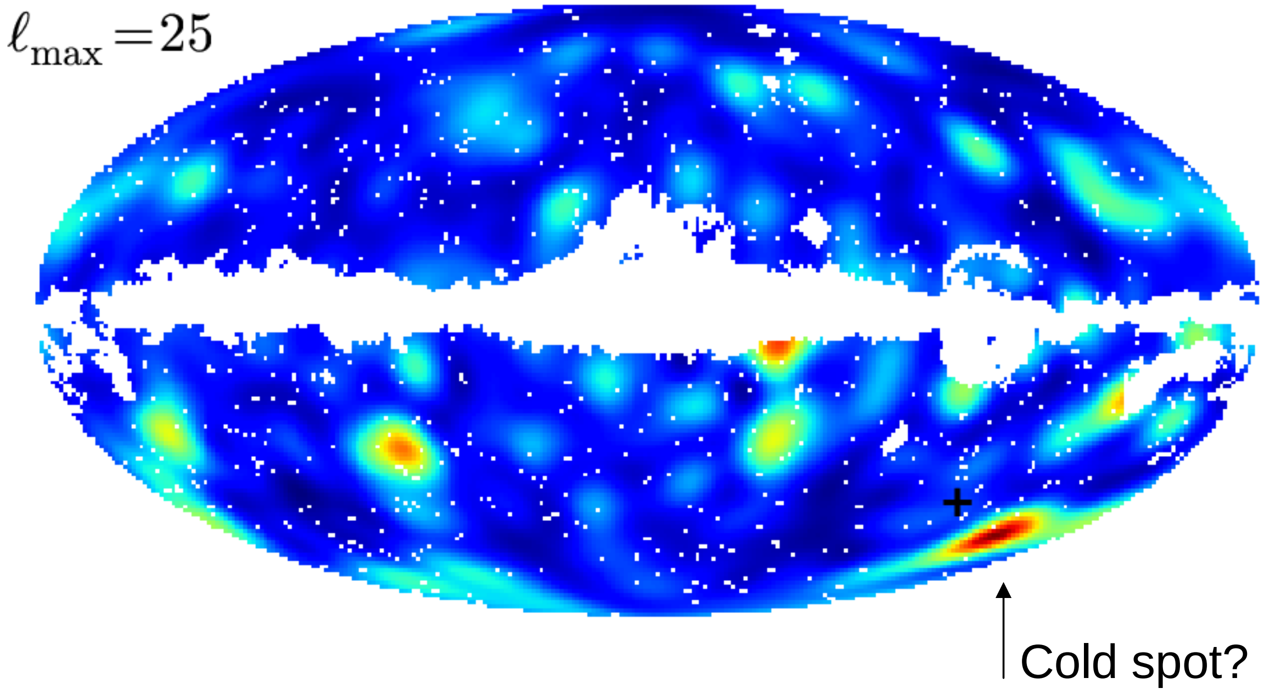
Simulate * many times to calculate $\langle \tilde{\mathrm{h}} \rangle$ (accounts for anisotropic noise/sky cut)

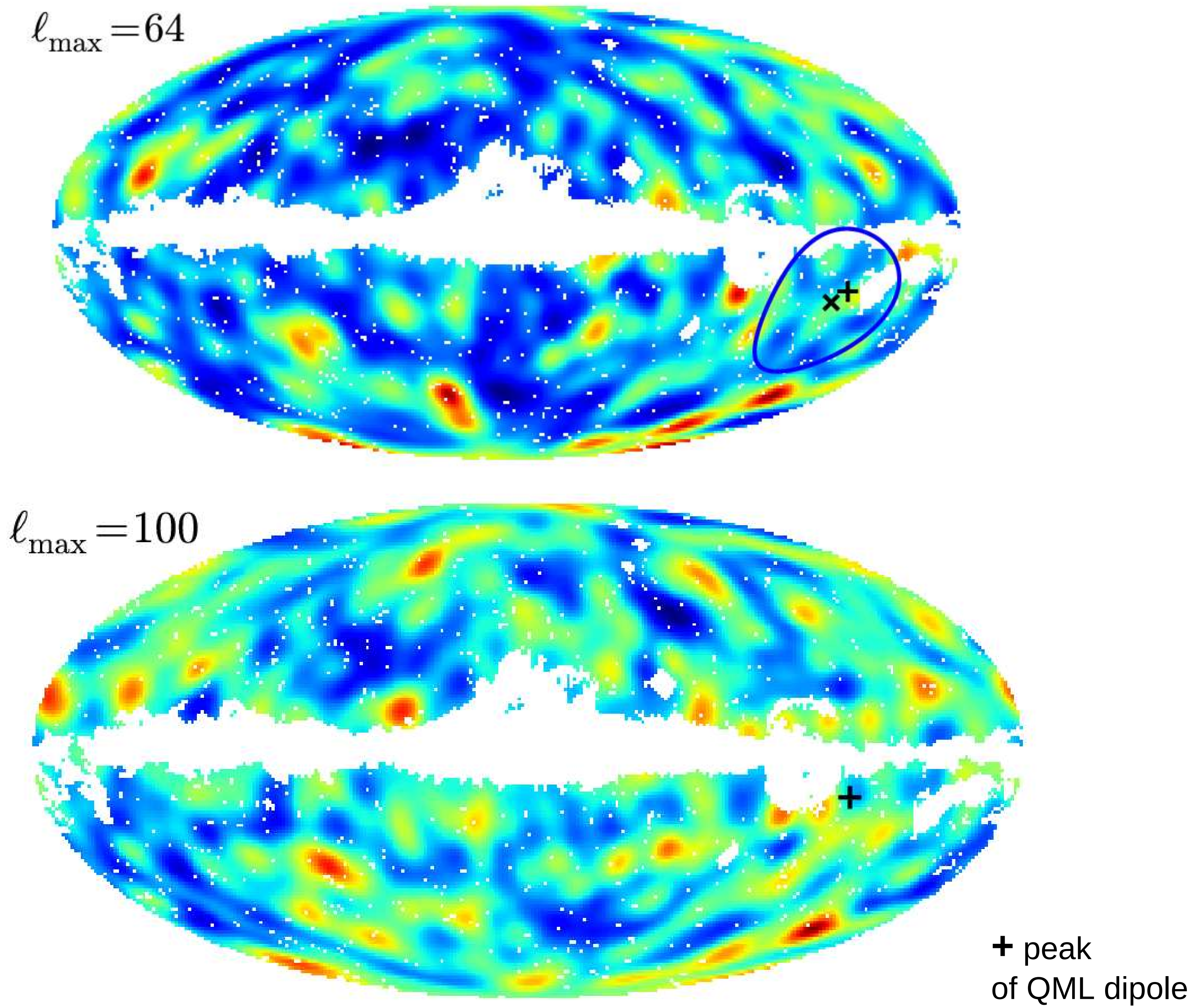
$$\Rightarrow \quad \hat{\mathrm{h}} = \mathcal{F}^{-1}[\tilde{\mathrm{h}} - \langle \tilde{\mathrm{h}} \rangle]$$

$\mathcal{F}$ Approximated or from sims

WMAP power reconstruction

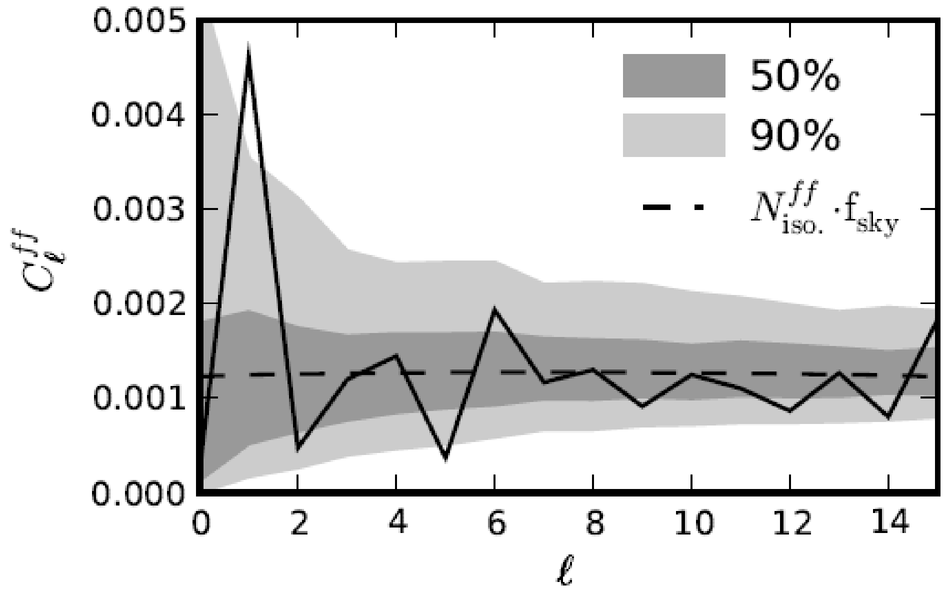
(V band, KQ85 mask, foreground cleaned; reconstruction smoothed to 10degrees)



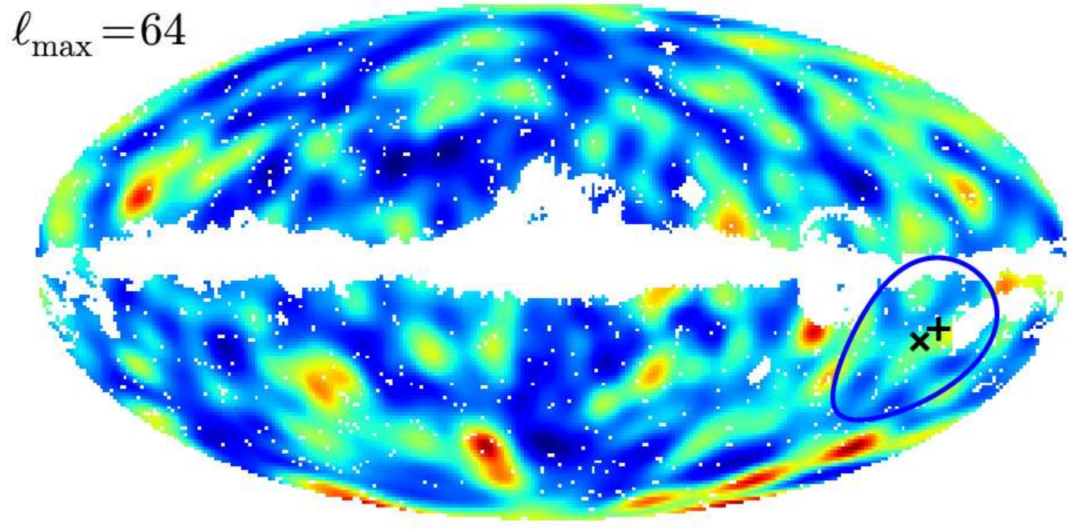


$\ell_{\rm max}=64$
$\ell_{\rm max}=100$
+ peak
of QML dipole

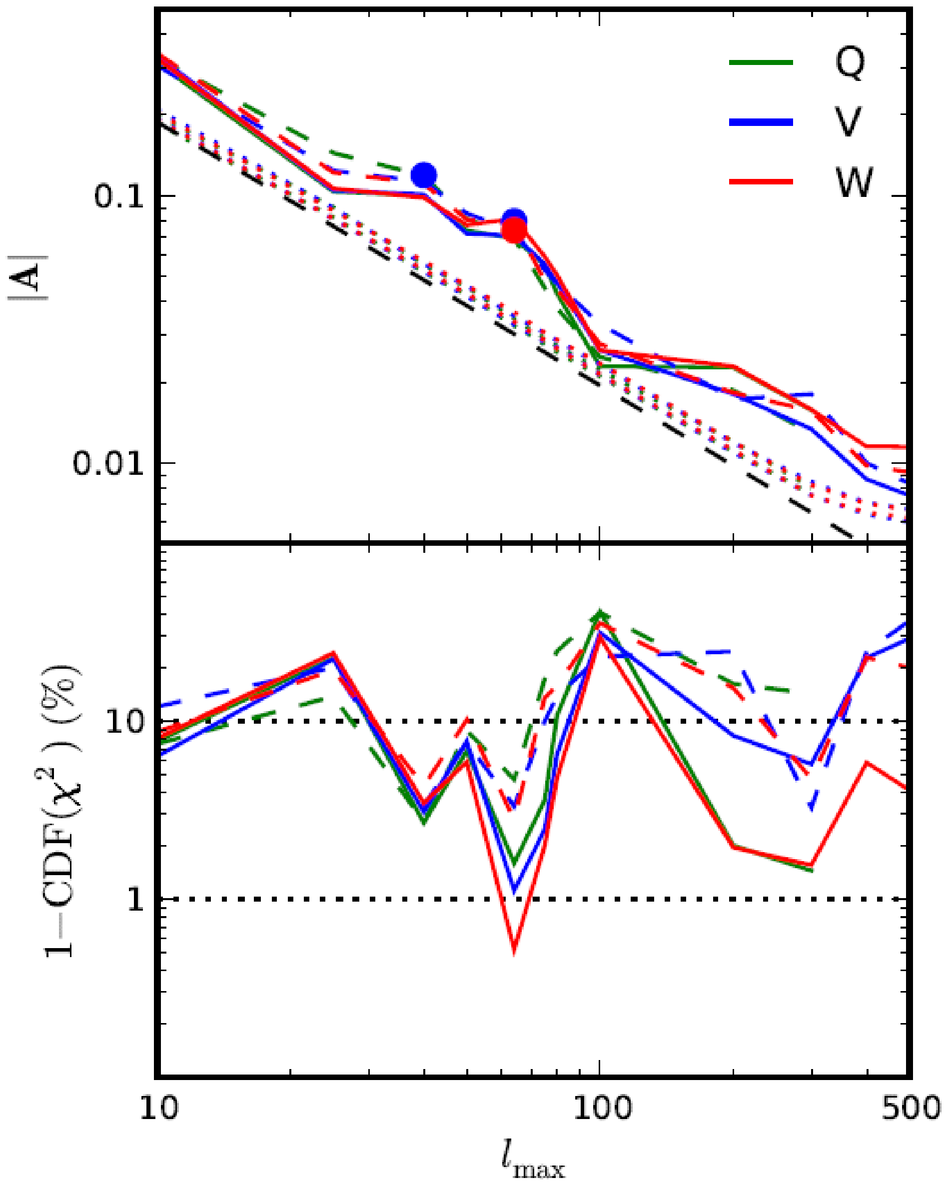
Modulation power spectrum l_{max}=64



Dipole power asymmetry?



Dipole amplitude as function of l_{max}



Only ~1% modulation allowed on small scales

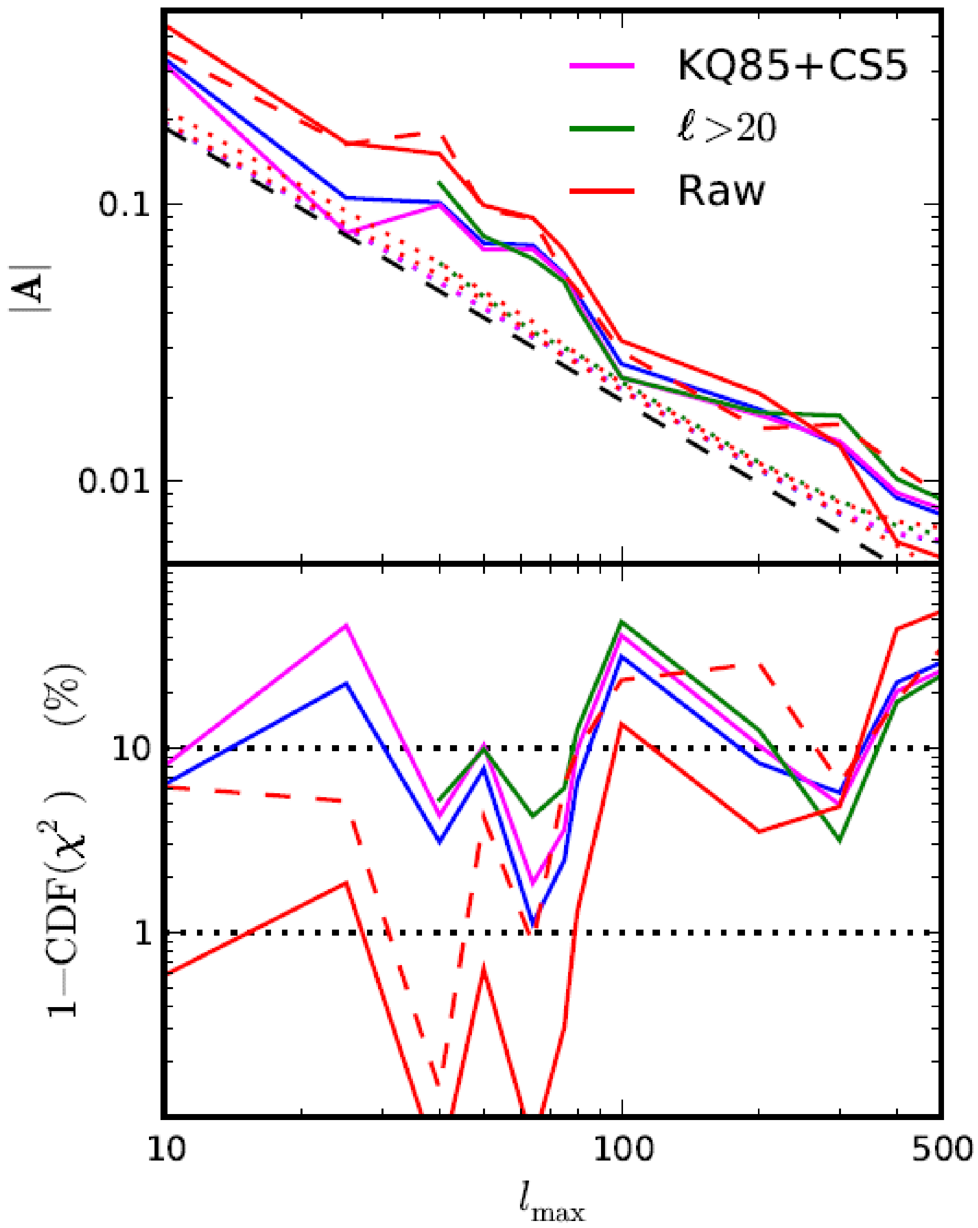
Consistent with Hirata 2009
- Very small observed anisotropy in quasar distribution

Is it just the cold spot?
Or just the low multipoles?
Or foregrounds?

- No

May be something interesting,
but only ~1% significance at most

Dashed is Raw+KQ75



Primordial power anisotropy

Look for direction-dependence in primordial power spectrum:

$$\langle \chi_0(\mathbf{k})\chi_0^*(\mathbf{k}')\rangle \;=\; (2\pi)^3\delta(\mathbf{k}-\mathbf{k}')P_\chi(\mathbf{k})$$

Assume late-time isotropization.

$$\Theta_{lm} = 4\pi i^l \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\Delta_l(k)\chi_0(\mathbf{k})Y_{lm}^*(\hat{\mathbf{k}})$$

Anisotropic covariance:

$$C_{l_1 m_1 l_2 m_2} =$$
$$i^{l_1-l_2}\frac{\pi}{2}\int \mathrm{d}^3\mathbf{k}P_\chi(\mathbf{k})\Delta_{l_1}(k)\Delta_{l_2}(k)Y_{l_1 m_1}^*(\hat{\mathbf{k}})Y_{l_2 m_2}(\hat{\mathbf{k}})$$

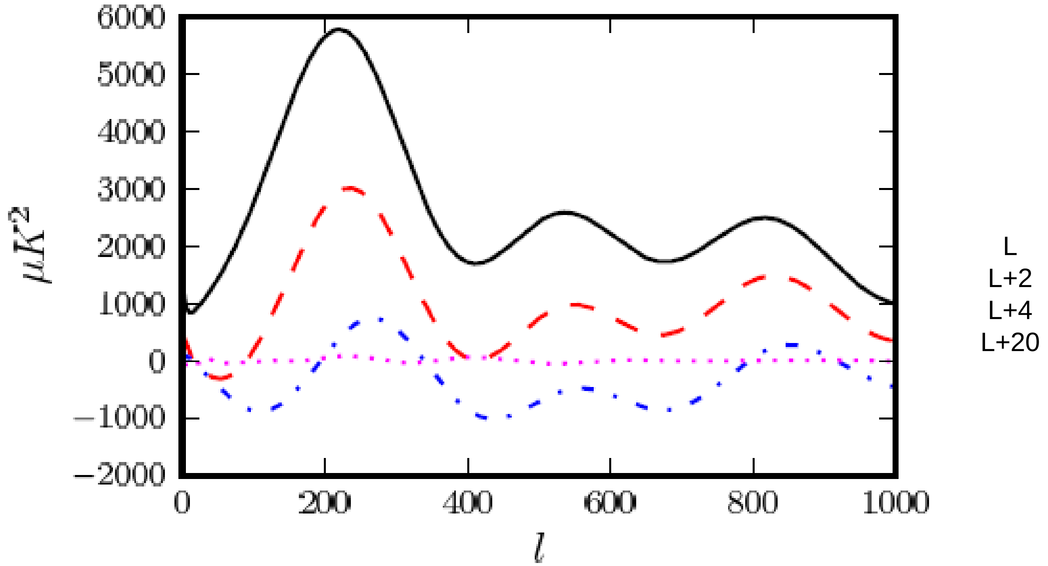
Simple case: $\mathcal{P}_\chi(\mathbf{k}) = \mathcal{P}_\chi(k)[1 + a(k)g(\hat{\mathbf{k}})]$

e.g.Ackerman et.al. astro-ph/0701357
Gumrukcuoglu et al 0707.4179

$$C_{l_1 m_1 l_2 m_2} = \delta_{l_1 l_2}\delta_{m_1 m_2} C_{l_1} + \sum_{lm} i^{l_1 - l_2} g_{lm} \int \mathrm{d}\Omega_{\mathbf{k}} C_{l_1 l_2} Y_{lm} Y^*_{l_1 m_1} Y_{l_2 m_2}$$

where

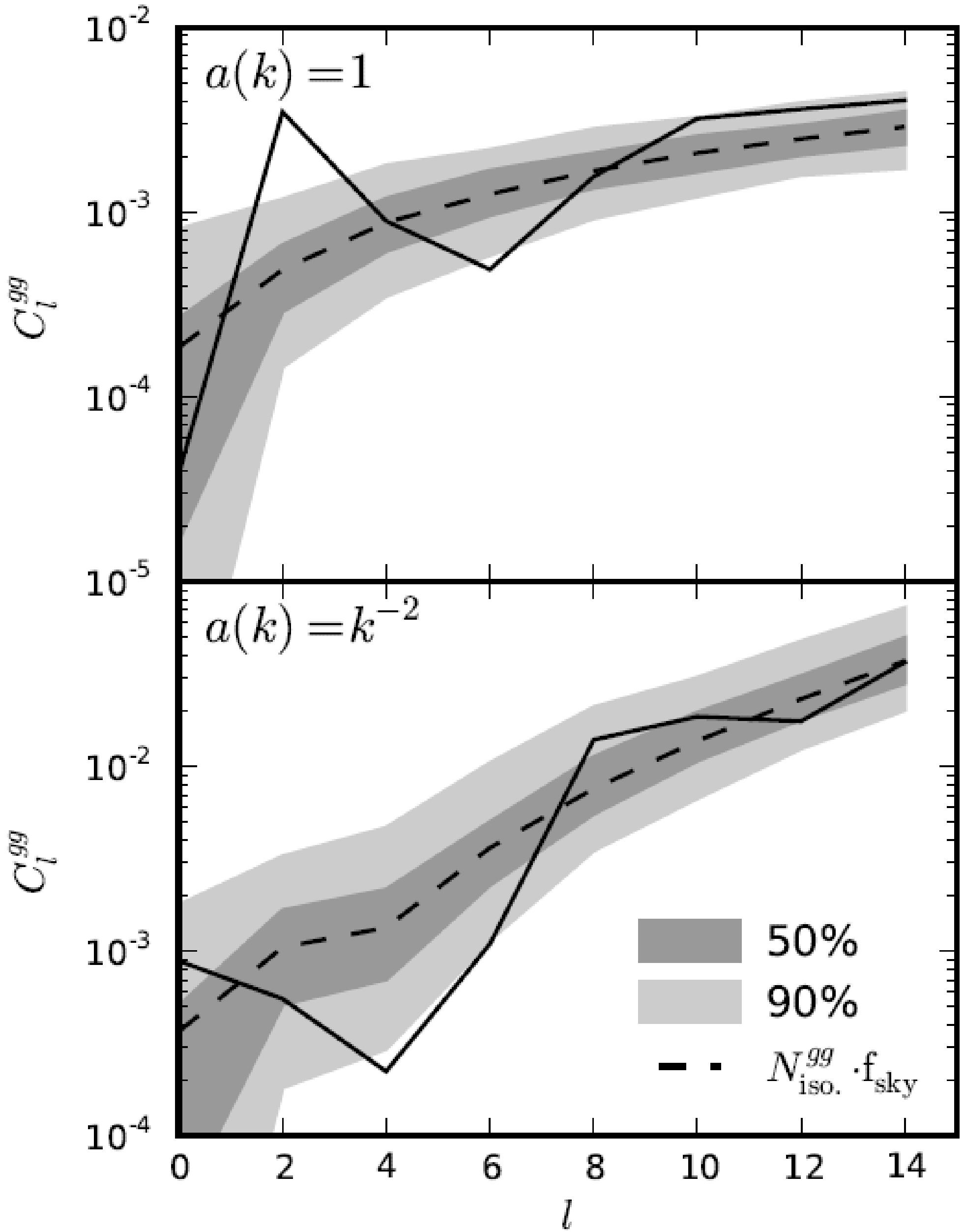
$$C_{l_1 l_2} \equiv 4\pi \int \mathrm{d}\ln k \mathcal{P}_\chi(k) a(k) \Delta_{l_1}(k) \Delta_{l_2}(k).$$



- Reconstruct g(k).

QML estimator:

$$\tilde{h}^g_{lm} = \frac{1}{2}\int \mathrm{d}\Omega Y^*_{lm} \sum_{l_1 l_2} i^{l_1-l_2} C_{l_1 l_2} \times \left[\sum_{m_1} \bar{\Theta}_{l_1 m_1} Y_{l_1 m_1}\right] \left[\sum_{m_2} \bar{\Theta}_{l_2 m_2} Y_{l_2 m_2}\right]$$

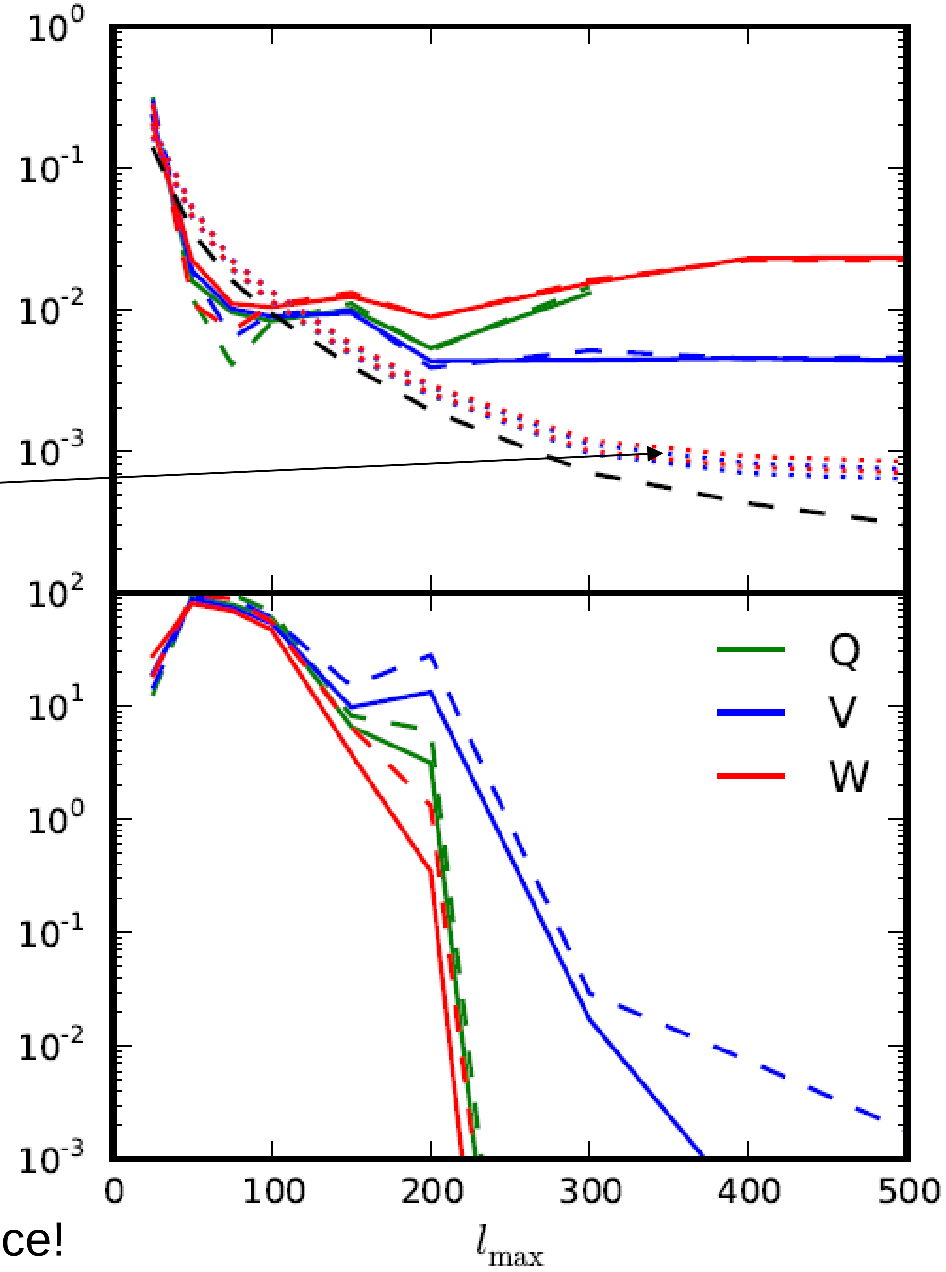


Quadrupole primordial power asymmetry??

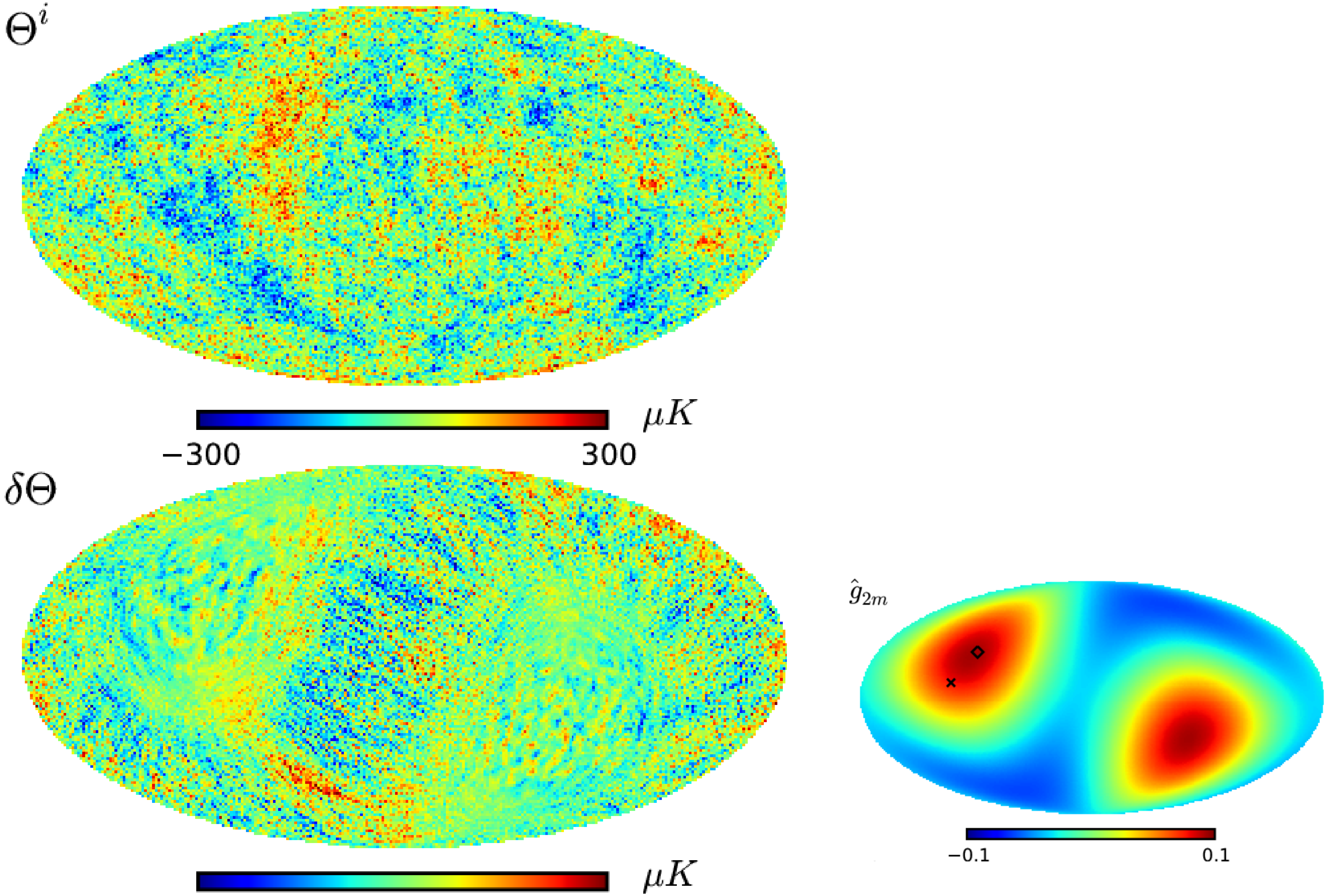
a(k)=1

Dashed: KQ75
Solid: KQ85

Variance from
simulations



Very significant evidence for
~ 10% quadrupole angular dependence!

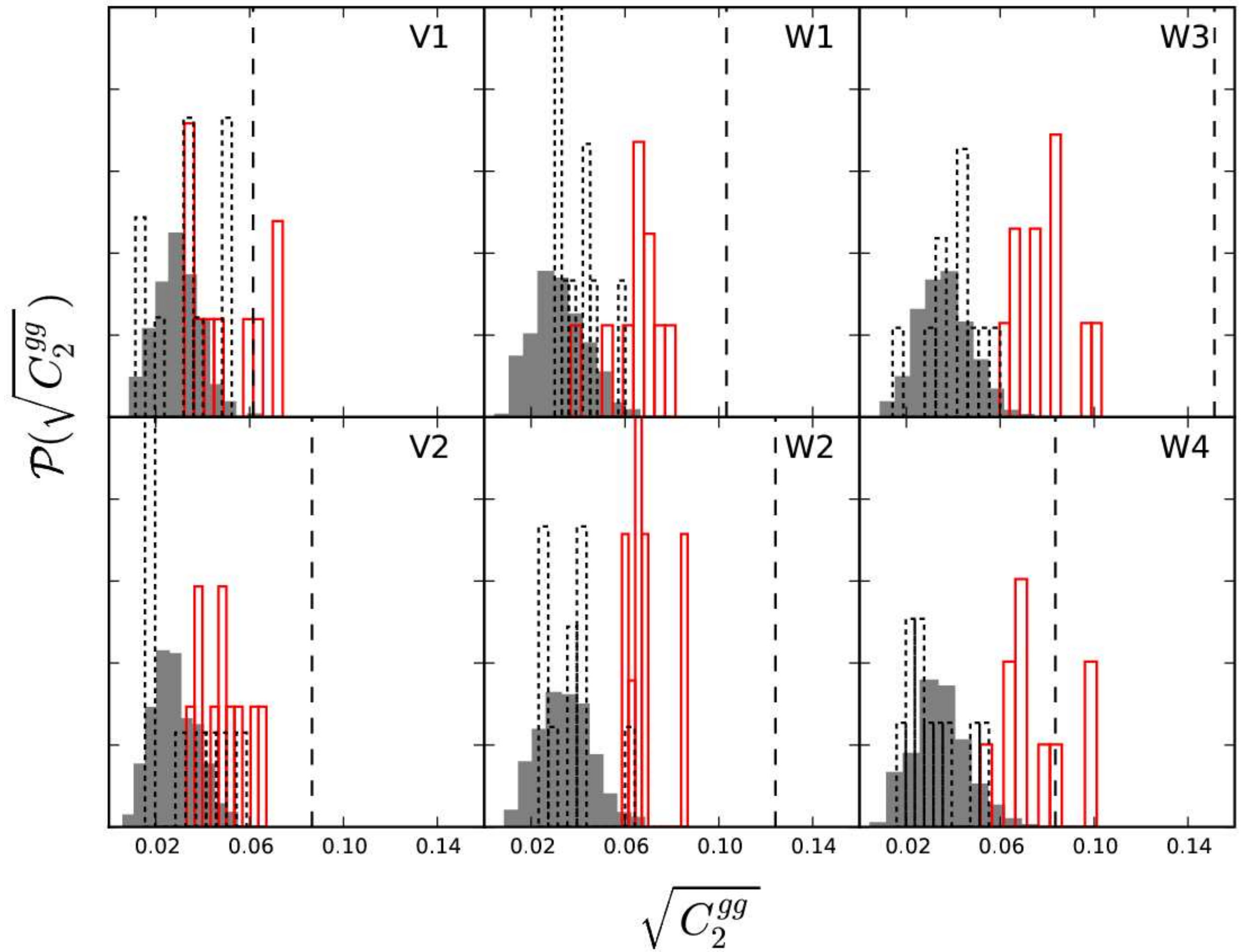


Direction close to ecliptic!

Could it be systematics?

- beam asymmetries? uncorrected in WMAP maps

Test with 10 asymmetric beam simulations of Wehus et al, 0904.3998



Intriguing, but probably not mostly primordial:

Signal varies significantly between detectors at the same frequency and aligned with ecliptic

- strong evidence for a systematic origin

Wehus simulations give effect of right order of magnitude

- beam asymmetry very important and must be accounted for
- but not consistent with data in all D/A, not complete explanation

Primordial spatial modulation

$$\chi(\mathbf{x}) = \chi_0(\mathbf{x})[1 + \phi(\mathbf{x})]$$

Modulation field

Gaussian and statistically homogeneous

$$\langle \chi(\mathbf{k})\chi(\mathbf{k}') \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') P_\chi(k) + \int \mathrm{d}^3\mathbf{x} e^{-i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{x}} \phi(\mathbf{x}) \left[P_\chi(k) + P_\chi(k') \right]$$

Expand: $$e^{i\mathbf{k}\cdot\mathbf{x}} = 4\pi \sum_{lm} i^l j_l(kx) Y_{lm}(\hat{\mathrm{x}}) Y^*_{lm}(\hat{\mathbf{k}})$$

Anisotropic covariance:

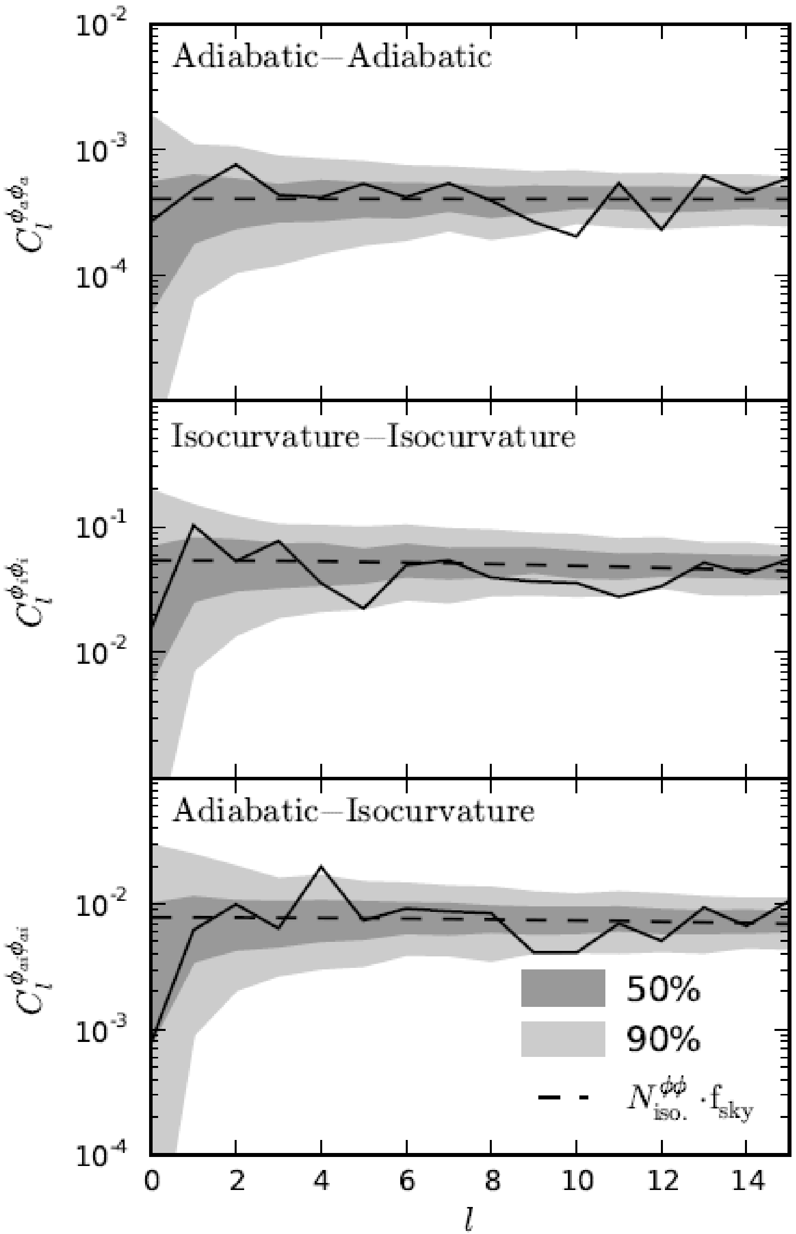
$$\begin{aligned} C_{l_1 m_1 l_2 m_2} &= \delta_{l_1 l_2}\delta_{m_1 m_2} C_{l_1} \\ &+ \int \mathrm{d}^3\mathbf{x}\phi(\mathbf{x})\alpha_{l_1}(x)\beta_{l_2}(x) Y^*_{l_1 m_1}(\hat{\mathrm{x}}) Y_{l_2 m_2}(\hat{\mathrm{x}}) \\ &+ \int \mathrm{d}^3\mathbf{x}\phi(\mathbf{x})\alpha_{l_2}(x)\beta_{l_1}(x) Y^*_{l_1 m_1}(\hat{\mathrm{x}}) Y_{l_2 m_2}(\hat{\mathrm{x}}). \end{aligned}$$

$$\begin{aligned} \alpha_l(r) &\equiv 4\pi \int \mathrm{d}\ln k\, j_l(kr) \frac{k^3 \Delta_l(k)}{2\pi^2} \\ \beta_l(r) &\equiv 4\pi \int \mathrm{d}\ln k\, j_l(kr) \Delta_l(k) \mathcal{P}_\chi(k) \end{aligned}$$

QML estimator for modulation field at distance r

$$\tilde{h}^{\phi}_{lm}(r) = \int d\Omega Y^*_{lm} \left[\sum_{l_1 m_1} \alpha_{l_1}(r)\bar{\Theta}_{l_1 m_1} Y_{l_1 m_1}\right] \times \left[\sum_{l_2 m_2} \beta_{l_2}(r)\bar{\Theta}_{l_2 m_2} Y_{l_2 m_2}\right]$$

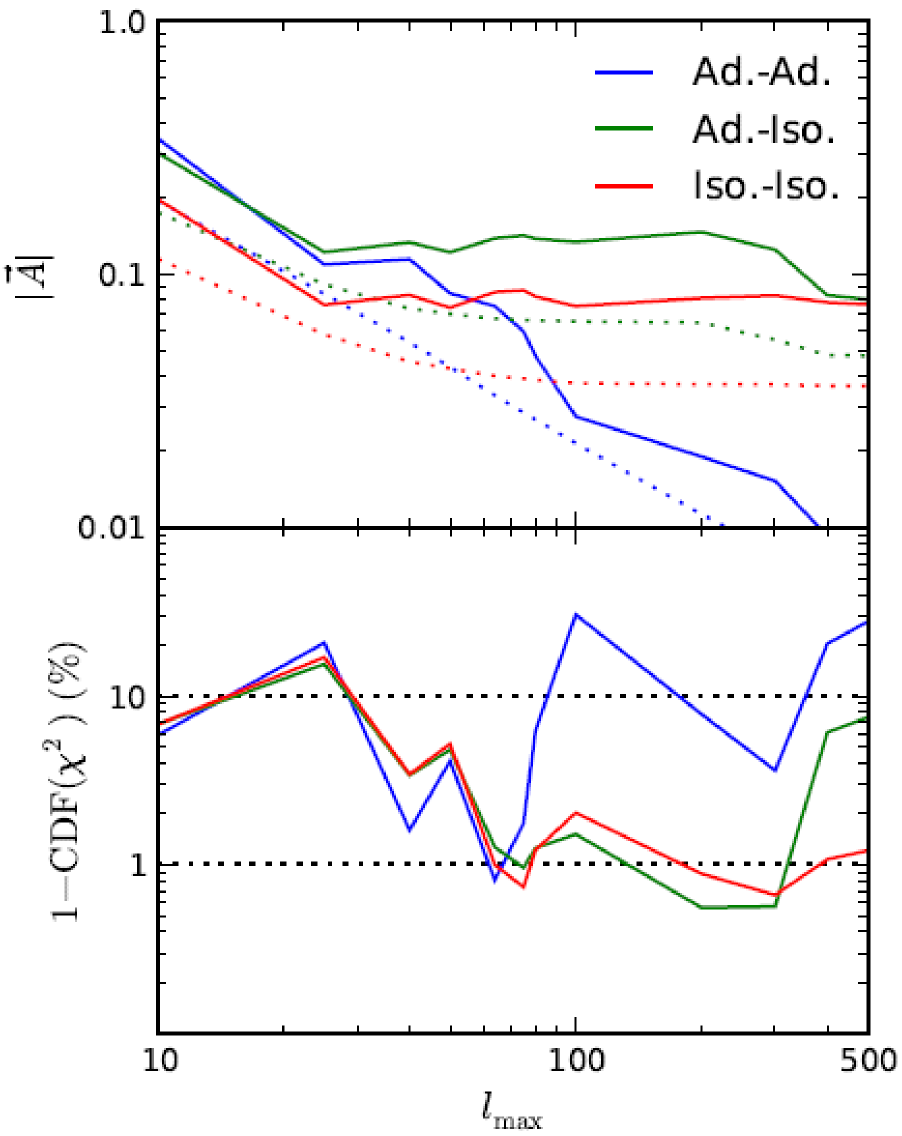
At recombination



Integrate over r, almost equivalent to spatial modulation model

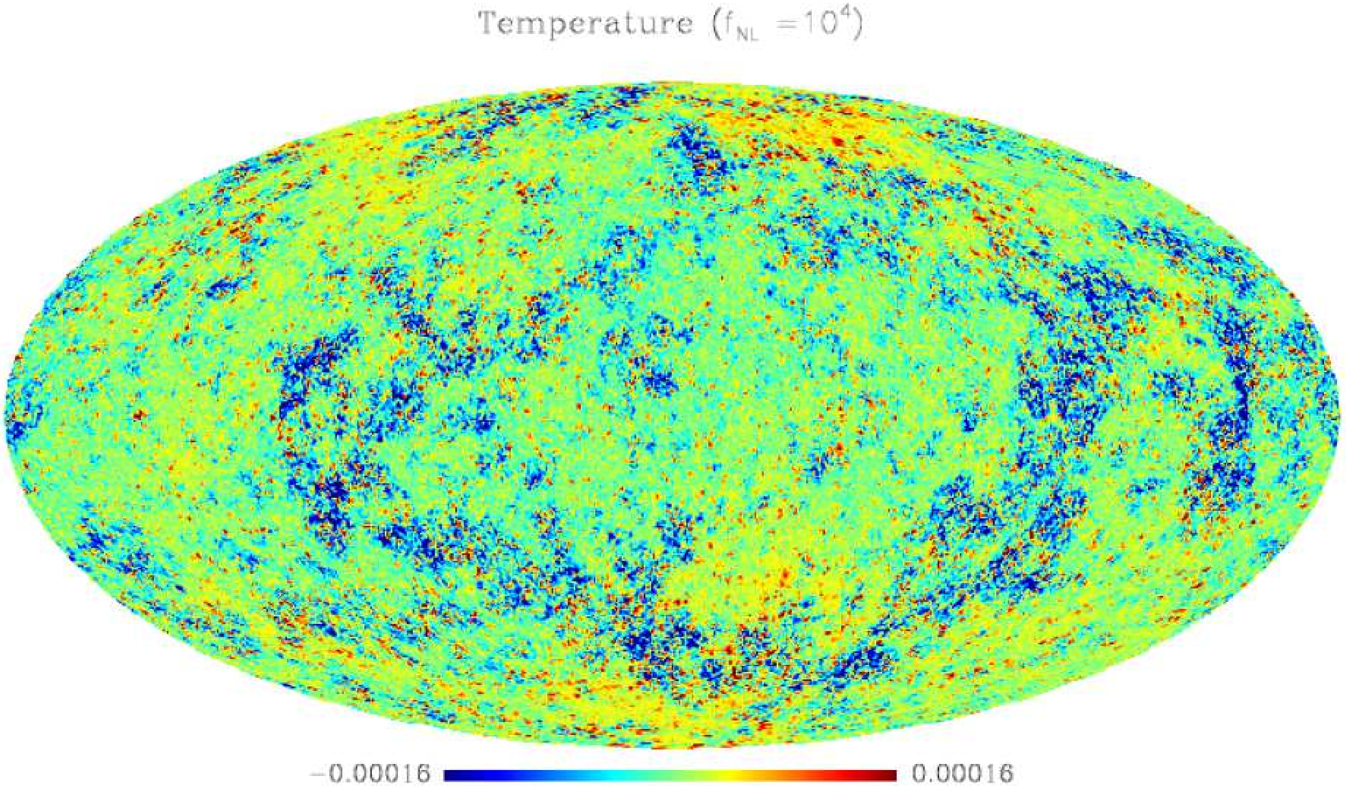
- Adiabatic model cannot explain dipole power asymmetry at ell <~ 60

- Isocurvature modes decay on small scales, a possibility



Bispectrum non-Gaussianity

- Local model: small scale power correlated with large-scale temperature
- Considering large-scale modes to be fixed, expect power anisotropy



Liguori et al 2007

Local primordial non-Gaussianity

$$\begin{aligned}\Psi &= \Psi_0 + f_{\mathrm{NL}}\Psi_0^2 \\ &= \Psi_0(1 + f_{\mathrm{NL}}\Psi_0)\end{aligned}$$

Just like the spatial modulation model
but modulation is the field itself

General bispectrum defined so that

$$\begin{aligned}\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}\rangle &\equiv B_{l_1 l_2 l_3}\begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3\end{pmatrix} \\ &= b_{l_1 l_2 l_3}\int \mathrm{d}\Omega Y_{l_1 m_1} Y_{l_2 m_2} Y_{l_3 m_3}\end{aligned}$$

Construct non-Gaussian field from Gaussian one:

$$T'_{lm} = T_{lm} + \frac{1}{6} B_{ll_1l_2} \sum_{m_1m_2} (-1)^{m_1} \begin{pmatrix} l & l_1 & l_2 \\ m & -m_1 & m_2 \end{pmatrix} T_{l_1m_1} T^*_{l_2m_2}$$

(assume B small)

How about reverse? Make Gaussian from non-Gaussian:

Write general quadratic anisotropy estimator:

$$\begin{aligned} 6X_{lm} &\equiv \sum_{l_1m_1,l_2m_2} B_{ll_1l_2} (-1)^{m_1} \begin{pmatrix} l & l_1 & l_2 \\ m & -m_1 & m_2 \end{pmatrix} \bar{\Theta}_{l_1m_1} \bar{\Theta}^*_{l_2m_2} \\ &= \int d\Omega Y^*_{lm} \sum_{l_1l_2} b_{ll_1l_2} \left[\sum_{m_1} \bar{\Theta}_{l_1m_1} Y_{l_1m_1} \right] \left[\sum_{m_2} \bar{\Theta}_{l_2m_2} Y_{l_2m_2} \right] \end{aligned}$$

Then $\Theta^G_{lm} \equiv \Theta_{lm} - f_{\rm NL}(X_{lm} - \langle X_{lm} \rangle).$ is isotropic and Gaussian

$f_{\rm NL}$ =1 if B has right amplitude

Bispectrum estimators are basically the cross-correlation of an anisotropy estimator with the temperature

$$\mathcal{E} = \frac{1}{F_{\mathcal{E}}}\bar{\Theta}^{\dagger}(\mathbf{X} - 3\langle\mathbf{X}\rangle),$$

In harmonic space

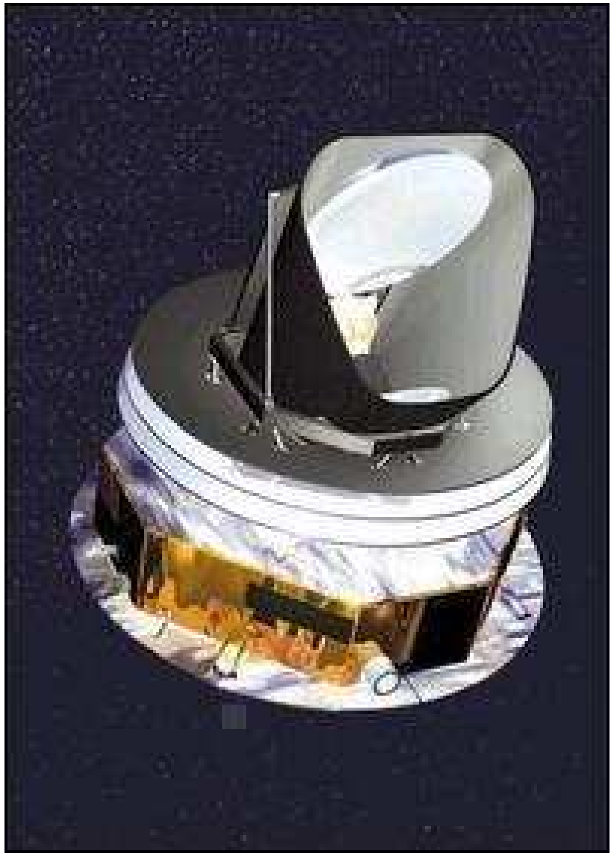
$$\mathcal{E} = \frac{1}{6F_{\mathcal{E}}}\sum_{l_i m_i} B_{l_1 l_2 l_3}\begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \times \left[\bar{\Theta}_{l_1 m_1}\bar{\Theta}_{l_2 m_2}\bar{\Theta}_{l_3 m_3} - 3C^{-1}_{l_1 m_1 l_2 m_2}\bar{\Theta}_{l_3 m_3}\right].$$

Creminelli et al 2005, Babich 2005, Smith & Zaldarriaga 2006

Planck and the future, 2009+

14 May 2009

High sensitivity and resolution
CMB temperature and polarization



Scope for better estimators:

- Polarization. More signal, very good check of primordial/local origin.

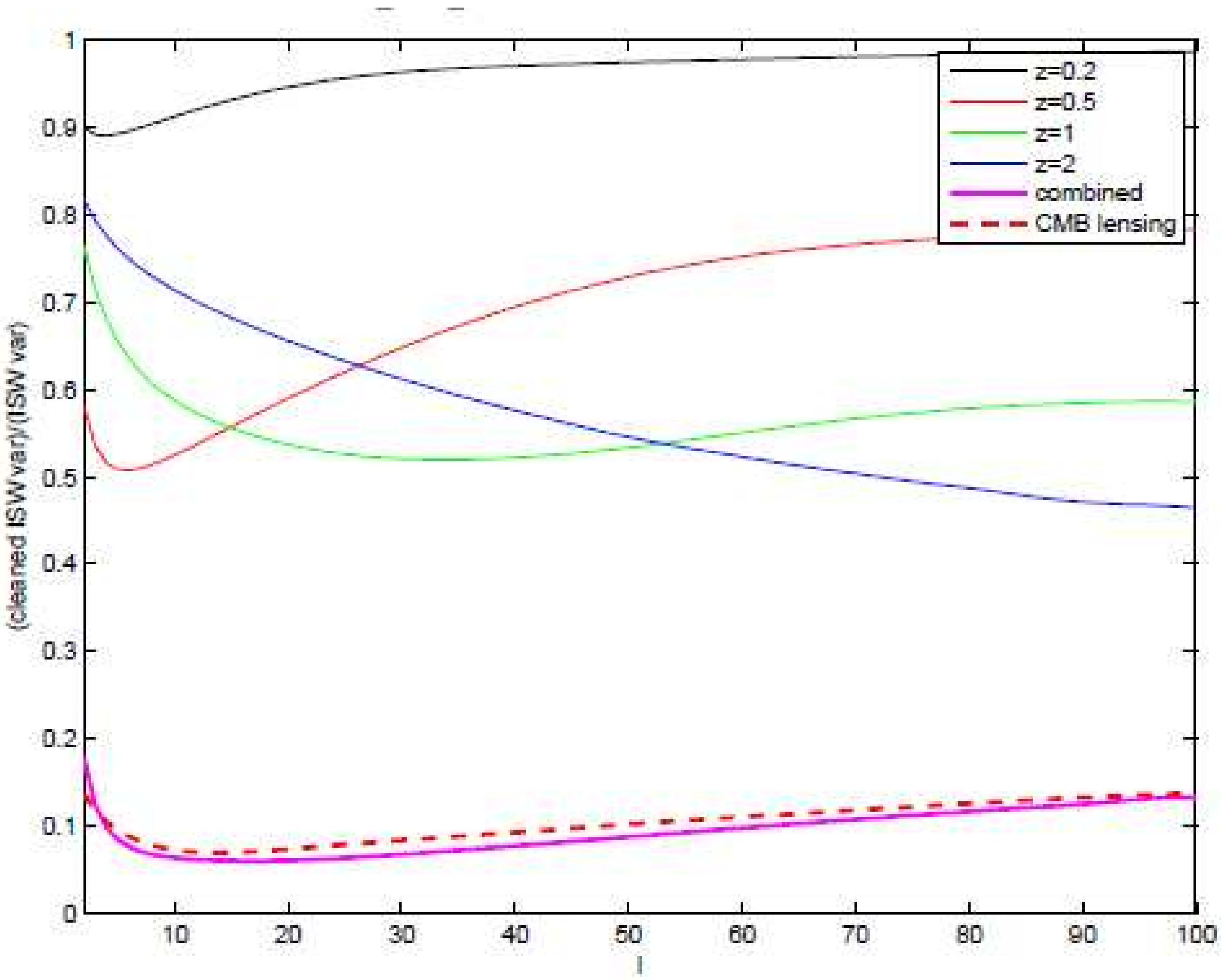
- If non-zero signal, need more complicated iterative estimators

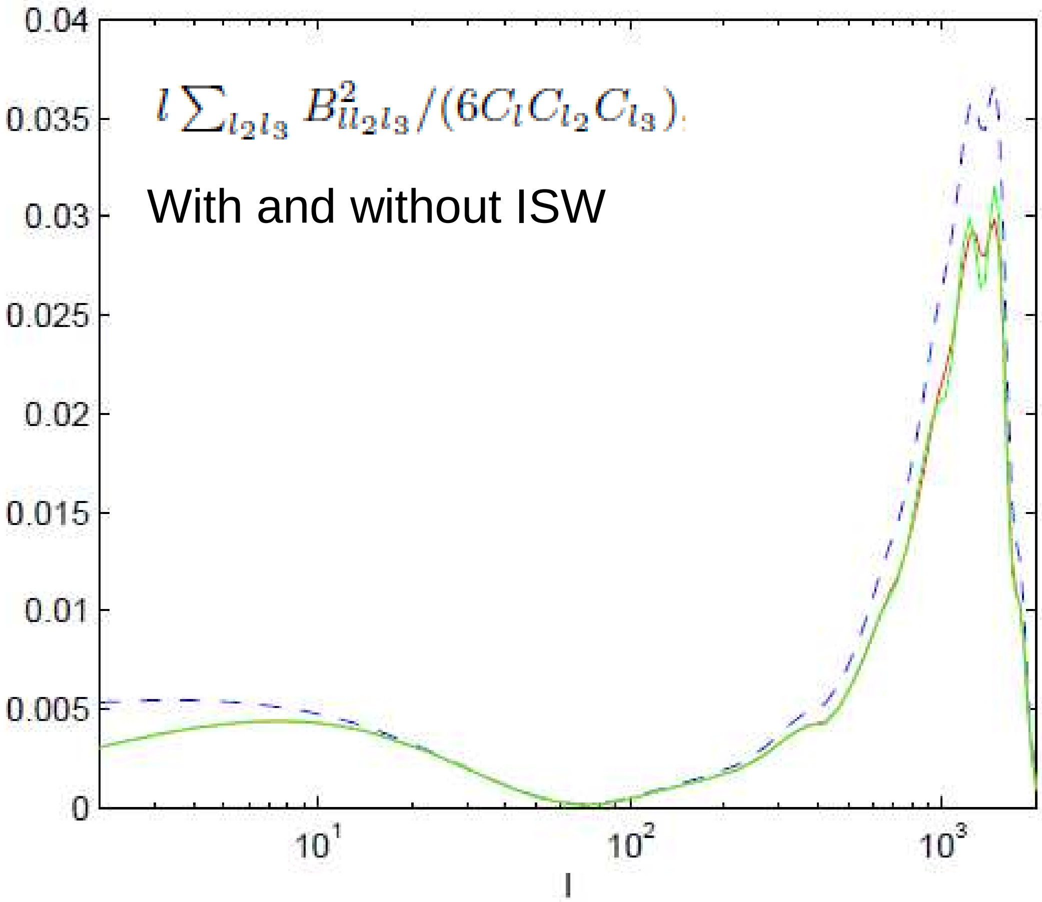
- Subtract effect of beam asymmetries and other systematics

- Account for uncertainties in cosmological parameters

- Use other probes of density/potential fields

- Remove ISW (e.g. Francis & Peacock 0909.2495)





~ 20% smaller error on f_{NL}

Conclusions

- Can easily constrain a variety of Gaussian anisotropic models using QML estimators
- Marginal evidence for dipole power asymmetry in WMAP
- Strong evidence for anisotropy with primordial anisotropy model

  - varies between detectors, ecliptic alignment
  - may be partly due to beam asymmetries (right order of magnitude)
  - not mostly primordial

- Can improve with Planck, polarization, ISW modelling

Calculate likelihood:

$$-2\log P(\mathbf{\Theta}^G) \sim \mathbf{\Theta}^{G\dagger} C^{-1} \mathbf{\Theta}^G + \text{const}$$

So $$P(\mathbf{\Theta}) = P(\mathbf{\Theta}^G) \left| \frac{\partial \mathbf{\Theta}^G}{\partial \mathbf{\Theta}} \right|$$

The maximum likelihood satisfies $\partial_{f_{\rm NL}} \log P(\mathbf{\Theta}) = 0$:

$$[\mathbf{\Theta} - f_{\rm NL}(\mathbf{X} - \langle \mathbf{X} \rangle)]^\dagger C^{-1} (\mathbf{X} - \langle \mathbf{X} \rangle) = \text{Tr}\left[(\boldsymbol{I} - f_{\rm NL} \mathrm{d}\mathbf{X}/\mathrm{d}\mathbf{\Theta})^{-1} \partial \mathbf{X}/\partial \mathbf{\Theta} \right]$$

The leading Newton-Raphson solution is then

$$\begin{aligned} \mathcal{E} &= \frac{1}{F_{\mathcal{E}}} \left\{ \bar{\mathbf{\Theta}}^\dagger (\mathbf{X} - \langle \mathbf{X} \rangle) - \text{Tr}\left[\partial \mathbf{X}/\partial \mathbf{\Theta} \right] \right\} \\ &= \frac{1}{F_{\mathcal{E}}} \bar{\mathbf{\Theta}}^\dagger (\mathbf{X} - 3\langle \mathbf{X} \rangle), \end{aligned}$$

$$F_{\mathcal{E}} \sim \langle F_{\mathcal{E}} \rangle = 3\text{Tr}\left[C^{-1} \text{cov}(\mathbf{X}) \right]$$

- the optimal estimator for weakly non-Gaussian fields

Take QML estimator for spatial modulation field at r

$$\tilde{h}^{\phi}_{lm}(r) = \int d\Omega Y^*_{lm} \left[\sum_{l_1 m_1} \alpha_{l_1}(r)\bar{\Theta}_{l_1 m_1} Y_{l_1 m_1}\right] \times \left[\sum_{l_2 m_2} \beta_{l_2}(r)\bar{\Theta}_{l_2 m_2} Y_{l_2 m_2}\right]$$

Local bispectrum: modulating field is the primordial anisotropy itself

Minimum-variance estimator for chi(r): $\beta(r)\bar{\Theta}_{lm}$

Integrate QML estimator weighted by r-dependence of expected signal:

$$\begin{aligned}\bar{h}_{lm} &= \int \mathrm{d}r r^2 \tilde{h}^{\phi}_{lm}(r)\beta_l(r) \\ &= \int \mathrm{d}\Omega Y^*_{lm} \sum_{l_1 l_2} b_{l l_1 l_2} \left[\sum_{m_1} \bar{\Theta}_{l_1 m_1} Y_{l_1 m_1}\right]\left[\sum_{m_2} \bar{\Theta}_{l_2 m_2} Y_{l_2 m_2}\right]\end{aligned}$$

$$b_{l_1 l_2 l_3} = \pm\frac{3}{5} f_{NL} \int r^2 \mathrm{d}r \beta_{l_1}(r)\beta_{l_2}(r)\alpha_{l_3}(r) + 5 \text{ perms.}$$

Correlating with $\bar{\Theta}_{lm}$ this is just the usual f_{NL} estimator